PROBLEM PAGE

It seems much too soon to give the solutions to my last pair of problems so, for now, here are two more. The first was suggested by Finbarr Holland who says that it arises naturally in the theory of edge functions.

1. Consider the 12x12 complex determinant

$$\begin{vmatrix}
a_1 & a_2 & a_3 & a_4 & a_5 & a_6 & a_7 & a_8 & a_9 & a_{10} & a_{11} & a_{12} \\
a_4 & a_5 & a_6 & a_7 & a_8 & a_9 & a_{10} & a_{11} & a_{12} & a_1 & a_2 & a_3 \\
a_7 & a_8 & a_9 & a_{10} & a_{11} & a_{12} & a_1 & a_2 & a_3 & a_4 & a_5 & a_6 \\
a_{10} & a_{11} & a_{12} & a_1 & a_2 & a_3 & a_4 & a_5 & a_6 & a_7 & a_8 & a_9 \\
b_1 & b_2 & b_3 & b_4 & b_5 & b_6 & b_7 & b_8 & b_9 & b_{10} & b_{11} & b_{12} \\
b_4 & b_5 & b_6 & b_7 & b_8 & b_9 & b_{10} & b_{11} & b_{12} & b_1 & b_2 & b_3 \\
b_7 & b_8 & b_9 & b_{10} & b_{11} & b_{12} & b_1 & b_2 & b_3 & b_4 & b_5 & b_6 \\
b_{10} & b_{11} & b_{12} & b_1 & b_2 & b_3 & b_4 & b_5 & b_6 & b_7 & b_8 & b_9 \\
c_1 & c_2 & c_3 & c_4 & c_5 & c_6 & c_7 & c_8 & c_9 & c_{10} & c_{11} & c_{12} \\
c_4 & c_5 & c_6 & c_7 & c_8 & c_9 & c_{10} & c_{11} & c_{12} & c_1 & c_2 & c_3 \\
c_7 & c_8 & c_9 & c_{10} & c_{11} & c_{12} & c_1 & c_2 & c_3 & c_4 & c_5 & c_6 \\
c_{10} & c_{11} & c_{12} & c_1 & c_2 & c_3 & c_4 & c_5 & c_6 & c_7 & c_8 & c_9
\end{vmatrix} .$$

Express this determinant as the product of four 3x3 determinants.

The next problem I heard of from Milne Anderson (University College, London) some years ago.

2. Prove or give a counterexample to the following statement.

If $a_n \geq 0$, for $n = 1,2, \ldots,$ and $\sum_{n=1}^{\infty} a_n < \infty$ then

$$\sum_{n=3}^{\infty} a_n^{\left(1 - \frac{1}{\log n}\right)} < \infty .$$

Phil Rippon,
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