SUPERBRAIN 1985

The Superbrain Competition is an annual mathematical examination open to all full-time students of University College, Cork, regardless of whether they are students of mathematics or not. The competition arose in 1984 as a result of a challenge from students of Electrical Engineering who claimed that because of their high points entry requirements, they were the best mathematicians in College. The topics of the examination are roughly those of the Honours Leaving Certificate in Mathematics so as not to give an advantage to students taking more advanced courses. However, the level of difficulty is sometimes considerably greater. The examination was set and corrected by Dr D. MacHale of U.C.C.

In its first year, the competition was won by an exceptional fourth year Science student, Stephen Buckley, recent winner of the Travelling Studentship in Mathematical Science, who is now pursuing a Ph.D. degree in mathematics at the University of Chicago. However, the next eight places in the order of merit were filled by students of Electrical Engineering, so they claimed a moral victory. This year the number of entrants was down from 44 to 30 and with the paper probably just a little less difficult than last year, the standard was a good deal higher. Interestingly, the winning mark was almost exactly the same as last year.

The competition was again a triumph for the Electrical Engineers who filled nine of the top ten places. The 1985 Superbrain is James Cunnane (EE4) (second last year) with a score of 71. In second place was Richard Kavanagh (EE,PG) with a score of 63. Third was Patrick Gaffney (EE2) with 61, while John O'Connell (Sc 2) with 57 took fourth place.

This year's contest was a straight fight between students of Electrical Engineering and Science, because students of Civil Engineering, Arts, Commerce and Medicine did not take part. Also, only two girls risked their mathematical reput-

ations. Are these facts signs of the times?

Fittingly, prizes for the top three were donated by Arthur Guinness and Co., in whose products many of the participants have a keen interest.

D. MacHale

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Answer all ten questions

1. In a darts competition, each dart scores 40, 39, 24, 23, 17 or 16 points. How many darts must be thrown to get exactly 100 points?

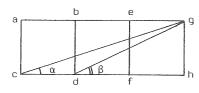
2. Five points lie inside an equilateral triangle of side 2 units. Prove that at least two of the points are no more than a unit distance apart.

3. Find all prime numbers that can be written in the form a^4 + $4b^4$, where a and b are positive integers.

4. If A, B and C are angles with $\sin A + \sin B + \sin C = 0 = \cos A + \cos B + \cos C$, prove that $\cos 3A + \cos 3B + \cos 3C = 3\cos(A+B+C)$.

5. In how many different ways is it possible to pay £100 using 50p, 10p and 5p pieces only?

6.



If abcd, befd and eghf are squares, find, with proof, the size of the angle α + β .

7. Given a triangle abc, show how to find a point p such that $|p,a|^2 + |p,b|^2 + |p,c|^2$ is as small as possible.

8. What is the maximum and the minimum number of "Friday the thirteenths" that can occur in any calendar year?

9. By considering

$$\int_0^1 \frac{x^4 (1-x)^4}{1+x^2} \, dx,$$

show that $\pi < \frac{22}{7}$.

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10.

In this long division question each asterisk stands for a whole number. Reconstruct all the calculations, given that there is no remainder.