BOOK REVIEWS

"MODERN DIMENSION THEORY" (Revised and Extended Edition)

By *J. Nagata*

Published by *Heldermann Verlag*, Berlin, 1983, DM68, pp. 284.
ISBN 3-88538-002-1

From the publisher's description:

"This book is a completely revised and extended edition of "Modern Dimension Theory" published in 1964. It succeeds the old edition in spirit and objective and gives a brief account of modern dimension theory in its present state. Although the book begins with elementary concepts not requiring any knowledge beyond elementary general topology it is not only of interest for the beginner but also for the working mathematician in particular because of its survey character and its rather complete bibliography of approx. 500 titles.

The developments during the last twenty years have been so remarkable and extensive that a thorough revision and a large number of additions had to be made. Especially the chapters on the dimension of non-metrizable spaces and of infinite-dimensional spaces had to be wholly rewritten. New sections on the Pontryagin-Schnirelman Theorem, on "Dimension and Ring", and on "Dimension and Metric Function" were included, to mention just a few. Furthermore, new characterization theorems of dimension are presented.

The reader will easily get a good bird's-eye view of the classical dimension theory, more recent approaches and latest developments, and is thus provided with a starting point for his own research. The working mathematician will appreciate the comprehensive treatment of the subject and the bibliography which together make the book an indispensable source."

"WILLIAM ROWAN HAMILTON. PORTRAIT OF A PRODIGY"

By *Sean O'Donnell*

Published by *Boole Press*, Dublin, 1983, $24.95, pp. xvi + 224,
ISBN 0-906783-06-2

This latest biography of William Rowan Hamilton deals much more with the man than with his work, but claims that in the end 'even the most recondite achievements' will seem less mysterious in the light of the study of Hamilton's personal development. Now the portrait of an artist as a man may sometimes throw some light on the way in which he came to exercise his imagination; but in the case of scientists, and of mathematicians especially, the roots of personality and invention are far too mysteriously entwined for us to disentangle them with the imperfect insights presently at our command. Whenever the evidence is available, it is good to have biographies of important mathematicians simply because we want to know what kinds of people our heroes were and what kinds of lives they led. Such a biography need not dwell in detail on the subject's achievements, so long as the reader learns enough of these to obtain a true measure of the intellectual stature of the subject, and so long as they are portrayed as vital components of his life (e.g. Mr Reid's biography of Hilbert). As T.L. Hankins writes in what Mr O'Donnell acknowledges as the 'definitive life' of Hamilton (published 1980) "it will not do to write the life of a mathematician excluding his mathematics". Unfortunately, that is what Mr O'Donnell has done - not intentionally, for he attempts to summarize Hamilton's scientific work; but inevitably, for he does not understand any of the mathematics well enough to convey its importance in any way other than by quoting from time to time what others have said of it. Probably Mr O'Donnell wanted this account of Ireland's greatest scientist to reach a much wider audience than did the monumental 'Life' of Graves or the scholarly book of Hankins, and of course he may succeed, for he has a lively style of writing; but if the object was to make more people aware of Hamilton's greatness, as all readers of this journal would wish, then he has not succeeded. Indeed, there is a sense in which he himself progressively seems to lose enthusiasm for the task he set himself. As the story unfolds, Mr O'Donnell becomes increasingly preoccupied with the defects of Hamilton's personality and chides him regularly, like a Victorian moralist, for having accomplished less than he might have done had he been more practical, punctual, abstemious, modest, sparing in language and decisive in action (also, Mr O'Donnell would have had Hamilton be a better Irishman!). Finally, if Hamilton's mathematics receives short shrift, Hamilton's personality in the end really does little better. A few psychological cliches applied to this or that episode that stand for 'modern' insights do not amount to a serious in-depth study of one of the greatest scientists of the nineteenth century. What survives the moralistic tirades is only a limp cardboard figure.

Mr O'Donnell is best on Hamilton's origins and family background - he makes a good case for the conjecture that Archibald Hamilton Rowan, the wealthy political rebel and Hamilton's godfather, was actually Hamilton's father; and he is good on Hamilton's up-bringing by his devoted Uncle James, the curate of Trim. There is also much of interest in the early chapters on life and education in the Ireland of Hamilton's youth. On the other hand, Mr O'Donnell's idiosyncratic stance manifests itself already at the very beginning: he is excited to dispose of the 'myth' of Hamilton's linguistic powers (as if anyone had ever taken this at its face value!) and refers to this discovery several times. From now on we shall have to be content with the knowledge that, aged thirteen, Hamilton could read easily 'only' in Latin, Greek and Hebrew. Also, we were invited to regret that Uncle James did not think to add Gaelic or Irish to young William's repertoire of languages! We learn of other ways in which Uncle James might have strayed with advantage from the classics-based education he imparted with such ardour, but we learn nothing new about

what must have been happening - the emergence of mathematical genius. And perhaps that's how it will always be regardless of who tries; we are left to wonder, without hope of ever being enlightened, what confluence of ideas and experiences of the young Hamilton could have brought him to the point where, as Hankins reports, on one page of manuscript preserved from student days, Hamilton writes down Newton's laws of motion in an obvious effort to memorize them for an exam, and on the same page he sketches his path-breaking ideas on geometric optics!

And from Hamilton's student days onwards, I find Hankins' description of his character and activities always more mature, serious and balanced. In the matter of Hamilton's drinking habits, Hankins is at once more explicit than Mr O'Donnell and more accurate in judging their impact; Hamilton's phenomenal computational ability never deserted him. When Father Richard Ingram checked the manuscript on Icosian cycles (paper LVIII on p. 623 of our edition of Vol. III of Hamilton's works), dated 1863, he found not a single error! In the case of a mathematician, that's what being able to hold one's drink means! Or take Hamilton's attitude to the Great Famine of '45-'48: Hankins lets Hamilton speak for himself and is altogether more informative on Hamilton in relation to the poverty then prevalent in Ireland; whereas Mr O'Donnell attacks Hamilton in strident terms that would be appropriate only at election hustings.

I could go on, and I could go on much longer in the context of Mr O'Donnell's descriptions of Hamilton's mathematics (but do look at the extraordinary disquisition on p. 146 on the significance of quaternions, ending with the moralist's injunction that 'to be first does not imply being best ...'!). However, perhaps enough has been said. Perhaps the greatest service I can render Mr O'Donnell is to urge readers to buy the book and themselves enjoy finding their own disagreements!

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"COMPLEMENTARY PIVOTING ON A PSEUDOMANIFOLD STRUCTURE WITH APPLICATIONS TO THE DECISION SCIENCES"

By *F.J. Gould*, Graduate School of Business, University of Chicago, and *J.W. Tolle*, Dept of Mathematics, University of North Carolina at Chapel Hill.

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ISBN 3-88538-402-7.

In these heady days when the influence of computing on mathematics waxes ever more strongly, a natural development is the enhanced status of constructive existence proofs. For example, the standard inf/sup argument used to prove the intermediate value theorem could well be supplanted by the slightly longer (but constructive!) proof which repeatedly bisects the interval under consideration. However, it is often difficult to obtain a constructive proof of a known result. Brouwer [2] published the proof of his famous fixed point theorem in 1912, and several alternative proofs appeared in later years, but not until 1967 did Scarf [5] give an explicitly constructive proof (which enabled one to find "approximate fixed points" z, in the sense that $||f(z)-z||_\infty$ is small). Incidentally, Hirsch [3] in 1963 published a proof of (a result equivalent to) the Brouwer theorem, the constructive nature of which was only noticed in 1976 by Kellogg, Li and Yorke [4].

Scarf's paper marks a watershed in the numerical solution of nondifferentiable nonlinear systems of algebraic equations. Its basic algorithmic procedure, that of *complementary pivoting*, was the inspiration for an explosion of research activity in

the 1970s. A description of this procedure follows.

First, an analogy. Consider a house having one entrance. The house consists of a finite number of rooms. All rooms have 0, 1 or 2 doors. All doors link exactly two rooms (the exterior of the house is regarded as a room). Then if one enters the house and obeys the rule that one cannot enter and leave a room through the same door, it is not difficult to see that one's path must terminate in a room inside the house having one door.

Let's translate the analogy into mathematics (I shall cut some technical corners here). Our house becomes a closed, bounded, simply connected subset D of R^n. Each room corresponds to an *n-simplex* (a 0-simplex is a point, a 1-simplex is a line segment, a 2-simplex is a triangle, a 3-simplex is a tetrahedron, etc.: an n-simplex is the closed convex hull of its n+1 extreme points or *vertices*). The set D is the union of these n-simplexes, and moreover the n-simplexes are required to fit together in a geometrically pleasing way: the intersection of each pair S_1, S_2 is either the empty set or an m-simplex, $0 \leq m \leq n$, whose vertices are vertices both of S_1 and of S_2. Every n-simplex has n+1 (n-1)-dimensional *faces* which are themselves (n-1)-simplexes. Certain of these faces will be designated as doors, in a way that corresponds to the analogy above.

To each vertex of each n-simplex we assign a *label* chosen from the set $T_n = \{0,1,\ldots,n\}$. (The way in which this assignment is carried out depends on the nature of the problem being solved.) Thus each n-simplex S has a set of n+1 labels associated with it, which may include some repetitions. If this set equals T_n, we say that S is *completely labelled* (cl). Similarly associate a set of n labels with each (n-1)-simplex S' which is a face of some n-simplex. We say that S' is *almost completely labelled* (acl) iff this set equals T_{n-1}.