individual differences which are related to mathematics performance, such as spatial and logical reasoning [8], cognitive style [5], and internal structure of mathematical knowledge [2] are less easily accommodated but may, with further investigation, become more accessible variables.

In conclusion, I would suggest that although the provision of mathematics education is a complex process, there is scope for substantial improvement through changes in instructional variables such as those described above, a willingness to try new approaches, and a cooperative spirit among mathematics teachers at primary, secondary and third levels of education.

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COMPUTERS IN THE TEACHING OF MATHEMATICS AT UCD

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In common with many other university mathematics departments, a considerable portion of our effort is involved with service teaching of first year courses. Much of this consists of rote techniques such as solving linear equations, inverting matrices, differentiating polynomial functions, and curve sketching of rational functions. These are all, with more or less ease, amenable to programming as tutorial sessions on a computer. The advantages are clear: a uniform procedure is taught to everyone, and each student can practice as much as necessary whenever he wishes. In this way the tutorial system can be extended, less dependence need be placed on tutors, and some formal tutorials can be replaced by sessions where

groups of students use the computer while the tutor gives individual help. This is all at essentially no extra cost to the College.

Whereas teaching software does exist at school level, it is extremely difficult to find good university level programs. The interactive nature of such programs is often limited to a choice of several options and, after a selection is made, the problem is then completely solved without further assistance from the student. One would prefer at every possible opportunity to ask the user to think about what step should be done next and let the machine perform the requested task, adding comments on the choice.

There is the apparently opposing desire to avoid any requirement for introducing special notation, teaching programming, or knowing the operating system. In practice, this means a student can only be asked either to choose from a set of options or type in a mathematical expression in its usual form. It is catering for this latter possibility that makes good programs exceedingly large: too long for most micros and very time-consuming to write and to make work correctly.

At the present time we have two such programs*: one for solving linear equations and the other for calculating determinants. In both cases the user may enter his own problem or have the machine generate one for him. He then types in elementary row (or column) operations until the conclusion of the problem, with a remark about his progress being made at most steps. Flow charts outlining each program are given in Figs 1 and 2. As exemplified by these diagrams, all algorithms can be described using a directed graph, each node or vertex corresponding to a step, which, in a program, becomes a block of statements. The most natural size of step for these programs is to take a step as consisting of one request for input from the student together with the subsequent calculations up to the next request.

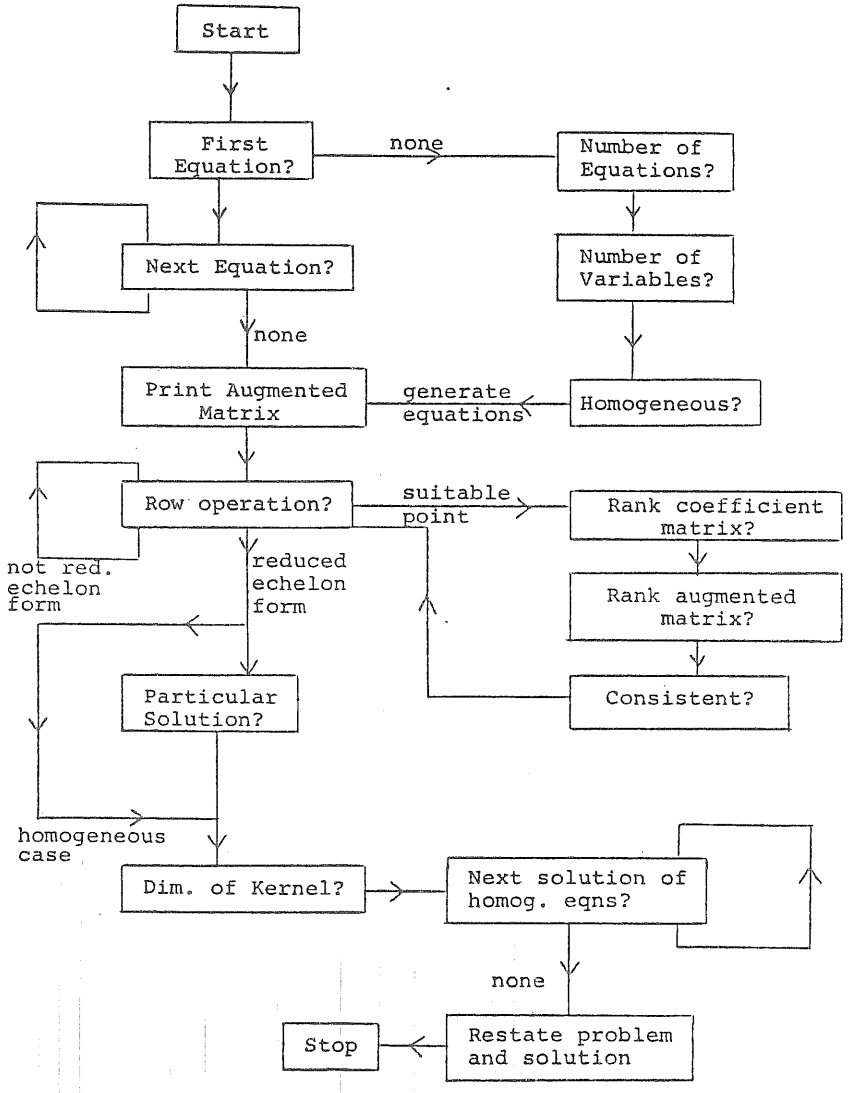

FIGURE 1: Sketch Flow Chart for "Solving Linear Equations" Program

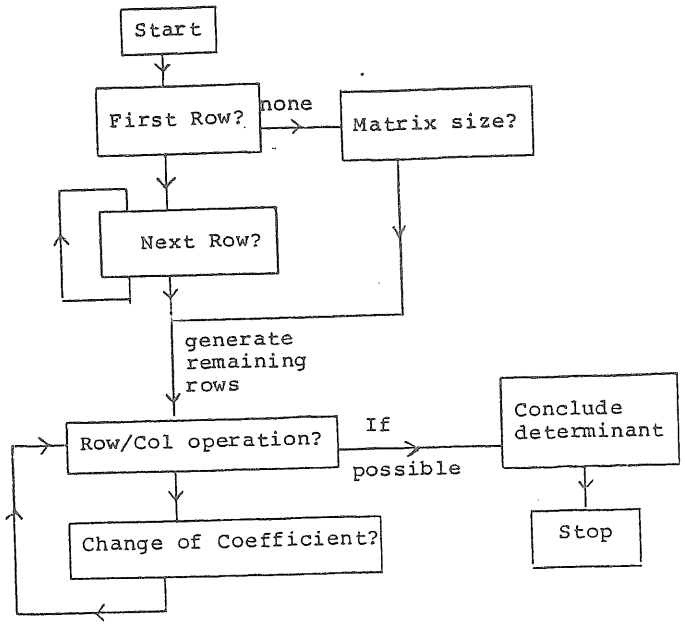

FIGURE 2: Sketch Flow Chart for "Calculating Determinants" Program

Each request from the computer can be answered either by an appropriate reply or one of a small set of letters which allow the user to move backwards or forwards through the exercise at will, ask for hints, or recap on the current state of the problem. These extra choices are as follows:

H (= Help) produces this list of letters,

F (= Forward) gives the answer to the current question and asks the next,

R (= Reverse) moves back to the previous question,

C (= Current) reproduces the present state of the problem with the current question,

I (= Information) gives a hint to answer the present question,

Q (= Quit) stops the session.

In particular, by repeatedly pressing F the student is able to obtain a complete worked example. This means that the program could be used before the material is covered in lectures. By pressing R, he can return to the previous step and make a different choice instead. The letter C is to ensure that the student is able to have sufficient information on the screen to answer a question; intervening hints or comments may have scrolled the material out of sight.

Expansion of such a service would obviously benefit students. The problems seem to be twofold. Firstly, users need a directory on a computer which allows interactive work. Restrictions on the number of terminals and on CPU time mean that careful scheduling of tutorial sessions is required. One solution is to have a login command file which logs the user off immediately if it is the wrong hour of the week and otherwise runs the program, with an automatic log-off when the program stops. This avoids any requirement to know about the system. However, examination time revision could certainly result in greater use of the programs to the detriment of the machine's performance: this is a problem of which we have no experience as yet.

The other main problem is with extending the software. The language used in the present programs is Basic, chosen because of the text editing facilities and the hope of using them on a microcomputer. A structured language such as Pascal would have been more appropriate because changes would be easier to make, existing subroutines could be used in new programs, and other authors might have a hope of adding to the existing programs.

Overall, many man-years of work are required to extend this to cover most first year material, probably several mathematicians directing some competent programmers.

It seems clear that funding for such a project would reap as benefit an improvement in our teaching so that the country would produce, for example, better engineering students and more capable scientists. At any rate the increasing abundance of available computer time is still something that we are only just beginning to appreciate and there is now a need for programmers as "laboratory technicians", not just in computer science departments but also in mathematics departments.

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* While the author is on leave of absence at the University of East Anglia, Norwich, copies of the programs are available from Dr J.B. Quigley, Mathematics Department, U.C.D., at a cost of £30 to cover the magnetic tape, packing and postage.

Minor modifications will be necessary if DEC Basic is not available.

BOOK REVIEWS

"SINGLE-VARIABLE CALCULUS"

By *Robert A. Adams*

Published by *Addison-Wesley*, 1983, £19.95 (sterling), 590 pp.

It must be a daunting task to set about writing a 600-page text book. It could be that the author wishes to introduce to a wider audience material not available in book form: however, in the case of a calculus text, a new author must search for new approaches, which will bring a greater unity or clarity to the subject matter. In the past twenty years, calculus texts have grown considerably in size (satisfying logistic growth rather than exponential growth, I hope), the increase in size being partly due to increase in page size for clarity in reading, but also partly due to additions of appendices to make each new edition more comprehensive. In the preface of Single-Variable Calculus, Professor Adams claims to have produced a book which "is not as massive or bulky in appearance as many other books available in recent years." Although this is certainly true, measuring $7\frac{1}{2}$"x $9\frac{1}{2}$"x $1\frac{1}{2}$" and weighing $2\frac{1}{2}$ lbs it is in no way a pocket calculus. The author is able to keep the number of pages below 600 because he regards calculus of several variables as suitable for a separate text (perhaps, he is writing a sequel himself).

Single-Variable Calculus is primarily designed for a two-semester course for science and engineering students. In Chapter 1, Functions, Limits, Continuity, the author introduces the ε-δ definition of a limit but relegates the proofs of the results about limits and continuous functions to an appendix. The brief treatment of inequalities I found unsatisfactory but the inverse of a 1-1 function is introduced without the confusing terminology of injections and surjections.