6. K.T. Smith, Smith's Primer of Modern Analysis, Bogden and Quigley, Tarrytown-on-Hudson, NY, 1971.

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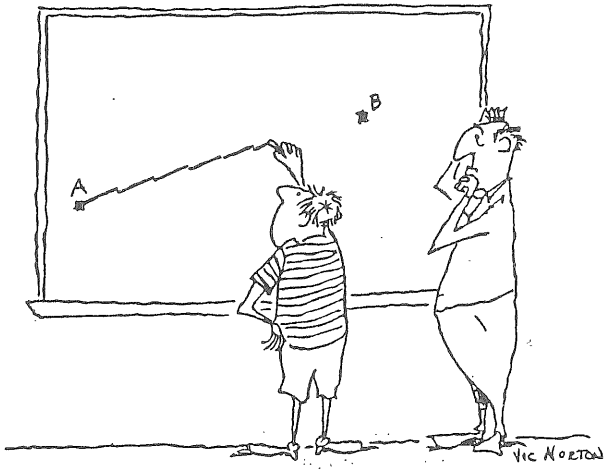
Cartoon without caption:
The computer-age generation



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THE ANALYTICAL REFORM OF IRISH MATHEMATICS

1800 - 1831

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The Origin of the Dublin Mathematical School

The mathematical tradition established by the Dublin Philosophical Society of William Molyneux (Fig. 1) had a major influence on the character of mathematics in Ireland [1]. The convergence of interests at the University of Dublin on specific aspects of mathematics, e.g. the theory of equations, optics, potential theory and variational principles [2], and the strong Irish tradition in statistics [3] had their origins in the interests and contributions of the members of the society.

STATISTICS	W. Petty, *Political Arithmetick* (London, 1690). F. Robartes, *An Arithmetical Paradox Concerning the Chances of Lotteries*, Phil. Mag. XVII (1693) pp.677-84.
GEOMETRY	St. George Ashe, *A New and Easy Way of Demonstrating Some Propositions in Euclid*, Phil. Mag. XIV (1684), pp. 672-6.
OPTICS	W. Molyneux, *Solution of a Dioptric Problem*, Bibliotheque Universelle et Historique, III (1686).
ENGINEERING MATHEMATICS	W. Molyneux, *A Demonstration of an Error Committed by Common Surveyors .....*, Phil. Mag., XIX (1677) pp. 625-31.
ASTRONOMY	W. Molyneux, *Concerning the Parallax of Fixed Stars*, Phil. Mag. VXII (1693) pp. 844-9. J. Walley, *Ptolemy's Quadripartite*, (Dublin, 1701).
ACOUSTICS	N. Marsh, *An Introductory Essay to the Doctrine of Sounds*, Phil. Mag. VIX (1684) pp. 472-88.

FIGURE 1: Mathematical Interest of the Dublin Philosophical Society Illustrated by Selection of Works.

The 18th century began on a high mathematical note at the University of Dublin with the first fundamental Critique of Newton by George Berkeley. He began tentatively his criticism of Newton's cosmology with his *Essays Towards a New Theory of Vision* (1709) and ended forcefully in his apology for theology, *The Analyst* (1734) which pointed out the unsatisfactory state of the underlying logical base of the Newtonian calculus [4]. Newton had disguised the inadequate logical base of his calculus by juggling away higher order terms which experiment had shown to be extraneous. "The good Bishop" took issue with Newton (and Leibniz) demanding proof of the truth of the calculus as a representative of material motion, rather than a utilitarian defence of the methodology whose inadequacies the founders of the calculus camouflaged by a good deal of mysticism. Berkeley was not the only man in Britain to criticize Newton [5] and indeed in Dublin there were anti-Newtonians during this period [6].

In the 18th century there was apparently little original work in mathematics at the University of Dublin except for the work of Hugh Hamilton (1729-1783) and Richard Murray (? - 1799) which was to lay the foundation of the Dublin Mathematical School. Hamilton wrote the elegant work *De sectionibus, conicis, tractatus geometricus* (1758) which earned the following accolade from Leonhard Euler:

> "There are three perfect mathematical works: these are by Archimedes, Newton and Hamilton."

In addition he wrote *The Analysis of the Infinities* (date uncertain) and *Lectures on Natural Philosophy* (1766). Murray was professor of mathematics from 1764 (two years after the chair was established) to 1795 and devoted his energies to the improvement of mathematics at the University of Dublin. From his patient teaching arose a broader mathematical base in the University and, in particular, during this period mathematics became the most important single subject in the Fellowship examinations which were then the only method of entry into this academic world [7].

Someone who was strongly influenced by Murray was Matthew Young (1750-1800) whose books such as *An Enquiry into the Principal Phenomena Of Sounds and Musical Strings* (1784) [8] contained some original researches. However, Young's most significant contribution to Dublin mathematical research was his role in the establishment of the Royal Irish Academy [9]. The Academy was dedicated to the development of new knowledge in contrast to the Dublin Philosophical Society's commitment to dissemination and diffusion of technical knowledge. William Hales also wrote several books on mathematical subjects [10] and his *Analysis aequationium* (1786) was complimented in a letter from Lagrange. Hales provided a thoroughgoing attempt to defend Newton's fluxian notion in his *Analysis Fluxionum* which he wrote after he had retired.

Young died leaving unfinished a work of great scholarship on Newton's *Method of Prime and Ultimate Ratios, Illustrated by a Comment of the "Principia"* [11]. The suppression of this work was unfortunate for mathematics in Dublin. Apparently it resulted from some feared heterodox doctrinal deviation by the pious polymath Young [12]. As for mathematical deviation, Rev. William Jones, the author of *Essays on the First Principles of Natural Philosophy*, claimed that in Dublin there were mathematicians who kept guard for the system of attractions "more severely that Newton himself did and would not suffer a heretic to land on their wast" [13].

The days of orthodox Newtonianism had, by the time of Jones' comment, already passed in Dublin and the old University mathematical establishment was to be superseded by a new generation of analytical reformers in the next period. The arrival of John Brinkley from Cambridge as the new Andrew's Professor of Astronomy in 1790 was the turning point. The growth of his influence in Dublin progressively subverted the old Newtonian tradition.

The Gentlemen of Science, Rev. John Brinkley and Rev. Bartholomew Lloyd and the Reform Movement

The Dublin School of Mathematics [14] was the creation of two men, Bartholomew Lloyd and John Brinkley. In 1792 Brinkley became the first Astronomer Royal of Ireland [15] and, while he waited patiently for more than a decade for the arrival of the great Ramsden Circle for Dunsink, he prepared himself with magnificent thoroughness for his later parallax observations with the Circle [16]. In 1800 he was already demonstrating a great awareness of the work of Lagrange and Laplace. It is evident that at the University of Dublin, Brinkley was the catalyst for the reform of mathematics there since he was the most senior University reformer in the period of great political reaction following the Rebellion [17]. He was isolated from University life because of his position at Dunsink and it was perhaps because of this that he was able to take the lead in introducing a knowledge of continental mathematics and physics into Dublin without raising a hornets' nest of opposition in the University. He was the first Dublin professor to use the analytic notation [18] early in the new century and this is of great significance in understanding the roots of the advanced analysis in Dublin.

The period 1798-1830 was an age of Tory hegemony at the University of Dublin, but one in which there must have been a working relationship existing between, on the one hand, the Tory Provosts and Tory majority on the Board and, on the other hand, the reformers. The reformers' roots were the old Whig establishment of the University. Lloyd, who was a member of a third generation of mathematicians at the University from this reforming tradition (Fig. 2), was evidently a trusted radical in that in 1813, when still a Junior Fellow and comparatively a young man, he was appointed to the Chair of Mathematics in an unprecedented promotion. As a reformer, Lloyd was in tune with the needs of his age and he has been credited with the single-handed reform of the University's mathematical curriculum [19], but this is far from the whole truth. The working

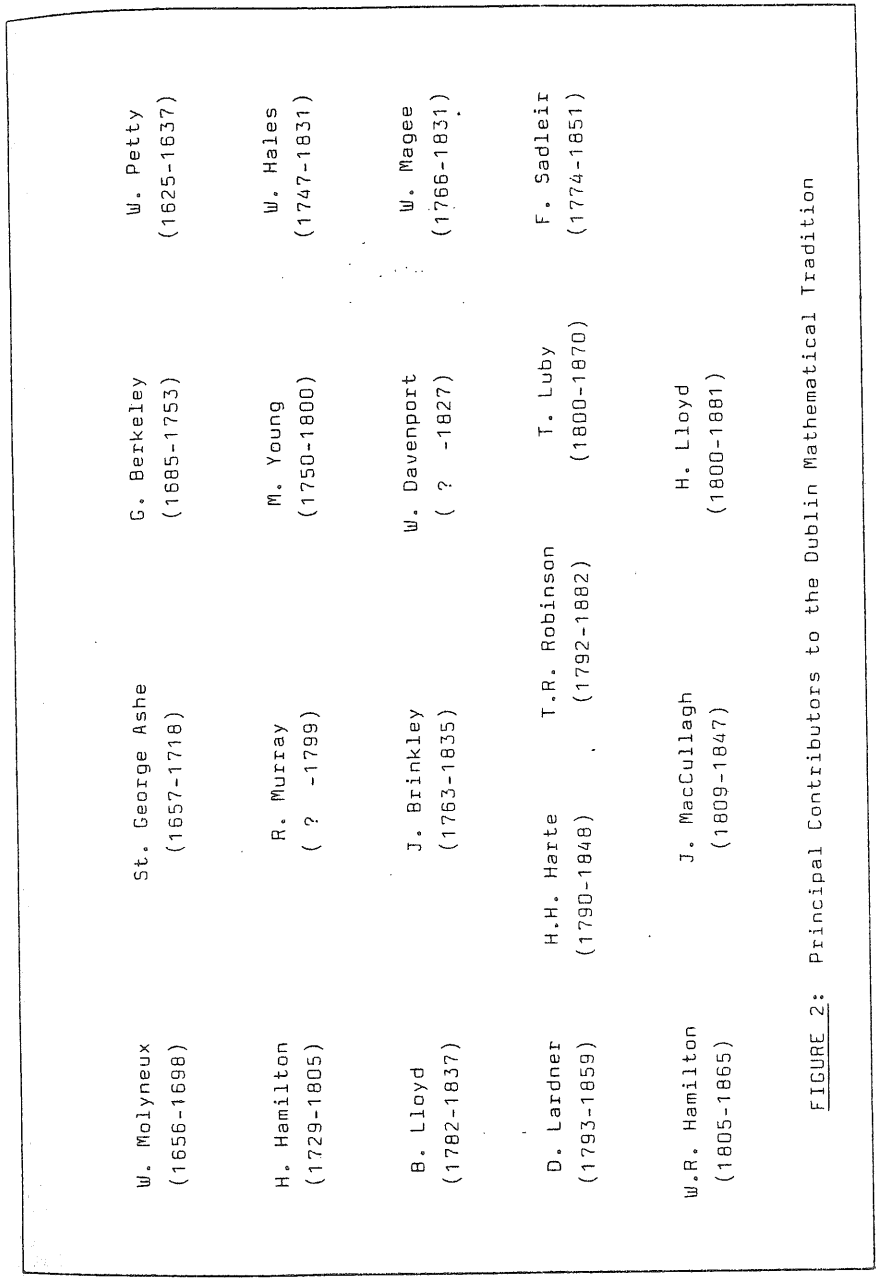

FIGURE 2: Principal Contributors to the Dublin Mathematical Tradition

out of the events leading to this important reform are far more complex than this "lone crusade theory" suggests, and it will require much patient scholarship to unravel it in any satisfactory manner. On his promotion, Lloyd immediately introduced the works of Lacroix, Poisson and Laplace into the medal examination [20], and more significantly perhaps also into the Fellowship examinations. He was in fact continuing the reform of the mathematical curriculum for undergraduates which had already been initiated with the earlier introduction of mathematics and algebra [21]. To effect his reform, Lloyd chose the method of writing a text-book. His *Analytical Geometry, a Treatise on the Application of Algebra and Geometry for Use of Undergraduates at T.C.D.* was published in 1815 seven years after the appearance of another "reformed" classic, Brinkley's *Elements of Plane Astronomy*.

The reform movement in Cambridge had begun with the work of Robert Woodhouse, Senior Wrangler in 1795, who has been described as the apostle of the analytical movement, since his *Principles of Analytical Calculation* (Cambridge, 1803) had apparently little contemporary influence in Cambridge. The official history of Cambridge Mathematics [22] dates the beginning of the reform as 1812, with the formation of the undergraduate "Analytical Society" of Babbage, Peacock and Herschel. These Cambridge reformers produced three publications in the period 1813-1820 [23]. However, William Whewell's *Elementary Treatise in Mechanics* (1819) was, perhaps, more influential in effecting this reform as he was appointed moderator of the university mathematics examinations in 1820 [24]. In 1820 Woodhouse was appointed to the Lucasian Chair in the university, copper-fastening the reform in Newton's university.

The real distinction between Dublin and Cambridge was due to the positions of Lloyd and Brinkley as professors, while the reformers at Cambridge were outside the establishment before 1820. The Dublin reformers had in addition published reformed text-books which, significantly, were not an uncritical

acceptance of French work, but rather were works of scholarship based to a recognizable extent on a University of Dublin tradition of mathematical text-books; thus Lloyd's was a replacement in part of Hamilton's book and Brinkley's a replacement of the book of Hales and Stack. Brinkley had, in fact, obtained the Andrew's Chair in competition with Hales, although Hales had strong support in his application from Fellows of the University.

The analytical notation was introduced into the curriculum at the University of Dublin in 1815, which was five years before Cambridge. During this period Lloyd moulded the minds of the leading undergraduates to produce an analytical school in Dublin. This school arose because of Lloyd's excellent teaching and boasted among its members some outstanding young lions. Lloyd's first generation disciples included Thomas Romney Robinson (1792-1852), Denis (Dionysius) Lardner (1793-1856), H.H. Harte (1790-1848), Thomas Luby (1800-1870) and perhaps Franc Sadlier (1774-1851) who later succeeded Lloyd as Provost. Those of this group were characterized by their commitment primarily to educational works. The Dublin reformers seemed to be classic examples of consolidations in a Kuhnian paradigm [25] and they boasted with their leaders an impressive list of educational publications in the period up to 1831 (Fig. 3). Their work was rewarded by the promotion of their mentor Bartholomew Lloyd to the Provostship in 1831 and the succession of Sadlier.

The reasons for the advanced position of Dublin with respect to Cambridge are simple to guess at, but almost impossible to substantiate. Cambridge was the centre of the British Newtonian tradition, while Dublin had a long legacy of critical acceptance of this tradition beneath which there lay a Cartesian current [26]. The connections between Ireland and France were very strong in the period up until 1798, and this was seen by those of influence in England as being treasonable. The principal reason for the advanced analytical position of Dublin

1815 : B. Lloyd, *Treatise on Analytical Geometry.*
Medals in Examination awarded on a new basis.

1820 : T.R. Robinson, *System of Mechanics.*
D. Lardner, *Central Forces*

1823 : D. Lardner, *Algebraic Geometry.*

1824 : D. Lardner, *Elementary System of Mechanics.*
D. Lardner, *A Series of Lectures on Locke's Essay.*
Brinkley receives Copley Medal of Royal Scoiety

1825 : D. Lardner, *Treatise on Differential and Integral Calculus.*

1826 : B. Lloyd, *Treatise on Mechanical Philosophy.*
Brinkley promoted to See of Cloyne.

1827 : W.R. Hamilton becomes Astronomer Royal at age of 21.

1828 : T. Luby, *Physical Astronomy.*
D. Lardner, *Discourse on the Advantages of Natural Philosophy* (London).
D. Lardner, *Treatise on Plane and Spherical Trigonometry* (London).
D. Lardner, *First Six Books of Euclid* (London).

1829 : D. Lardner, *Mechanics and Pneumatics and Newton's Optics* (SDUK).

1830 : H.H. Harte, *The System of the World* (2 Vols) (London). Translation of Laplace's *Systeme du Monde.* Later translates Laplace's *Mecanique Celeste* and Poisson's *Mecanique.*
D. Lardner and H. Kater, *Treatise on Mechanics* (London).
D. Lardner, *A Treatise on Hydrostatics and Pneumatics.*

1831 : H. Lloyd, *Treatise on Light and Vision.* (SDUK).
B. Lloyd becomes Provost of T.C.D.
H. Lloyd Professor of Nat. Phil.
Moderatorship Examinations (Honours) introduced.

FIGURE 3: Chronology of Reform Text Books by University of Dublin Authors Before 1831.

was, however, almost certainly due to the profoundly middle-class nature of the University and its educational methods. The educational methods were based on text-books, lectures, grinders to support the college tutors, well defined syllabi, and competitive *viva-voce* examinations with questions requiring extremely precise responses. The examination was the sole arbiter of the degree award. This unique system led the University to be crowded with "back stairs men" at the quarterly examinations, who otherwise did not attend as they studied in their own chosen manner and time outside the University. These much despised students, however, had the supreme virtue of greatly increasing the revenue of the Fellows, whose incomes were dependent on examination fees. The nature of mathematics lent itself very well to this examination system, and no doubt Lloyd and the reformers exploited this advantage using mathematics as a wedge to open the way for their more ambitious plans whose implementation followed Lloyd's election as Provost [27].

Brinkley and Lloyd appeared to be radical reformers in 1815, but these gentlemen were not a revolutionary force. They did aspire, however, to be an intellectual leadership of the nation and were consequently enthusiastic supporters of Coleridge's idea of a "clerisy" [28]. The pattern of French secular professionalization of science was not repeated in Britain because the leaders of the reform movement such as Lloyd and Brinkley were professional only in a vocational sense. The Dublin leaders' expertise in their subject specializations gave then a real advantage in the scientific fraternity of their day which was demanding professional standards. "The gentlemen of science" as Morrell and Thackray [29] named them, were only one competing group for the leadership of the British scientific fraternity. In 1831 when Lloyd was elected Provost, the outcome of the analytical reform was still in the balance with the main battle front centred on the wave corpuscular controversy in optics. The reformers had captured the high ground in Dublin and Cambridge, but the initial battle for the Royal

Society had been lost in 1830. The provincial scientific societies, which had emerged along with the Mechanics' Institutes with the development of the industrial revolution, were largely led by Newtonians such as Edinburgh's Henry Brougham and David Brewster.

The process of professionalization in France [30] had been associated with a political struggle for ascendancy between "the savants" (theorists) and "manants" (experimentalists) and by 1815 "the savants" had effectively established an ascendancy over the "manants". In Britain the mathematicians had set out to repeat this process by rendering the new Baconian sciences mathematical. This threatened seriously to undermine their dominance, as these new sciences appeared to have at that time destroyed the dominance of the Newtonian sciences. This "analytical revolution" as it has been called was therefore a reform out of necessity for the University of Dublin, as it was threatened by the emergence of a burgeoning Baconian institute, the Royal Dublin Society and later the new Dublin Mechanics Institute from 1825. The paradox was that the practical men largely defended the old Newtonian orthodoxy which the theoreticians, who had formerly been stoutly Newtonian, now attacked.

The emergence of a new mathematical physics which developed from this ideological struggle was consequently modelled on the French 'physique'. That development has been recently investigated from an English perspective and the study identified a strong Irish involvement in this process [31]. The first group of Lloyd reformers at the University of Dublin with the two leaders, effected a thorough going change in the old Newtonian tradition of Dublin with their wide-ranging educational works. Perhaps more significantly this prepared the ground for a new second generation of mathematicians, who were differentiated from the first by their commitment to the generation of new knowledge. The change can be summed up by saying that the ideals of the Royal Irish Academy in this period gained

ascendancy over those of the Dublin Society.

Concluding Comments

The analytical reform carried through by Lloyd was a major break with the old tradition at the University of Dublin which, as his disciple Lardner said, allowed "the study of mathematics to leap a chasm of one hundred years" [32]. Lloyd brought into the centre of Irish mathematics not only an awareness of the contemporary French works of Lagrange, Laplace, Poisson, Fourier, Monge, Legendre and Lacroix, but it also enabled the Dublin mathematicians to assimilate the earlier work of the 18th century continental mathematicians such as that of Euler and the Bernoullis.

An objective measure of Lloyd's educational reform was the 1822 undergraduate science Medal Examinations. These were based on Woodhouse's *Trigonometry*, Lardner's *Algebraic Geometry*, Lacroix's *Calcul Differentiel et Integral* and *Theorie des Lignes Courbes*, Lloyd's *Mechanical Philosophy*, Poisson's *Mecanique* and selections from Newton's *Principia* and Laplace's *Mecanique Celeste*. This curriculum was followed by the best of the undergraduates in the subsequent period. Before Lloyd's reform the undergraduates body "were employed fathoming the mysteries of Decimal Fractions" [33]. In 1822 they boasted among their number William Rowan Hamilton who was then pursuing his research into mathematical optics [34]. Lloyd's principal achievement therefore was that his work opened up for the first time in a British University the great range of continental discovery and marked the introduction to these islands of higher analysis. He led his disciples into an alliance with Cambridge based mathematicians, initially on the mutual commitment to what Babbage called the pure principle of "D-ism", that is the Leibniz notation, against that of Newton which Babbage characterized as the "dot-age", but later this alliance had other major and far-reaching implications for British mathematics.

One of Lloyd's devoted admirers, J.H. Singer, delivered the eulogy at his Memorial Service. This eulogy provided a contemporary assessment of the significance of his reforms.

> "Our University bears proof of the skill and prudence with which he could adapt the institutions of venerated antiquity to meet the demands of modern improvement, and the rapid and accelerated advance which our Institution and our country have made in all the various Departments of Science is connected essentially with the name and labours of our gifted and venerated Provost."

The Rev. Bartholomew was undoubtedly the most successful reforming "gentleman of science" of his age and it was through such reforms that Britain was transformed into a great power of the 19th century. Mathematics had as part of this process of modernization to be reformed to meet the new demands of a nation rapidly industrializing and widening continuously its spheres of activity. The only way for Britain to meet the challenge of the French in the early years of the 19th century was by emulation and Lloyd was the man at this time who organised the stealing of the French cloths and thereby prepared the way for the advance of Irish mathematics which began its first real independent flowering in the period of his short Provostship. The Dublin Mathematical School was Lloyd's creation although initially inspired by Brinkley. Brinkley was also the man who provided the model for the work of the second wave of Lloyd's reformers on whom the fame of Irish mathematics still securely rests.

References

1. K.T. Hoppen, *The Common Scientist in the Seventeenth Century, A Study of the Dublin Philosophical Society 1683-1708.* (London, 1970).

2. Personal Communication, Professor D. Spearman, 9-9-82.

3. Personal Communication, M. Hegarty, 13-10-83. Ireland has a strong tradition in the field of statistics of whom perhaps W.S. Gosset is best known for his t-distribution, but in more recent times, R.C. Geary has continued this tradition by his internationally acclaimed researches.

4. C.B. Boyer, *A History of the Calculus and its Conceptual Development*, (Dover, New York, 1959).

5. C.B. Wilde, *Matter and Spirit as Natural Symbols in Eighteenth Century British Natural Philosophy*, British Journal for History of Science, Vol. 15, Part 2, No. 50, July 1982, pp. 99-131.

6. P. Browne, *The Procedure, Extent and Limits of Human Understanding*, (1728).

7. The dominance of mathematics in T.C.D. in the 19th century must be attributed largely to the Fellowship Examinations emphasis on the subject.

8. This work connects with the early study of Marsh in the D.P.S.

9. D.N.B. entry states that his society for the study of Syriac theology and philosophy founded in 1777 became a germ of the R.I.A.

10. Hales' scientific works were on sound (1778), planetary motion (1782), Analysis of chronology (1809-1812). He was Rector of Killeshandra from 1787.

11. In T.C.D. in manuscript form are two books in Latin: *Principorum Newtoni*, apparently prepared for publication. T.C.D. manuscript N.4. 14-17.

12. Young's amended version of the Psalms was also suppressed by Fellows.

13. Kenyon MSS, (Hist. MSS. Comm.), 540.

14. A.J. McConnell, *The Dublin Mathematical School in the First Half of the Nineteenth Century*, Proc. R.I.A., Ser. A, Vol. 50, (Nov. 1944), pp. 75-88.

15. N.D. McMillan, *The Dublin Focus of Astronomy*. The Irish Astronomical Journal, submitted for publication, details at great length the work of the Dunsink Observatory and points to the central role of Brinkley in the analytical reform movement's work in this field.

16. Brinkley's work on parallax was a development of Molyneux's earlier researches.

17. McDowell and D. Webb, *Trinity College in the Age of Revolution and Reform (1794-1831)*, Hermathena, No. LXXII, 1948.

18. *op cit.*, Note 2.

19. D. Lardner, Preface, *Algebraic Geometry* (Dublin 1822). It is that in addition to Brinkley, Lloyd had the active assistance of Davenport, professor of natural philosophy and probably others such as Franc Sadleir.

20. The origin of the honours examination lies in these competitive examinations.

21. Algebra first appeared in the T.C.D. curriculum in 1808 according to R.B. McDowell and D.A. Webb, *Trinity College Dublin 1592-1952; An Academic History*, (Cambridge University Press, 1982), p. 90. Mathematics first appeared in the curriculum in 1793, p. 69, although arithmetic it was assumed was taught in schools.

22. W.W. Rouse Ball, *A History of the Study of Mathematics in Cambridge*, (Cambridge, 1889), pp. 118-137.

23. Cambridge reform works before 1821 were *Memoirs of the Analytical Society* (1813) Babbage and Herschel, *Translation of Lacroix Traite* (1816) Babbage, Herschel and Peacock, *A Collection of Examples of the Application of the Differential and Integral Calculus*, (1820) Peacock, Herschel and Babbage (2 vols).

24. Whewell was moderator in 1820 and was succeeded in 1821 by Peacock.

25. T.S. Kuhn, *The Structure of Scientific Revolutions*, University of Chicago Press, 1962).

26. The Molyneux influence in Dublin was enduring and his early Cartesianism perhaps is most clearly expressed in the Dublin tradition of vortex studies in mathematics which were central to the work of James MacCullagh in his search for the Cartesian synthesis in the 1830s and 1840s.

27. Lloyd's reforms in Trinity are described in detail in Chapter 6 of R.B. McDowell and D.A. Webb's *Trinity College Dublin 1592-1952; An Academic History*, (Cambridge University Press, 1982).

28. S.T. Coleridge, *On the Constitution of the Church and State, According to the Idea of Each*, (London, 1830).

29. J. Morrell and A. Thackray, *Gentlemen of Science*, (Oxford, 1981).

30. For a discussion of this ideological struggle between the theoretical and experimental tradition in France see I. Grattan-Guiness, *Mathematical Physics in France, 1800-1835*, in H.N. Jahnke and M. Otte.

31. M. Crosbie and C. Smith, *The Transmission of Physics from France to Britain: 1800-1840*.

32. D. Lardner, Preface to *Elements of Theory of Central Forces*, (Dublin 1820).

33. *Ibid.*

34. Hamilton's optical researches led directly to the discovery of quaternions.

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