A Final Remark on Infinite Exponentials

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The last issue of the Newsletter contained an article on infinite exponentials in which the following problem (dating back to 1907) was stated.

For any complex number a let a_1 = a and

$$a_{n+1} = \exp[a_n \log a], \quad n = 1, 2, \ldots ,$$

where the principal branch of log a is taken.

Is the sequence a_n convergent whenever a lies in

$$R_C = \{e^{\zeta e^{-\zeta}} : |\zeta| \leq 1\} \ ?$$

This problem has now been given a more-or-less complete solution by Dr. I.N. Baker of Imperial College, London, who uses the classical theory of iteration as developed by Fatou and Julia. In fact a_n is convergent when a is an interior point of R_C and when a is of the form $e^{\zeta e^{-\zeta}}$, ζ being a root of unity. However for most points of ∂R_C the sequence a_n is divergent.

Details should appear.

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School Mathematics – Knocking on Heaven's Door

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You who are reading this most likely know what's wrong with secondary school mathematics in Ireland [1]. The only question you need to have answered now is: "When is something going to be done about it?"

At the time of writing (October 1982) the Department of Education Mathematics Syllabus Committee is supposed to be in session. The agenda, as far as I know, does not mention Syllabus change but the Irish Maths. Teachers Association representative on the Committee will be asking that the IMTA's Draft Syllabus submitted last year be considered as a basis for a new scheme. For some IMTA members this session of the Syllabus Committee is tantamount to a sitting of the Delphic Oracle. Years of deliberation and consultation of members have gone into preparing the IMTA's case. Their brief is now in the hands of God. Or rather, it has been for about 18 months.

The IMTA's case is for a reduction by about one-third in the Higher Leaving Cert. courses – which should result in large percentages of examinees scoring high marks (why not?) in their exams. The IMTA are firmly behind an anti-abstract groundswell among teachers.

The groundswell has been there for years but with a fall-off of interest by Maths. teachers in their subject (as evidenced by enrolments in their Association) and the onset of the micro-computer, the Maths. teachers dummytit, there is no more hope of the groundswell coming to anything than there is of the Exchequer financing radical change.

This last consideration might prove to be a decisive one. Proponents, for example, of three Leaving Cert courses (lower, middle and higher) are almost certain to have their ideas frozen in the chill of public cut-backs. Indeed it may even be that any change which requires expenditure (e.g. on teacher retraining courses) which run into five (would you believe four?) figures will die a lonely death in the corridors of Marlborough Street where Department of Education finances are controlled. But that's only speculation.

Yet how dire is the need for change? Pretty dire, even viewed from the outside. That dirty word "fail" has not yet been erased from the minds of this country (surely an object lesson in how to brain-wash a whole people if ever there was one) even though no such category exists officially. One cannot fail the Leaving Certificate. It is an official impossibility.

Unofficially, and outside in a brain-washed world, if you don't get 40% in Maths you've got nothing: if you don't get 65% in Maths you've only "passed" (another illegitimate word). There are even things called "red honours" in popular lore: a "red honour" is 55% on a Lower Leaving Cert paper.

Unofficially there is concern in high places. Higher Course Maths numbers fell dramatically during the last 8 years or so as children began to make the sensible decision that the Higher Course wasn't worth the time it took up, even for extra University points. The Department responded (or was it coincidence?) by making the Higher Course papers easier - and by propping up lower course grades at marking-time whenever there was a danger of catastrophic numbers falling into the "fail" category or of a sensational divergence from the usual proportions in the other marks-categories.

At the heart of the matter one suspects that the make-or-break importance of the Leaving Cert which draws huge numbers into Maths classes (e.g. all aspirants for nursing, of which there is a never-ending supply) has placed a strain on the syllabus which it was never intended to take. A syllabus after all should be no more than a coatstand on which we hang our coats, a steel-mesh onto which we pour our concrete, a line on which we hang out our washing. Far from these humble stations, the secondary school syllabus has become the Word of God. Teachers teach the syllabus now. There is little time left to teach mathematics. If there are chinks in the syllabus, there are chinks in the Word of God and it's bound to show. Any starryeyed nurse-aspirant will tell you, her mind filled with pictures of bedside manners, emergency drips, tucking in the children, switching off the lights. The word is Irrelevance.

Further back, at Inter Cert, all is not well either. In editions of the IMTA Newsletter in May and September of this year I detailed faults in the present Geometry course as well as faults in proofs of theorems given in standard commercial textbooks. Within its own rationale the Geometry course is a chaotic bag of difficult (to me) jumble. It lacks order and elegance and to this day I am not satisfied with answers given to key questions such as "why is it important for pupils to prove that the image of a line under a translation or under a reflection in a point is a line, although they are asked to take no notice of the fact that the image of a line under reflection in a line is a line?" (They are encouraged to use this latter result, without comment. Worse, they have already had it motivated to them that a translation and a reflection in a point are composi -

tions of reflections in lines!!).

Axioms are sloppy and there are other faults - not least being the thinking that methods which were found good in the older School-Euclid Geometry should be exhumed and inserted into the present course. Was our changeover to "Papy's" Geometry no more than a classical native solution to having to make a choice?: "We'll make it look as though we have Papy but in reality we'll have Euclid". For example, some teachers have taken the step of teaching proofs of congruence of triangles before beginning the Geometry proper and maintain that inter alia the Side-Angle-Side (SAS) and SSS and ASA results give more machinery to the pupils for solving problems [2].

For my part, I will not present a course of Geometry to pupils which purports to be based upon a transformation approach but depends for its life upon a course of Geometry which it replaced. For one thing I don't agree with a deus ex machina group of congruence theorems to ease the present difficulties: for I couldn't make it work in class, another, the double-think behind this state of things - which has lasted now for 14 years - is bad for our integrity as Maths teachers.

Indeed, all our present country's problems are not economical. If one allows that the changed Geometry was first moved around 1968, consolidated in 1973 and will continue to be examined at least until 1984 (what else!), then there are a quarter-million people walking around this country who have been brought up on a diet of bad Geometry - a Geometry worse in quality than that fed to their parents.

Of course some of the material is so difficult that luckily perhaps 80% of the students have not digested it. Mr. Fred Holland writing in 1977 said "Try as I will I cannot convince my first and second year students of the necessity for equipollence in the definitions of the transformations of the plane. Their minds turn to and run along their preconceived knowledge of length. They appear to accept equipollence just to humour me. If it were in reverse and equipollence followed equality of length, everything would be fine. But to them I am putting the cart before the horse" [3].

How far all of this is from what one Maths Professor said at an IMTA Conference recently: "We do not require of entrants to our courses that they have smatterings of this, that, or the other topic. The main requirement is a clear mind, an ability to reason and beyond the basics we'll teach them the rest. In fact, we''ll even teach them the basics".

Dark ages indeed. The IMTA has campaigned publicly (well before the present hubbub) about the need for syllabus reform; it has set up its commitees, held meeting upon meeting, poured out its sweat, fleshed out its ideas and after four years work submitted its Proposals for new Maths.Syllabi.

Here outside the Oracle the weather is as bleak as ever. We could do with good news to cheer us up.

References:

[1] Over 5 years ago John Kelly of Wicklow Vocational School listed the main faults as follows: (Irish Maths Teachers Association, Newsletter, No. 31, January 1977).

(a) Inadequate teacher training; (b) Demotion of traditional skills (mental arithmetic, graphs, estimation, logs(!), algebra);
(c) Distancing from Science curricula; (d) Over-abstraction (irrelevance to life of laws of associativity, etc., Axiomatics and Symbolism too soon); (e) Superficiality, e.g. treatment of groups;
(f) Unsuitability, general orientation towards University Mathematics.

[2] Article by Gerard Coogan, IMTA Newsletter, No. 30, October 1976. See also "Computer Mathematics 1". G. Coogan. Folens 1982.

[3] IMTA Newsletter, No. 31, January 1977.

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MATHEMATICS HORSES FOR ELEMENTARY PHYSICS COURSES

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Recent publicity about the 1982 summer examinations at U.C.C. has served to draw attention to the fact that for a number of years significant numbers of first year students have been having difficulties in their first university Physics course. While this problem is not confined to U.C.C., nor indeed to Irish Universities, it does seem to exist in first science classes in U.C.C. in a particularly severe form. Given that the first year Physics course in U.C.C. is taught at a low level (significantly lower than U.K. A-levels and certainly not at any higher standard than prevails in most North American universities) it is important that we attempt to ident-

ify the reasons for this problem.

We reject totally the argument that the course is intrinsically too hard for the students. We feel that it is a counsel of despair to suggest that Irish university students are less able than their counterparts abroad. Further, those students who attempt either the exam or the course a second time have a high success rate. Thus one must conclude that the primary reasons for the student's failures must lie in their background and preparation.

One contributory factor is that more than 40% of the first science Physics class in U.C.C. have not done Physics at Leaving Certificate level. We accommodate this by teaching a course which assumes no previous knowledge of the subject. The course is very compact (only about half the Leaving course is covered) and nothing is included that is not on the Leaving Certificate syllabus. We believe, however, that an absence of a background in Physics is not the primary source of the difficulties. The failure rate among students who did Physics but obtained a poor grade (e.g. D,E or F on the higher paper) in the Leaving Certificate is significantly worse than the failure rate among students starting the subject from scratch.

We have become convinced that a major contribution to the difficulties that students are having comes from their grasp, or rather lack, of the fundamentals of Mathematics. We recognize that Physics is Physics and Mathematics/in that each discipline has its own ethos and understanding. (is Mathematics) On the other hand it is clear that Mathematics provides the language of Physics. In all Physics courses at this level the insights and ideas involved are conveyed by mathematical relationships which both codify and illuminate the physical processes. In turn, we expect students to manipulate these formulae and be able to extract numbers from them.

Both the Science Faculty at U.C.C. as a whole and the Department of Physics have carried out a number of detailed surveys into the background and skills of incoming students during the past five or six years. Dianostic tests in basic mathematics have been a regular part of such investigations. We have discovered an appalling lack of the most elementary mathematical preparation among the first year science students (the problem is much less severe among the pre-medical, pre-dental and first engineering classes). The problem is not an absence of knowledge but rather a total lack of facility with even the simplest operations. It is not possible here to list all skills found lacking, but a few examples might help to