FOR YOUR DIARY

I.M.S. Group Theory Conference. U.C.G., May 11-12, 1979. (Details from Professor M. Newell, U.C.G.).

Conference on the Numerical Analysis of Semiconductor Devices. Trinity College Dublin. June 27-29, 1979. (Details from Secretary, NASECODE 1, 39 T.C.D., Dublin 2).

Conference on Current Problems in General Relativity. Dublin Institute for Advanced Studies, July 2-6, 1979. (Details from The Director, School of Theoretical Physics (Working Seminar 1979), D.I.A.S., 10 Burlington Road, Dublin 4).

Dundee Biennial Conference on Numerical Analysis, Dundee, June 26-29, 1979.

L.M.S. Conference on Aspects of Contemporary Complex Analysis. Collingwood College, Durham University, July 1-20, 1979.

L.M.S. Conference on Progress in Analytic Number Theory, University of Durham, July 22-August 1, 1979.

L.M.S. Conference on Noetherian Rings and Rings with Polynomial Identity. University of Durham, July 22-August 1, 1979. (Details from Dept. Math., University of Durham, Durham DH1 3LE).

A.M.S. Summer Research Institute Finite Group Theory. University of California, Santa Cruz. June 25-July 20, 1979 . (Details from Dr. W.J. Le Veque, A.M.S., P.O. Box 6248, Providence, R.I. 02940).

CALCULATING PRODIGIES

James Callagy (U.C.G.)

During the eighteenth and nineteenth centuries some remarkable individuals appeared from time to time whose phenomenal powers of rapid mental calculation excited the interest of mathematicians and psychologists. Those who possessed such powers from an early age and were mostly self-taught are the most interesting:

> Buxton (1707 - 1772), an English farmhand; Fuller (1710 - 1790), a negro slave; Whateley (1757 - 1863), later Protestant Archbishop of Dublin; Colburn (1804 - 1840), a farmer's son of Vermont, U.S.A.; Bidder (1806 - 1878), son of an English stonemason; Dase (1824 - 1861), of Hamburg; Safford (1836 - 1901), of Harvard; and three shepherds, Mondeux, Mangiamele and Inaudi, about 1867 - 1900.

It is significant that all but two have been completely ignored by historians of mathematics who always tell us of the extraordinary calculating powers possessed by great mathematicians like Euler and Gauss at an early age. Apparently, there exists a consensus of expert opinion that ability to do mental arithmetic rapidly has very little, if any, correlation with mathematical insight and creativity. Whatever about the converse, this view probably derives from a tradition originating with classical Greek mathematicians who clearly defined 'arithmetica' or number theory, as a liberal pursuit, and 'logistica', or practical computation, as unworthy of inclusion in the 'mathemata'.

We find this view reflected in Renaissance writers like Leonardo Bruni (1536) who expressed nothing but contempt for the medieval 'calculatores' and in our own times the historian of science, George Sarton, protesting that press reporters and other ignorant people attribute mathematical genius to these lightning calculators who can perform fantastic computations.

When one studies the methodology of some of those mentioned in achieving amazing feats of numerical skill, however, the classical argument appears insufficient nor does it justify the attitude adopted by historiographers towards them. Sarton's statement that it represents mathematical ability of a very low order loses much of its credibility too, when one considers the unique analytical processes (often devised ad hoc) of calculators like Bidder, Colburn and Inaudi, apart altogether from the time factor. We are fortunate in this respect to have complete explanations, notably from Bidder and Colburn on their own methods and from Binet and

Darboux on those of Inaudi. They indicate mathematical ability of no mean order and at times achieve the quality of elegance which we may define as the direct attainment of a foreseen end. Modern mathematics, to be sure, owes a supreme debt to the Greeks for having consistently substituted ideas for calculations, but one feels that those calculating prodigies belong to a more ancient tradition, the wealth of which has only recently been tapped by archeological research. This tradition goes back over four millenia to the indefatigable Babylonian calculators who compiled those admirable tables of reciprocals, powers and 'Pythagorean triples', using a curbrous cuneiform notation to achieve amazingly accurate results. Since their sexagesimal system was not readily adaptable to abacal computation, and for that matter being positional it does not require it, the art of rapid mental calculation must have been highly cultivated a good thousand years before the Greeks appeared on the scene.

Perhaps the best lightning calculator of recent times was Professor A.C. Aitken of Edinburgh who described what goes on in the mind of a rapid mental calculator. After a lecture in 1954 he referred to the discouraging influence of modern machines on people with talents like his, remarking that "mental calculators may, like the Tasmanians or the Maori, be doomed to extinction, but you may be able to feel an almost anthropological interest in having seen one".

The humanists amongst us, however, still continue to feel a lively mathematical interest in their careers and methodology and at our History conference in U.C.C. we looked at the achievements of some of those heroic individuals of the eighteenth and nineteenth centuries already mentioned. The following discussion necessarily excludes some relevant complementary material which was projected on transparencies at the conference (30/9/75).

JEDEDIAH BUXTON (1707 - 1772) born at Elmton, Derbyshire, was the son of the village schoolmaster who completely neglected his education. He never learned to read or to write figures and his mental faculties were of a low order. Numbers however had such a fascination for him that if any object was stated he began at once to compute how many inches it measured or what its area or volume was. His only practical accomplishment was to estimate by inspection the acreage of an irregular field very accurately but his calculations were slow. He could find the time for sound to travel 5 miles at 1142 ft. per second mentally in 15 minutes, but in 1751 he calculated the product of three numbers of seven, eight and five digits respectively carrying on the computation over several days in his mind.

In 1754, he was examined by some members of the Royal Society while in London, where some of his friends brought him to Drury Lane to see a play of Garrick's. Curious to study his reaction, they asked his opinion which was negative except for the fact that he could tell them the exact number of words uttered by the various actors and the number of steps in the dances. It was found also that he worked on bases 60 and 15 and had invented a notation of his own using terms like 'tribe' for 10^{18} and 'cramp' for 10^{36} when recording large numbers. A curious and perhaps unique feature in his case was his ability to handle two or more different calculations simultaneously.

THOMAS FULLER (1710 - 1790) was a Negro, born in Africa. He was captured in 1724 and exported as a slave to Virginia, U.S.A. Like Buxton, he never learned to read or write and his abilities were confined to mental arithmetic. He could multiply more rapidly than Buxton two numbers of up to nine digits each and give the number of grains of corn in a given mass or the number of seconds in a given period of time. Problems involving proportion, or the 'rule of three' as it was known, were his speciality. He was a slow calculator compared with others who follow.

RICHARD WHATELY (1757 - 1863) was remarkable for his ability to calculate rapidly at the age of five or six years and the fact that he later became Archbishop of Dublin. We shall allow him to speak for himself:- "Soon" he wrote, "I got to do the most difficult sums always in my head, for I knew nothing of figures beyond numeration, nor had I any names for the different processes I used. But I believe my sums were chiefly in multiplication, division and the rule of three. I did these much quicker than anyone could do on paper and I never remember having committed an error. I was engaged either in calculation or 'castle-building' morning, noon and night but when I went to school, at which time the passion wore off, I became a perfect dunce at ciphering, and so have continued to be ever since".

ZERAH COLBURN (1864 - 1840) was perhaps the most celebrated of the lightning calculators who could claim some recognition from historiographers because of his paramount influence on the career of William Rowan Hamilton at the age of fifteen. To quote Sir Edmund Whittaker (May 1954) "Young Hamilton loved poetry and the classics but his interests and the whole course of his life were completely changed when he met one Zerah Colburn, an American youngster, who gave an exhibition in Dublin of his powers as a

lightning calculator in 1820". Referring to the occasion in later life, Hamilton recalled: "For a long time afterwards, I liked to perform long operations in arithmetic in my mind, extracting square and cube roots and everything that related to the properties of numbers".
Colburn was the son of a small farmer of Cabut, Vermont, U.S.A.; and at the age of six showed extraordinary powers of mental calculation which he exhibited on tours of America. In 1812, he performed in London where he was repeatedly examined by competent observers. It was clear that the child of eight years operated by certain rules and while doing his calculations his lips moved as if he was expressing the process in words. He was able to explain his method in some cases but his speed in finding factors of large numbers, square and cube roots almost instantaneously was amazing. When asked once for the square of 4395, however, he hesitated but on the repetition of the question he gave the correct answer 19,316,025. Questioned about his hesitation, he replied that he disliked multiplying two four-figure numbers but he had thought of another way. In his own words: "I got the factors of 4395 as 293 x 15. I multiplied 293 by 293 and then the product twice by 15".
In 1814 he was in Paris but his exhibition fell flat amid the political turmoil of the time. Some English friends and Americans like Washington Irving raised a fund for his education and he was admitted to the Lycée Napoleon and later to the Westminster School in London. With education, however, his powers of rapid calculation declined and he lost the boyish frankness which had charmed his audiences. Subsequently, he commenced on the stage, tried his hand at teaching school, then became an itinerant preacher and, finally, a 'professor of languages'. It would seem that he was destined to have saved William Rowan Hamilton from a similar fate. Colburn did however find time off to write his autobiography containing an account of his methods before he died at the age of 36. His longest time for any calculation seems to have been about three seconds, otherwise he gave correct answers instantly to such problems as 8^{16} ; $n^{10}\ \forall n \leq 9,\ n \in N$; $106929^{\frac{1}{2}}$; $268,336,125^{\frac{1}{3}}$ so rapidly that the gentleman who was taking them down had to ask him to repeat them. But, although unable to explain his method of factoring, his ability to deal with primes was marvellous - as Cajori records - in readily showing that $2^{2^5}+1 = 4,294,967,297$ had prime factors $6,700,417 \times 641$. This was the example given by Euler as invalidating Fermat's Prime Number formula $2^{2^n}+1$, of which Colburn as a boy was unaware.

GEORGE PARKER BIDDER (1806 - 1878) a contemporary of Colburn, however, is by far the most interesting of all self-taught calculators, because he subsequently received a University education, retained his lightning power of calculation to the end of his life, and gave us a full analysis of the methods he invented and used. Born at Moreton, in Devonshire, at the age of six he was taught by his father to count up to 100. Although he knew nothing of the symbols nor the meaning of arithmetical terms, using the counting process only he taught himself the results of addition, subtraction and multiplication of numbers up to 100 by arranging and re-arranging marbles, buttons or pebbles in patterns. In later life he attached great importance to this experience and believed that his powers of calculation were strengthened by the fact that, ignorant of the written symbols, he had to rely on concrete vocal representation of numbers only. When he was nine years old his father took him on tours about the country to exhibit his extraordinary powers. In less than a minute he gave the time it would take sound to travel a distance of 123,256 miles at 4 miles per minute, in days, hours and minutes. By the age of 10 he could find the square root of 119,550,669,121 in 30 seconds. In 1817, two distinguished Cambridge graduates, Jephson and Herschel, were so impressed by his general intelligence as well as his calculating abilities that they raised a fund for his education and persuaded the father to abandon the role of showman. But in a few months the father had changed his mind and insisted upon his son's return. In 1818 he was matched against Colburn who was then two years older than Bidder, and proved to be the better calculator. Finally, the father and son came to Edinburgh where some members of the University succeeded at last in persuading the father to leave the boy in their care. In due course, Bidder graduated with distinction in Civil Engineering, became a successful railroad engineer and designed, and also supervised, the construction of the Victoria docks in London. He retained his amazing powers of rapid calculation to the end of his life, which proved a valuable asset to him as a frequent parliamentary witness in engineering matters. Just before his death, at the age of 72, he gave an illustration of them to a friend who, in connection with the then recent discoveries in physics, remarked that if 36,918 waves of red light, occupying only one inch, are required to give the impression of red to the eye, how immense must be the number of rays striking the eye in one second, if light travels at 190,000 mls per second; "You needn't work it out", said Bidder, "the number will be 444,433,651,200,000".

Bidder had two elder brothers, one an actuary, the other in religion, and also a son who later became a distinguished barrister - all of them had similar powers of rapid mental calculation but none of them developed their powers to the same extent. Even in the third generation, a grandson and a granddaughter inherited the same talents.

HENRI MONDEUX and VITO MANGIAMELE, both of whom were born in 1826, are an interesting pair about whom there was a suspicion that they had been exploited by others who taught them rules enabling them to simulate powers they didn't possess. Both were shepherds, in very poor circumstances and after short careers as exhibitioners returned to their sheep once more. In 1839 and 1840 they were brought to Paris and tested by Arago, Cauchy and others. Mondeux's performances were striking. One question put to him was the solution of the equation $x^3 + 84 = 37x$, to which he answered 3 and 4, not detecting the third root -7; another was to find integral solutions for $x^2 - y^2 = 133$, to which he replied at once $(67, 66)$ and when asked for a simpler solution he said instantly $(13, 6)$. As children they were indeed remarkable, but Mondeux if he were really self-taught like the other prodigies, would have to be credited with the discovery of algebraic theorems making him a mathematical genius. In that case, he would certainly have achieved far more than he did.

JOHANN MARTIN ZACCHARIAS DASE (1824 - 1861) of Hamburg, on the other hand, was the calculating prodigy who made some special contributions to the history of mathematics. Having had a fair education and every opportunity to develop his powers he made little progress beyond reckoning and numerical calculation. He was dull-witted, knew only German, and remained ignorant of geometry to the end of his life. He held various small official posts in Germany from time to time and also gave exhibitions of his skill in Germany, Austria and England. While in Vienna he met Strasznicky who urged him to apply his powers to scientific purposes and was introduced to Gauss, Schumacher and Petersen. As a lightning calculator he holds the unbeaten record for having found the square root of a 100-digit number in 52 minutes. Like the others he had a phenomenal memory and could repeat all the numbers mentioned in a performance one hour afterwards. His peculiar gift, somewhat like Buxton's, was ability to calculate at a glance the number of sheep in a flock, books on a shelf or the number of letters in a line of print chosen at random. His calculations on paper were incredibly rapid but always correct, and at the age of 16, Strasznicky taught him the formula :

$$\frac{\pi}{4} = \arctan\frac{1}{2} + \arctan\frac{1}{5} + \arctan\frac{1}{8}$$

and asked him to calculate π approximately. In two months he carried the approximation to 205 places of which the first 200 were correct - a result which was published in Crelle's Journal 1844. From 1844 to 1847 he was employed in the Prussian Survey and in his leisure time calculated the natural logarithms of the first 1,005,000 numbers to seven decimal places. On the recommendation of Gauss, the Hamburg Academy of Sciences gave him a special grant for the purpose of making tables of factors for all numbers from 7×10^6 to 10^7 but he lived only long enough to finish about half the work - he was aged 37 when he died.

TRUMAN HENRY SAFFORD (1836 - 1901) of Royalton, Vermont, U.S.A., although always a rapid calculator in his youth, gradually lost his exceptional powers with education at Harvard where he became a professional astronomer. At the age of 10 he was examined by experts answering readily such questions as: 'What is the surface area of a regular pyramid of slant height $17'$ and whose base is a regular pentagon of side $33\frac{1}{2}'$ — which he answered correctly as 3354.5558 sq. ft. in two minutes. Like Colburn, he factorized large numbers with ease but could not say how.

JACQUES INAUDI (1867 -) who started life as a shepherd spent the long hours of his watch pondering on numbers but, unlike Bidder, used no concrete representation like marbles etc. - except perhaps 'variants' such as sheep. At the age of six his brother, an organ-grinder, took him on tours through Provence, during which he earned a few sous in street exhibitions. His extraordinary ability in calculation attracted the attention of some showmen who took him to Paris in 1880. Still ignorant of reading and writing, he was very impressive in his performances and his powers steadily improved until he could multiply two numbers of ten digits rapidly. In Paris, Binet and Darboux posed him many problems such as the square root of $\frac{1}{6}$ the difference between the square of 4801 and unity which he found immediately. He could find integral roots of equations and integral solutions of problems by a method of trial and error, but his most remarkable feat was the expression of numbers less than 10^5 in the form of a sum of four squares, taking no more than a minute or two. This power was unique and most mental calculators found considerable strain attempting it - as indeed have most mathematicians attempting to prove Bachet de Méziriac's (1612) theorem that any number may be expressed in this form, which was first proved by Lagrange in 1770.

We are indeed indebted to Binet and Darboux for detailed descriptions of a typical performance of Inaudi's, published in the Comptes Rendus and the Revue des deux Mondes of 1892, and a complete analysis of his own methods by Bidder, given in a lecture to the Institute of Civil Engineers in 1856. It was generally thought that as well as having phenomenal memories most lightning calculators visualized numbers, but both Bidder and Inaudi relied altogether on articulation and hearing. Inaudi always repeated numbers proposed to him slowly to his assistant who wrote on a blackboard, and relied on his speech muscles and his ear during calculation; he never glanced at the written symbols which would only confuse him, he said. Bidder said it would have taken him four times longer if numbers were written for him as they would not be so vividly impressed on his imagination, and never used symbols at all in his mental procedure. He thought of a number, like 984 for instance, in a concrete way as a collection which could be arranged in 24 groups of 41. Dase on the other hand appears to have visualized the numerals as on paper. Another feature common to many of them was the ability to repeat the numbers occurring in a performance hours afterwards, and also to give the correct sequence of digits forwards or backwards from any selected point. Bidder once did this, one hour afterwards, in the case of a number of 43 digits. He had developed an associative principle for multiplication at the age of eight, by which he could multiply two numbers of six digits each in about seven seconds - a facility he acquired by practice at the village blacksmith's forge where the locals congregated in those days to match feats of skill. He multiplied from left to right - contrary to school practice - adding his partial products as he proceeded so that he never had more than two numbers to add at a time. It is remarkable that Inaudi also multiplied in this way and sometimes used negative quantities as in $27 \times 729 = 27(730 - 1)$. In division Bidder used a process peculiar to himself which he termed a digital process, where a 'digital' is defined as 'the sum of the digits (mod 9)'. That the digital of a number is equal to the product of the digitals of its factors (mod 9) is a theorem which can be applied to find if 73, for instance, is a factor of 23,141.

A curious question has been raised as to whether a law can be found for the rapidity of mental working of calculating prodigies. Bidder stated that in multiplying a number of n digits by itself he believed the strain on his mind varied as n^1 (assuming strain proportional to time). In the case of Dase it seems to have been n^3, but more detailed information and observation

would be needed to support any theory on this subject. Perhaps, some day, computer scientists, interested in cybernetics, and behavioural psychologists, interested in numerics, may be able to enlighten us when they come up with the calculating prodigy par excellence - an automatic self-programming digital computer.

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