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# Thomas Waters: The Four Corners of Mathematics, CRC Press, 2025. ISBN: 978-1-032-59498-9, GBP 19.99, 277 pp.

REVIEWED BY PETER LYNCH

This book presents a panoramic view of mathematics – pure and applied – spanning three millennia. It is clearly written with ample illustrations and diagrams and there is an excellent selection of topics, many of which may be new to readers. The book is in four Parts covering the fields of Geometry, Algebra, Calculus and Topology. Each Part comprises four chapters and each is about sixty pages in length. The high point of the book is the final Part, and I will focus most attention on that.

Part I deals with Geometry. The axiomatic approach, which originated in Ancient Greece, was used to great effect in Euclid's *Elements*. Some examples clarify the step-by-step procedure of this approach. The work of Apollonius on conics is reviewed. The development of geometry through the work of Islamic and Indian scholars is traced. The parallel postulate remained a contentious issue until finally it was shown to be inessential. Non-Euclidean geometry sprang independently and simultaneously from three sources, Lobachevsky, Gauss, and Bolyai, greatly broadening our understanding of the subject. Through his study of general curves and surfaces, Gauss initiated differential geometry. Chapter 3 gives an excellent presentation of how the line element of Gauss encapsulated the essence of geometry and led on to the $n$-dimensional manifolds of Riemann, so crucial for Einstein's later work. Fractals are treated in Chapter 4 in enough detail to enable readers to generate images never seen before.

The second Part is on Algebra. Various primitive number systems are described, ultimately displaced by the Hindu-Arabic numerals. Diophantus used symbols for numbers and introduced some key ideas that are of interest today, but the first systematic account of algebraic methods was that of al-Khwarizmi. This work eventually reached Europe, triggering a flurry of mathematical activity in Italy, where solutions of cubics and quartics were found. The intrigue and skulduggery accompanying these mathematical advances is recounted in the book. From this work there emerged complex numbers and, eventually, the Fundamental Theorem of Algebra. The relationship between the winding number and the roots of a polynomial is well described in the text.

Next comes an account of the difficulties in solving quintic equations, the findings of Abel and the tragic story of Galois, whose work gave rise to modern group theory. There follows an account of the fundamental group of a manifold, and the foundational work of Emmy Noether on abstract algebra.

Chapter 8 opens with a bold claim: "While the $19^{\text{th}}$ century was the century of Calculus and the $20^{\text{th}}$ the century of Topology, the $21^{\text{st}}$ century will be the century of Linear Algebra". The rationale for this is the rapidly increasing importance of Artificial Intelligence and the need to handle ever-larger data sets. Quaternions and the many developments following from Hamilton's discovery, are discussed. Several interesting applications illustrate the importance of the resulting mathematical advances.

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Part III is on Calculus, described in the opening sentence as "the greatest idea Mathematics ever had". Archimedes made some vital early contributions, building on ideas of Eudoxus. Two great problems passed down from Greece were to find areas bounded by curves and to determine tangent lines to curves. Descartes and Fermat made substantial inroads but it was the work of Leibniz and Newton that established the fundamental basis of the subject. The lack of rigour in handling infinitesimals, highlighted by Bishop Berkeley, was ultimately resolved in the $19^{\text{th}}$ century. Two little letters, $\epsilon$ and $\delta$, have struck terror in maths students' hearts ever since.

Chapter 10, on the Solar System, is an interesting whistle-stop tour from the Greeks, via Ptolemy, Copernicus and Kepler to Newton's *Principia*. The $n$-body problem is introduced, and the discoveries of Poincaré and his homoclinic tangle lead us into chaos. The chapter ends with a brief look at General Relativity.

Maxima and minima are considered in Chapter 11. Here we must consider functions of several variables, and partial derivatives are introduced. The problem of Johann Bernoulli, to find the curve on which a particle will slide to the lowest point in minimum time, was solved by Newton but, more importantly, it later inspired Euler to develop what he called the Calculus of Variations. Both Euler and Lagrange found the equations for a general solution of such problems – the Euler-Lagrange Equations – which are central in analytical dynamics. Poincaré's analysis of the 3-body problem is reviewed, as is the remarkable and delightful theorem of Emmy Noether that links mechanical invariants and symmetries. The chapter ends with a discussion of geodesics on a triaxial ellipsoid, a problem that was first solved by Jacobi.

Partial Differential Equations, which form the subject matter of Chapter 12, are "at the very heart of modern mathematics". The origins of the three classical PDEs – the wave equation, Laplace's equation and the heat equation – are treated. The solution of these stimulated profound mathematical developments. A fourth order PDE featured in the research of Monsieur Antoine Le Blanc, aka Sophie Germain, whose tale is told. The chapter ends with a return to General Relativity, the wave equation emerging from Einstein's Field Equations and the detection in 2015 of gravitational waves, which were triggered by an orbiting pair of black holes more than a billion years ago.

Part IV, on Topology, opens with the usual example of the Bridges of Königsberg, but moves quickly to an excellent discussion of the Gauss-Bonnet Theorem, which provides a strong connection between geometry and topology. A clear sketch gives a good idea of the proof of this beautiful result. Then the key topological equivalence relations, homotopy and homeomorphism, are introduced.

In Chapter 14, entitled "Degree", curves in the plane and in higher-dimensional spaces are discussed. The winding number and the rotation number are defined and used to classify plane curves. Then the Poincaré-Hopf Index Theorem is described and some surprising and delightful connections are made linking Euler's characteristic ($V - E + F$), the Hairy Ball Theorem and the Gauss-Bonnet Theorem. This sounds formidable, but the treatment in Waters' book is a model of lucid exposition. The chapter ends with the statement that "A hairy 18-sphere would have to have a 'tuft' somewhere, but we could comb a hairy 19-sphere flat". You cannot tell when you might need to know that!

Chapter 15 is on Homology or "using algebra to count holes". The going gets tougher but the author manages well to strike a good balance between clarity and rigour. The ideas here are not often found in a "popular maths" book and are all the more welcome for that. Betti numbers are defined and evaluated for the sphere and torus. The Euler-Poincaré characteristic is introduced and shown to be equal to the alternating sum of Betti numbers. This explains why the closing statement of the previous chapter must hold! Finally, simplicial complexes are introduced. Waters recalls a suggestion of Emmy Noether: to define the quotient group of the group of cycles by the group of cycles homologous to zero as the Betti Group. This led to the general formulation of homology groups and the emergence of Algebraic Topology as a major branch of maths.

Having warmly praised this worthy book, I have three minor grumbles. The title is poorly chosen; the branches of mathematics in the four parts are not *corners*, but *pillars* upon which rest many other results. The references are excellent but, with 137 entries, an additional short list of "highly-recommended" sources would be very helpful. The index is comprehensive, but many entries point to in excess of a dozen pages, plunging the reader into multi-dimensional space; these topics need to be sub-divided.

In summary, I can heartily recommend this well-researched and well-written book as a valuable and accessible introduction to some of the principal branches of mathematics. In my youth I read every maths book in Dun Laoghaire Public Library, and many more from elsewhere, but none compared, in quality or scope, to the book under review. I believe that it could be recommended to any young student hoping to embark on a mathematical career. He or she is sure to learn some new and delightful mathematical truths, and should thoroughly enjoy the process of discovery.

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