

## PROBLEM PAGE

J.P. MCCARTHY

### PROBLEMS

Thanks to all those who responded to Problem Page 95's call for more problems. However, *no* solutions to Problem Page 94 have been received: we will keep those problems open until Summer 2026.

The first of this edition's problems comes courtesy of Des MacHale of University College Cork.

**Problem 96.1.** Prove, using group theory, the following results in number theory:

- (1) If  $m$  and  $n$  are natural numbers, then  $m!n!$  divides  $(m+n)!$ .
- (2) If  $p$  is prime and  $n$  a natural number, then  $n!$  divides

$$(p^n - 1)(p^n - p) \cdots (p^n - p^{n-1}).$$

The second problem was sent in by Yagub N. Aliyev, of ADA University, Baku, Azerbaijan.

**Problem 96.2.** Let  $a > 0$ . Suppose that two distinct normals to the parabola  $2y = ax^2$  intersect the parabola again at  $A$ . Prove that the  $y$ -coordinate of  $A$  is strictly greater than  $4/a$ .

Finally a problem from Finbarr Holland of University College Cork.

**Problem 96.3.** Where  $\Gamma$  is the gamma function, determine the limit

$$\lim_{p \rightarrow 0^+} \frac{1}{p^2} \left( 1 - \frac{p \Gamma^2(p)}{2 \Gamma(2p)} \right).$$

We invite readers to submit problems and solutions. Please email submissions to [imsproblems@gmail.com](mailto:imsproblems@gmail.com) in any format (preferably L<sup>A</sup>T<sub>E</sub>X). Submissions for the summer Bulletin should arrive before the end of April, and submissions for the winter Bulletin should arrive by October. The solution to a problem is published two issues after the issue in which the problem first appeared. If possible, please include solutions with your submissions.

DEPARTMENT OF MATHEMATICS, MUNSTER TECHNOLOGICAL UNIVERSITY