

PROBLEM PAGE

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Thanks to all those who responded to Problem Page 95's call for more problems. However, *no* solutions to Problem Page 94 have been received: we will keep those problems open until Summer 2026.

The first of this edition's problems comes courtesy of Des MacHale of University College Cork.

Problem 96.1. Prove, using group theory, the following results in number theory:

- (1) If m and n are natural numbers, then $m!n!$ divides $(m+n)!$.
- (2) If p is prime and n a natural number, then $n!$ divides

$$(p^n - 1)(p^n - p) \cdots (p^n - p^{n-1}).$$

The second problem was sent in by Yagub N. Aliyev, of ADA University, Baku, Azerbaijan.

Problem 96.2. Let $a > 0$. Suppose that two distinct normals to the parabola $2y = ax^2$ intersect the parabola again at A . Prove that the y -coordinate of A is strictly greater than $4/a$.

Finally a problem from Finbarr Holland of University College Cork.

Problem 96.3. Where Γ is the gamma function, determine the limit

$$\lim_{p \rightarrow 0^+} \frac{1}{p^2} \left(1 - \frac{p \Gamma^2(p)}{2 \Gamma(2p)} \right).$$

We invite readers to submit problems and solutions. Please email submissions to imsproblems@gmail.com in any format (preferably \LaTeX). Submissions for the summer Bulletin should arrive before the end of April, and submissions for the winter Bulletin should arrive by October. The solution to a problem is published two issues after the issue in which the problem first appeared. If possible, please include solutions with your submissions.

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