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Gesture in the University Mathematics Classroom

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ABSTRACT. One of the enduring challenges in undergraduate mathematics education is understanding how students have internalised complex mathematical concepts. Often, however, students do reveal their conceptualisations – though not through speech or writing, but through their bodily actions. In this way, aspects of students' mathematical meaning-making become externally visible in the *gestures* they produce. In this paper, I outline the process of gestural analysis through examples drawn from undergraduate mathematical interactions. These episodes illustrate how attention to embodied cognition can enrich our understanding of mathematical thinking, and offer educators new ways to perceive and support students' conceptual development.

1. Introduction

When we teach mathematics, what is going on in our students' heads? This might be a question that every mathematics educator has wrestled with. Descartes argued the mind and body are dichotomous: distinct, though connected, entities [7]. This would imply that to speak to the mind, we should put aside the body – however recent advances in cognitive science challenge this partitionist view. Neurobiologists have found that even in the absence of external (physical) stimuli, our sensorimotor systems can remain active (e.g. [6]) – suggesting the brain may operate *thought* and *action* through the same neural circuitry. This idea is central to the theory of *embodied cognition*, which proposes that abstract thought (including mathematical reasoning) is grounded in bodily experience. Neurophysiology is one component of embodied cognition – though, admittedly, a hard one to observe in the classroom. In contrast, for the teaching and learning of mathematics, a very accessible manifestation of embodied cognition are the *gestures* of students and teachers.

Gestures are spontaneous, idiosyncratic, symbolic movements of the hands and arms that accompany speech [17], offering a nonverbal perspective into a speaker's mind. Gesture has been widely studied in teaching and learning contexts (particularly in mathematics education for children; see [9] for review) as a tool to aid education. Gestural analysis can be especially valuable in the context of service courses for STEM undergraduates (such as calculus, linear algebra, and differential equations for non-mathematics majors) where large numbers of students encounter complex, layered mathematical ideas for the first time, and must navigate them carefully. These courses offer an opportunity to bring gesture research into practical use: reading students' gestures can help instructors spot conceptual difficulties, and support clearer communication between teacher and learner. Gesturing itself can also help students think through ideas, and furthermore serve as a foundation for abstraction and formalisation in higher mathematics. By reflecting on some pedagogical experiences at a large, public, primarily undergraduate

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institution on the U.S. Western coast, I will expose gestural analysis as an active and engaging (and enjoyable) educational aid for teachers and students alike.

2. Taxonomy

Researchers classify the gestures used in mathematical discourse by a number of different criteria. One could frame McNeill's taxonomy [17] as the 'standard' framework, insofar as it has been examined and expanded on by several researchers in mathematics education (e.g. [8, 19]). In keeping with the holistic perspective of embodied cognition, McNeill proposes dimensions – rather than rigid categories – for labelling gestures: *deictic*, *iconic*, and *metaphoric*. (Edwards [8] later refined this framework for mathematics, proposing a split of the *iconic* dimension into *iconic-physical* and *iconic-symbolic.*)

Deictic or *pointing* gestures indicate objects, locations, and can include abstract referents: for example, after solving a problem, *pointing* upwards while saying "I have it!". (Here the gesture is not aimed at a physical object, but the abstract concept of the problem's solution.) In mathematics, we all use deictic gestures frequently: pointing at a step of a proof or calculation (to link in a verbal explanation), to highlight information in text or on graphs, to focus attention, or to indicate confusion. To add emphasis, a more pronounced gesture may be useful – a repeated tap on the board, a revolution of the index finger, or any simple, rhythmic hand movement synchronised with speech [3]. These "*beat*" gestures are used to communicate mathematical significance, which can be lost if only expressed verbally, and in general might not be recognised by undergraduates [16]. Given the difference in proof comprehension, reading and engagement between experts and novices (e.g. [12]), providing emphasis and direction using gesture might provide what Goldin-Meadow calls a "window to the mind" – but of the *lecturer*, for the student.

3. Iconic-Physical Gestures

In a first course on multivariable calculus, I structured a lesson around the use of *iconic-physical* gestures to probe students' concept images of *surfaces* – an approach I ultimately found more effective than relying solely on the students' mathematical language. Iconic-physical gestures "depict semantic content directly via the shape or motion trajectory of the hand(s)" [3], directly evoking characteristics of "a concrete object or event" [17]. (For example, using one's straightened hand as a knife to 'cut' an imagined circle into portions, when discussing fractions [8].) After a lecture on traces and contour maps, my students were presented with an in-class exercise from *Calculus, Eighth edition*, J. Stewart (2015) replicated in *Figure 1 (left)*: *A contour map of a function is shown below. Use this to make a sketch of the function.* A sketch of the function is included in *Figure 1 (right).*

Contour maps are a vital tool for representing three-dimensional data in a two-dimensional format, and will be frequently employed by these students in their mathematics, physics, and engineering courses. In presenting them first as a collection of cross sections of a surface in $\mathbb{R}^3$, students can approach them as a physically realisable and gesturable object; even if the contour map represents abstract data, the students can benefit from grounding the problem in terms of their bodily experiences [18]. The learning outcome of this section is to reconstruct a surface in three-dimensional space from the data of a two-dimensional contour map: whether students have done so correctly can be determined by the teacher from the *iconic-physical* gestures they produce in support of their description of the surface.

Most commonly, students incorrectly identify this surface as a *cone*, which one can identify from the iconic-physical gesture in *Figure 2 (left).* Here the student traces the shape of the surface in their imagined gesture space, and the straight trajectory of the

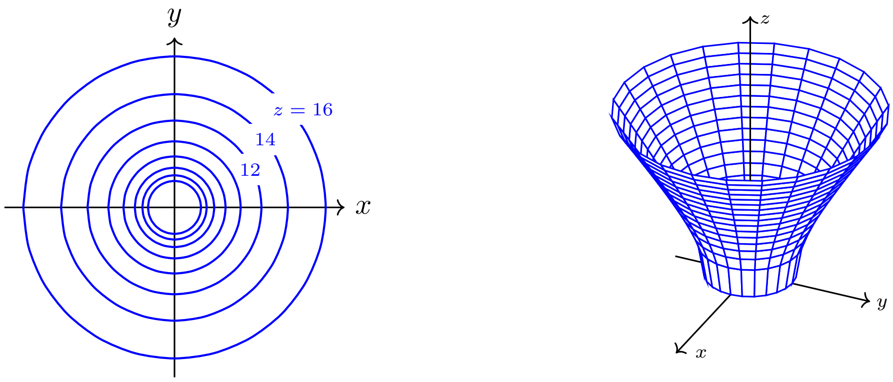

FIGURE 1. Contour map (Left) and graph (Right) of a hyperboloid of one sheet.

hands indicates a cone. This is immediately noticeable as different to the gesture one naturally produces when tracing the shape of a hyperboloid of one sheet, in *Figure 2 (right)*. As suggested by Alibali and colleagues [4], this misconception (or 'trouble spot') can be readily identified by the instructor, and – instead of further verbalisation, *corrected by gesture* – to re-establish shared understanding. This *gestural mimicry* [13] between student and teacher uses a resource accessible to the entire classroom to develop mathematical meaning, at a more intuitive level for the mathematical novice.

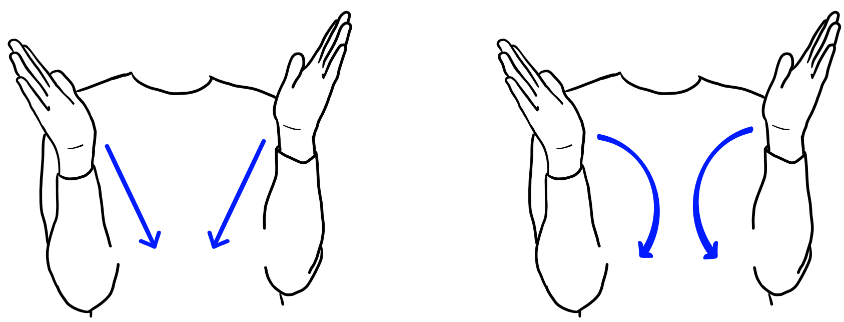

FIGURE 2. Iconic-physical gestures corresponding to the surface in *Figure 1*: a cone (Left) versus a hyperboloid of one sheet (Right).

The student's subsequent – often subconscious – *grounding action* of embodying the mathematical data of *Figure 1* when corrected beneficially influences future attempts at such problems: according to the theory of *action-cognition transduction* [1], thoughts and actions are a two-way street, with one influencing the other. By connecting *Figure 1 (left)* with *Figure 2 (right)*, I teach non-verbally – a practice that, as Hord and colleagues suggest, might be particularly salient for students with learning disabilities, such as dyslexia [11]. Undergraduate students with dyslexia report high-level difficulties (re)organising and (re)structuring information mentally [22]; difficulties that can make processing mathematical information verbally or in text especially demanding. Given a 364% increase over the last 15 years in the number of students with disabilities registered for support services in Irish HEIs (the largest faction of which are students with dyslexia) [2], this is not an abstract issue, but an urgent and growing concern for Irish higher education.

4. Iconic-Symbolic Gestures

When teaching material on matrices, elementary row operations, and row-echelon form, watch for an *iconic-symbolic* gesture used by students when reasoning to each other their row-reduction steps (see *Figure 3*). Iconic-*symbolic* gestures, as opposed to iconic-*physical* gestures, refer to “a symbolic, written inscription, which in turn represents a specific mathematical entity or procedure” [8]. They are a nod by Edwards that, in mathematics, a ‘concrete referent’ may not be a physical object or event – the ‘crossing out’ gesture when simplifying fractions in numerator and denominator [8], for example, is iconic-symbolic.

In my linear algebra class, for a displayed matrix (on the board, in their prewritten notes, or in a book — a setting where the students could not or would not physically write their calculations or steps), when nullifying all elements of the matrix below a pivot element, students would move their finger down in a “C” shape while describing the row reduction operation they are performing (see *Figure 3*). This was taken by all parties to mean ‘perform (verbally expressed) operation to (verbally expressed) row’.

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 5 \\ -3 & 2 & -1 & 0 \end{bmatrix} \xrightarrow[R_3 \to R_3+3R_1]{R_2 \to R_2-2R_1} \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & -1 & -2 & -3 \\ 0 & 8 & 8 & 12 \end{bmatrix}$$

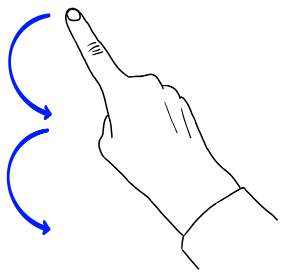

Figure 3. (Left) Steps towards reducing a matrix to row-echelon form. (Right) Accompanying iconic-symbolic gesture.

This is a deictic gesture, but can have another meaning as well – in *Figure 3*, often students would not be pointing towards R_1 (at the top of the “C”) nor did the gesture conclude pointing specifically at R_2 or R_3. In addition, the accompanying speech often specified the rows involved more accurately than pointing from a distance. This gesture captures the *action* of performing an elementary row operation – moving from one step of the row reduction algorithm to the next.

It is also noteworthy that this gesture was sometimes performed *without* accompanying speech, when a student was self-explaining. This suggested the gesture has value as an *offloading mechanism* [9]: instead of mentally performing individual, isolated calculations, the student can store somatically the overall goal of “$R_3 \to R_3 + 3R_1$” through pointing at R_1, and with the “C” gesture, incorporating R_3. The intensity of activity in the university classroom – thinking, listening, writing, reasoning – would suggest that any opportunities to ‘free up’ mental space and reduce the cognitive pressure on students should be taken by the educator!

5. Grounded Blending

After drawing a curve on a blackboard x-y plane, there is a gesture that often appears when discussing the *tangent line* to a point on the curve: a straightened hand, hovering above the drawn curve at a roughly appropriate slope (*Figure 4, right*). Usually I do not need to specify that the hand is a stand-in for the line: there are a sufficient number of common features (shape, gradient, position, direction) between the mathematical concept and the iconic-physical gestural representamen that the association is automatic and natural. The formal mechanism to systematically analyse such internal mental representations is known as *conceptual blending*, though when applied to hand gestures tends to be labelled *grounded blending* [20]. 'Blending' is an internal process where an abstract mental space is combined with (the mental representation of) a real, physical space, along generic features common to both spaces. The visual schema commonly used (e.g. [8, 20]) to represent the conceptual blend of (1) mathematical objects and properties with (2) a mental representation of the hand and its location in space, is shown below (*Figure 4, left*). By decomposing the grounded blend into its constituent mental spaces, I specify a *meaning* for the gesture: the conceptual structure that underlies the physical hand movement.

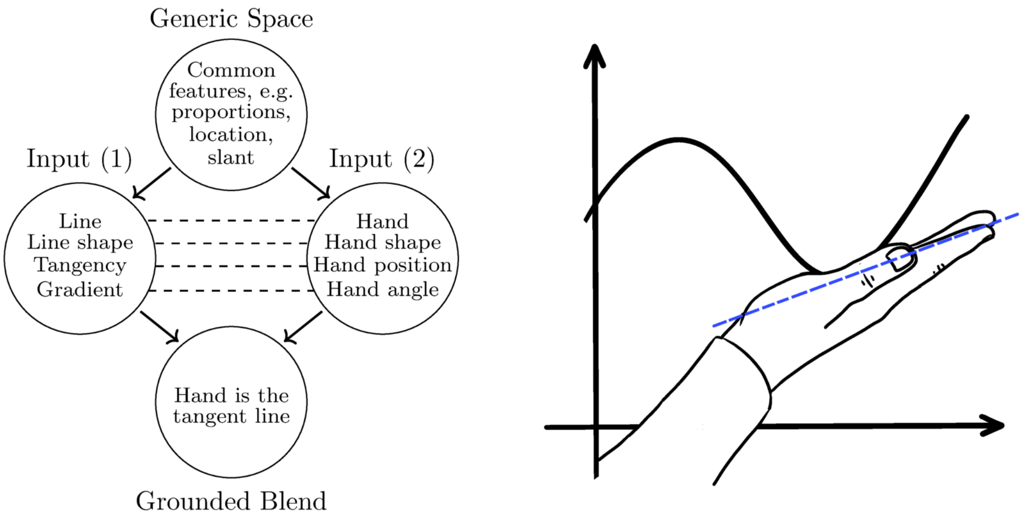

FIGURE 4. (Left) Grounded blend of (Right) the iconic-physical gesture of the hand representing the tangent line to a curve.

In any pedagogical setting – a lecture, tutorial, office hours, etc. – gestures are frequently used as communication tools between the student and the educator. Interpreting the meaning of a gesture is thus crucial to effective teaching and learning. As the final example shows, there are sometimes multiple layers behind a gesture, and teasing out the details becomes a fascinating exercise into students' mathematical conceptions.

6. Metaphoric Gestures

The discussion of *series* (and their convergence or divergence) constitutes a significant portion of the second or third course in single-variable calculus which STEM undergraduate students complete in the U.S. Successful execution of convergence arguments requires the student developing *analytical proof schemes*, of the Harel and Sowder hierarchy [10]: for a given problem, students must first determine an appropriate series convergence test (by "anticipating the outcomes of proposed transformations" [19] – *operational thought*) then perform a chain of *logical inferences* to utilise the test and draw an appropriate conclusion. While a particular example is not wholly *general*, considering the use and execution of a convergence test in abstract does require an awareness that these arguments may be applied across several members of some class of series. Thus, determining whether a student is engaging analytical proof schemes, as opposed to *empirical* or *external conviction* schemes, becomes essential to promoting successful and rigorous engagement with the material.

So, can one 'read' from a student's gestures that they have developed a sophisticated understanding of series? Similar to Nathan and colleagues [19], I argue that certain gestures can indicate when a student is operating analytical proof schemes. A recurrent gesture I noticed when discussing material one-on-one with a student, appearing across several examples, was a sequence of descending horizontal slices (depicted in *Figure 5*). This was typically presented by the student after selecting a series convergence test to employ; the gesture served to substitute (or supplement a summary of) verbal calculations and confirmations necessary to apply the test – and notably *did not* reference prior work or written inscription. This can be categorised as a *metaphoric* gesture – the slices had no physical referent in the students' speech, and no inherent mathematical relevance to series – representing to the student the concept of *proof*, through the metaphor "proving is motion" (cf. [20]).

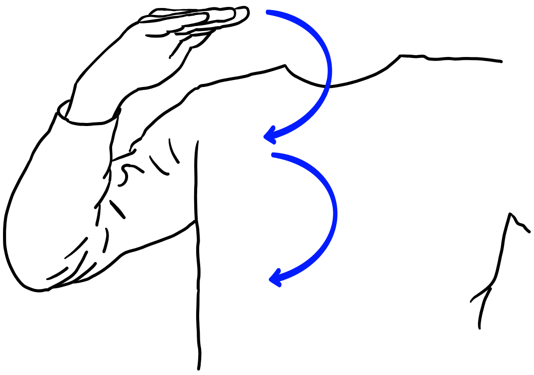

FIGURE 5. Descending horizontal slices made while discussing series convergence.

Metaphoric gestures depict semantic content through metaphor, presenting abstract concepts in a tangible, embodied form [3]. Parrill and Sweetser propose that metaphoric gestures can be modelled as the composition of two conceptual blends: the first linking an iconic gesture with a concrete subject (the *source* of the metaphor), and the second linking this subject with an abstract concept (the *target* of the metaphor) [20]. In this way, a metaphoric gesture is in fact iconic – but the icon the gesture references has no immediate referent correlating to a physical object, inscription, mathematical entity, or procedure, and instead symbolises the tangible component of an abstract relationship which the speaker has internalised through a metaphor. These types of gesture are commonly observed in mathematical discussion [8] as the language used to describe mathematical ideas can be highly metaphoric [15].

By providing a grounded blend decomposition of the *Figure 5* gesture, I argue the gesture provides an *iconic* reference to the process of navigating a mathematical proof – presented in text as a sequence of paragraphs, each outlining and validating a step in a single cohesive argument. *Figure 6, top left* illustrates how the (iconic) grounded blend between this gesture and the textual presentation of the proof may be established. This textual format also serves as a convenient vehicle for grounding the metaphor "proving is motion": the premises, intermediate deductions, and conclusion of the abstract logical argument can be mapped onto physical locations within the text, with premises appearing first, deductions unfolding in subsequent paragraphs, and the conclusion stated at the end. In this way, the student may establish a mental space (*Figure 6, top right*; "Conceptual Blend (Metaphoric)") where the source and target of the metaphor are integrated. The result (*Figure 6, bottom*) is a conceptualisation of the gesture as representing the process of proof.

Using the *Figure 6* blending model to analyse the *Figure 5* gesture subsequently allows us to argue that the student, in utilising this gesture, is employing analytical proof

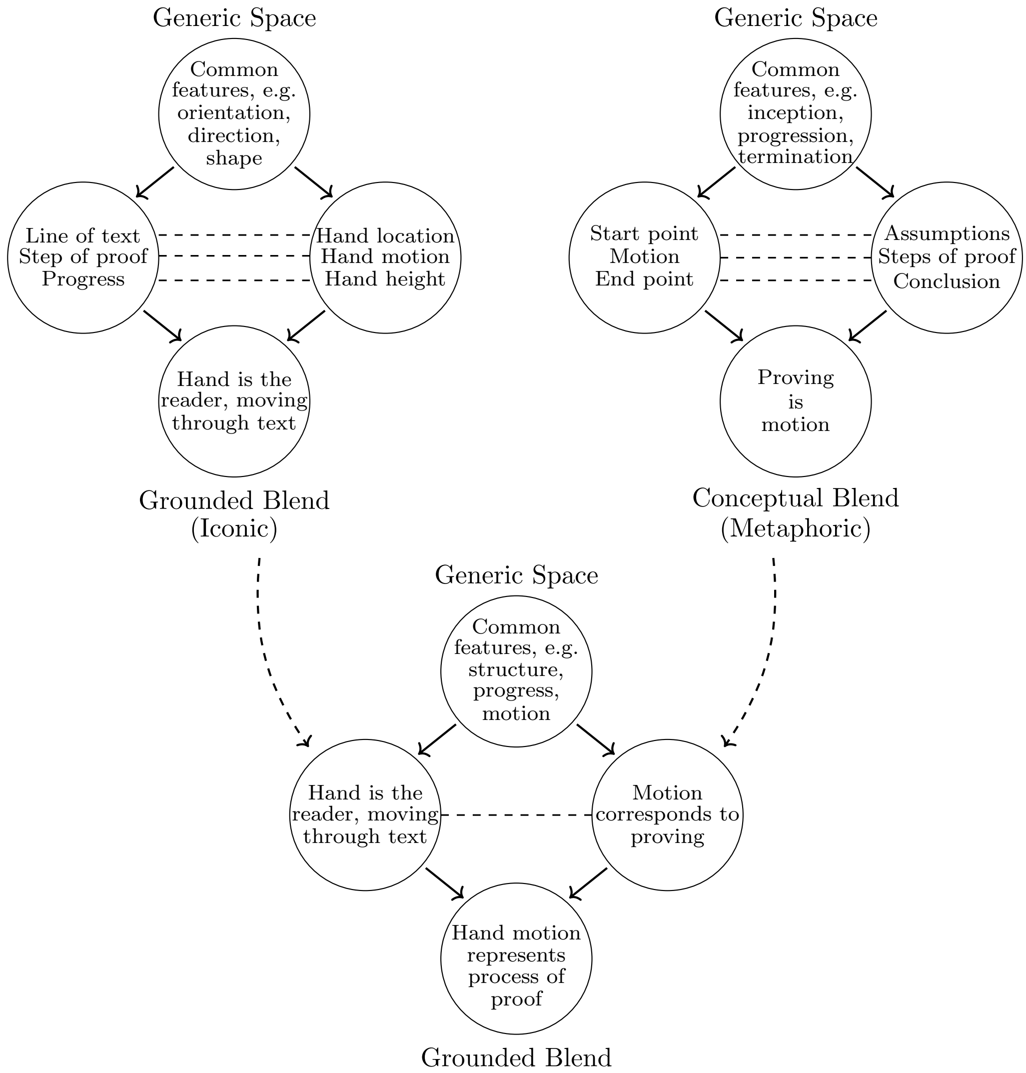

FIGURE 6. Assignment of meaning to the *Figure 5* gesture, using Parrill and Sweetser's blending model [20].

schemes. The *Figure 5* gesture illustrates a *chain of logical inference* by representing individual steps in a proof as consecutive horizontal slices. The gesture supports *operational thinking* (an "application of mental operations that are goal oriented and anticipatory" [10]) though its direction and dynamism: the gesture has a definite termination point (near the abdomen), and embodies expected incremental progress to that point through a sequence of steps. Finally, the prevalence of the *Figure 5* gesture across multiple examples of series and convergence tests indicates that the student considers this process a *general* one.

7. Discussion

The brain is the locus of physical movement, perception, and abstract thought: gestural analysis gives educators a unique perspective on the silent (and possibly subconscious) thinking and reasoning processes of undergraduate students. I have strived to make gestural analysis a standard part of my teaching routine, and will actively encourage my students to make gestures to facilitate their learning: to hold ideas in their hands, to offload cognitive demands, and as a means of confidence building and self-expression. Gesture can be used to bridge a mathematical language barrier: to recognise the idea a student wishes to communicate, when the student's mathematical register is incomplete. It further provides a "window to the mind" [9], offering insight into students' reasoning processes and the ways they construct mathematical knowledge, as understood through the theory of grounded blends. It is a particular joy to tease out the details of why a student may frequently make a certain (metaphorical) gesture: what metaphor does this embody? What understanding of mathematics does this reveal? And how may my own understanding of mathematics change? Even at research-level talks in abstract mathematics, one may find that the speaker's hands can say more than their voice.

Gesture is one element of a complex system where multiple modalities intersect to facilitate interpretation, integration, and action [18]. These examples further the argument that teacher-student communication is dependent on extra-verbal interactions, and mindfulness of such expressions can lead to more effective and fruitful instruction, as reviewed by Roth [21]. Embodiment can be seen as one tool a student and teacher have innate access to, when expressing thoughts to themselves and to others – especially useful if that expression is pre-emptively hindered by communication issues or learning disabilities.

In a related article, Kokushkin and I examine some of the cognitive challenges and navigational strategies that undergraduate students with dyslexia identify, when reflecting on their mathematical experience [22]. From interviews, we found a persistent theme of *gesturing* as a strategy students identify in mathematics to navigate cognitive load, and organise and structure information at a high level. These strategies are not unique to students with learning disabilities (e.g. [14]), hence when we make use of the embodied resources in the classroom's *semiotic bundle* (the resources used to communicate meaning – such as symbols, gestures, phrases, or relationships – in use by the classroom; [5]), we can engage more students in mathematics at a level where reading and writing skills are not the focus, opening up more inclusive ways of learning. I encourage the reader to reflect on their embodied mathematical experiences and incorporate their own insights – alongside the examples presented in this article – into future teaching, thinking, and learning.

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