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Philippe Zaouati: Perelman's Refusal: A Novel, American Mathematical Society, 2021. ISBN: 978-1-4704-6304-5, $29.00 ($23.20 to members), 133 pp.

REVIEWED BY PETER LYNCH

The background to this novel will be known to many *Bulletin* readers. For a century, a conjecture made by Henri Poincaré in 1904 eluded all attempts at proof. In 1982, William Thurston, a Princeton mathematician, proposed a taxonomy for classifying three-dimensional manifolds. His theory, known as the geometrization conjecture, describes all such manifolds. Over a period beginning in November 2002, Grigori (Grisha) Perelman, who had been completely out of contact with the mathematical community for seven years, posted three papers on `arXiv.org` with a proof of Thurston's geometrization conjecture. Perelman's papers did not mention Poincaré but, in fact, the Poincaré conjecture is a special case of Thurston's conjecture.

The Poincaré conjecture is that all closed, simply-connected three-dimensional manifolds are topological 3-spheres. It is a key result in topology and also has important implications for cosmology: the universe is perhaps the largest three-dimensional manifold, so the conjecture is relevant to the "shape of the universe".

In 2006 the International Mathematical Union (IMU) nominated Perelman for a Fields Medal. The award was to be made at the quadrennial International Congress of Mathematicians (ICM) in Madrid in August 2006. The IMU Newsletter predicted that the congress would be the occasion when Poincaré's conjecture would become a theorem. However, Perelman indicated his intention to decline the award and IMU feared that this would cast a shadow over the congress. The IMU President of the time, Professor Sir John Ball, travelled to St. Petersburg to meet Perelman, in the hope of persuading him to accept the prize.

The above sketch sets the scene for *Perelman's Refusal*. The action of the book takes place over a few days in June 2006. The author, Philippe Zaouati, met with Professor Ball in 2014 to discuss the entire affair. While Ball was positive about the plan for a book and provided valuable input, he did not comment on Perelman's personal circumstances or on the content of their conversation, which he said was strictly confidential. The extensive conversations in the book are products of the author's imagination, but they have a great semblance of authenticity and credibility.

John Ball and Grigori Perelman met on 11th June 2006. They spent the morning in a conference centre by the Neva River and the afternoon walking around the magnificent city of St. Petersburg. The two characters interacted with empathy, each man fully aware of the sincerity and honesty of the other. Ball tried, using a number of clever and persuasive arguments, to convince Perelman that, in everyone's interest, he should come to Madrid and accept the Fields Medal. Failing that, he should permit it to be awarded *in absentia*. However, it seemed evident from the outset that Perelman's decision had already been made. This was not the first time he had declined a prize: in 1996, he had refused a prestigious award from the European Mathematical Society, and he would later reject the $1 million Millennium Prize of the Clay Mathematics Institute.

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Late in the evening of June 11th, John Ball, relaxing in an easy chair in his hotel room, falls into a reverie, imagining the thoughts of Perelman. Clearly, he has a deep respect for, understanding of and sympathy with the complex Russian. But why would Perelman turn down the honour of a Fields Medal? There seemed to be several reasons. Fame meant nothing to Perelman. He had resigned from the Steklov Institute and no longer considered himself a mathematician. He felt that he could not accept a prize intended to encourage mid-career mathematicians. He wanted nothing to do with the ICM, which he regarded as a circus, or to accept an award from the King of Spain. A more domestic reason bubbled up in Ball's reverie, forcing him to conclude that Perelman was determined: "Mamma won't go to Madrid; I won't ask Mamma to go to Madrid. No, I won't go."

The description of Ball's reverie is a worthy, and successful, attempt to provide a window on the mental workings of a mathematical genius. But is the genius Perelman or an archetype conjured up by the author? In either case, the italicized passages in the chapter make for fascinating reading. The reverie strives to plumb the mind of Perelman, to understand what enthuses him, what irks him, what infuriates him.

Mathematics was the spiritual force that impelled Perelman. As a Jew, he faced major obstacles to his mathematical development: in the Russian university system, there was systematic discrimination against Jews. However, competing in the International Mathematical Olympiad in 1982, Perelman achieved a perfect score, winning a gold medal. This gained him access, at the age of sixteen, to the School of Mathematics and Mechanics at the Leningrad State University, without the requirement to take the discriminatory admission examinations.

The Fields Medal held no value for Perelman. Money was of little interest to him; indeed, he feared it. The 1990s was a time of great economic upheaval in Russia, and he witnessed some of the unsavoury consequences: "In Russia, money always leads to violence". This alone was reason enough for him to decline the $1 million Millennium Prize.

The following morning, the two men met once more and walked together again through the streets of St. Petersburg. Anyone planning to attend the ICM in July should enjoy the narrative detail provided by the author in his descriptions of that splendid city. Although the prospects seemed remote, Ball wondered whether there was any circumstance in which the Russian would come to Madrid? Before they parted, he put one last question to Perelman; he proposed an imaginative, if highly improbable, scenario. He batted off Perelman's objection that it was hypothetical, asking him to treat the proposal in a *reductio ad absurdum* way, at which point Perelman finally said "Yes". However, the condition — which I shall not reveal — was never satisfied.

This book contains little about the mechanics of the Poincaré conjecture. It discusses Ricci flow only in a general way and readers seeking details must look elsewhere. However, an excellent popular account, with many endnotes pointing to further sources, is available [1]. The mathematical development of Grigori Perelman, his career in America, his return to Russia and his withdrawal from the mathematical community are touched upon but again a more detailed source is available [2]. Finally, an extensive article in *The New Yorker* [3] includes a detailed account of the sorry story of a paper published by mathematicians Cao and Zhu in the *Asian Journal of Mathematics*. They claimed credit for the proof of Poincaré, but their claim did not survive scrutiny: passages of their paper were plagiarised and it brought no honour to its authors.

The stimulating re-imagination of the encounter between Grigori Perelman and John Ball makes this book well worth reading. I enjoyed it greatly and can recommend it to *Bulletin* readers and, indeed, to anyone interested in the world of mathematics.

References

[1] O'Shea, Donal, 2007: *The Poincaré Conjecture: in Search of the Shape of the Universe*, Walker and Co., New York, 292pp. ISBN: 978-0-8027-1654-5.
[2] Gessen, Masha, 2011: *Perfect Rigour*, Icon Books, 242pp. ISBN:978-1-8483-1301-9.
[3] Nasar, Sylvia and David Gruber, 2006: Manifold Destiny: A legendary problem and the battle over who solved it. *The New Yorker*, August 28, 2006. `https://www.newyorker.com/magazine/2006/08/28/manifold-destiny`
[4] Podcast, 2006: In Our Time: the Poincaré Conjecture. Melvyn Bragg and guests discuss a puzzle that may explain the shape of the universe. `https://www.bbc.co.uk/sounds/play/p0038x8l`

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