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Steven H. Strogatz: Infinite Powers: The Story of Calculus, Atlantic Books, 2019.
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REVIEWED BY PJ O'KANE

In James Brennan's review [1] of Donald Marshall's book on Complex Analysis in the summer 2019 Bulletin, he discussed how there may be many different opinions on the best starting point for introducing students to a new subject. For complex analysis, they ranged from Cauchy's to Riemann's views before Weierstrass' position that it should be power series prevailed. In this book, Steven Strogatz argues that the concept of infinity should be used when introducing them to calculus. That doing so is not the norm is exemplified by Serge Lang's classic text, *A First Course in Calculus* where, while infinite decimals are mentioned in the first chapter, the word "infinity" and its fuller import for calculus does not appear until page 184, and only four times thereafter.

This is not a textbook that demonstrates that new approach, nor does it claim to be. In fact, it is not a book of mathematics. Rather, it is a book about mathematics that appeals to those, like the reviewer, who may have a reasonable grounding in the subject but not enough to be a practicing mathematician. It takes the reader through the evolution of calculus, from the why to the how, weaving in the motivations, challenges, successes and sometimes pettiness along the way, before talking about its everyday use and future direction. They are all interesting — especially the pettiness.

At the outset, he distinguishes between the concepts of "completed infinity" and "potential infinity" using the repeating decimal representation of $\frac{1}{3}$ as an example. Interpreting $0.333\ldots$ as an infinite succession of threes is an example of the former, while interpreting it as a limit that you can get progressively closer to by just adding further digits is an example of the latter. Potential infinity, he says, is more elegant and easier to work with than completed infinity because it stays in the world of the finite. He illustrates this with a quick explanation of how it neatly resolves Zeno's paradox of Achilles and the Tortoise. The trick is using limits: "With limits and infinity, the discrete and the continuous become one", he remarks. (He also resolves the paradox using straightforward algebra, by the way.)

An interesting trait of the author is that he sometimes pursues a line of thought, seemingly ignoring a potential challenge that may trouble the reader, only to address it later and instantly remove that nagging concern in a very convincing way. The effect is to achieve full closure, not only on the point being discussed but also on all the intervening points where the concentration may have been slightly distracted. It makes an interesting form of argument. He does it when emphasising the power and success of calculus based on the principle that everything can be continuously subdivided and subsequently reassembled. For a number of pages the background concern is, how is that compatible with the concept of the quantum, the antithesis of continuous, but then

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he gets to it, citing infinite decimal representation, while still but an approximation, as the reconciliation. The same clinching argument on approximations is presented later when discussing the accuracy of Newton's conclusions on orbits despite having imagined entire planets condensed to single points using infinitesimals. He sums it up nicely: "How's that for a lie that reveals the truth?"

It's when talking about the evolution of calculus that the human dimension arises. We are inclined to think of calculus as beginning with Newton and Leibniz in their two different flavours, but Strogatz goes back as far as Archimedes for its roots in his attempts to quantify Pi through successive division and reassembly, according to what became the so-called "Method". He notes that Archimedes wrote to Eratosthenes around this time saying that he used the technique more often as a means of verification than of discovery. He then makes a passing but interesting comment on how common that is in mathematical discovery: "First comes intuition. Rigor comes later." It's a phenomenon that should be familiar to former Maynooth graduates who benefited from an open invitation to bring in previously unseen problems and watch them being solved by a practicing mathematician. It was always a revelation to see how an approach could be taken, abandoned, revisited and then combined with another to eventually get to a solution whose final, succinct write-up belied the longer path to first reaching it.

Back to the history of calculus, he explains its origins as integration coming first, and differentiation following, the reverse of how it is typically taught. He explains how differentiation progressed through a contemporaneous but independent linking of algebra and geometry by the imperious Descartes and the more genial Fermat. The greats, it would seem, were not all, or at least not always, quite so noble. Descartes certainly wasn't. He considered Fermat a provincial upstart and took umbrage at his questioning of his results. Worst of all he became vengeful on discovering that what he thought was his creation, Analytic Geometry, had been formulated ten years earlier by Fermat. Unfortunately, though, it would seem that arrogance prevails on occasion, as what people are introduced to today as Cartesian coordinates were actually first developed by Fermat.

The insights into the practical applications of calculus are interesting, and one, virus treatment, is particularly topical, as he talks about how it was used in the battle against HIV. The early days of its treatment was marked by three phases: initial infection, a period of apparent remission and then, unfortunately, the terminal impact. "Apparent" is the operative word here as during what was thought to be an asymptomatic phase, sometimes lasting up to ten years, an ongoing battle was raging between the body's self-defence and the mutation of the virus. What appeared outwardly calm disguised a delicate balance between the rate at which the virus was growing and the rate at which it was being killed off by the immune system, something in the order of ten billion virus particles per day. That discovery and its modelling using calculus led to a complete revision of its medical treatment and the adoption of a three-drug regime that provided a better defence against the ever-mutating virus, turning a once-fatal disease into a manageable one. At the time of writing a major concern is the delta strain of Coronavirus. If the same approach is being taken today, how ironic that calculus may be trying to get rid of delta!

Other applications of calculus he discusses include how the FBI used it to shrink the size of fingerprint file storage by a factor of twenty, how it explains the rule of 72 used in finance as a ready reckoner for how long it takes for money to double in value (if the rate multiplied by the term equals 72), how Thomas Jefferson employed it to make a better plough, how CAT scans work and more bizarrely how, using just some shredded cheese, a ruler and a microwave oven, you can calculate the speed of light (don't bother, just Google it).

The numerous tidbits of information he discloses are wonderful. For example, related to the last point above, he casually remarks that the idea of using microwaves to heat food arose from an engineer who noticed that after an extended period beside a magnetron the peanut bar in his pocket had begun to melt. He doesn't mention what else might have been affected.

On the future of calculus, or at least on future applications, he becomes more philosophical. He talks about its role in the investigation into the incredible efficiency of DNA packaging using supercoiling even though it is definitely based on the discrete rather than the continuous; about its use along with machine learning and AI in creating a possibly unbeatable chess player and the implications of the apparent intuition involved; the Gödel-like implications for complex nonlinear systems arising from a proof by Kovalevskaya that some outcomes are fundamentally unsolvable, leading to the jarring conclusion that "determinism does not imply predictability."

It is said that John Nash was inspired to be a mathematician by reading Bell's *Men of Mathematics*, and Andrew Wiles cites stumbling across a book in a library as the catalyst for devoting his life to proving Fermat's Last Theorem. The author speaks fondly of Ms Santon who introduced him to the concept of infinity and ultimately his love of mathematics. Everyone deserves a Ms Stanton, someone who kindles a youthful fascination in a subject, any subject. Mine was Seamus McTague, our headmaster, who introduced magic squares to a group of 10-year olds and challenged them to fill one in overnight (and because it was a challenge, not homework, it was all the more inviting). It fostered a life-long interest in the subject and shows how small actions, passing remarks and casual comments by educators, especially early ones, can have profound effects that sadly, they may never be aware of.

Today we have much more at our disposal for seeding an interest in any topic or activity. Giving access to books like this may be one such catalyst for the next generation of mathematical creators, thinkers - and reviewers.

References

[1] James Brennan: *IMS Bulletin 83 (2019) 37–40*, Donald E. Marshall: Complex Analysis. `http://www.irishmathsoc.org/bull83/Brennan.pdf`

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