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Cumann Matamaitice na hÉireann

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The aim of the *Bulletin* is to inform Society members, and the mathematical community at large, about the activities of the Society and about items of general mathematical interest. It appears twice each year. The *Bulletin* is published online free of charge.

The *Bulletin* seeks articles written in an expository style and likely to be of interest to the members of the Society and the wider mathematical community. We encourage informative surveys, biographical and historical articles, short research articles, classroom notes, book reviews and letters. All areas of mathematics will be considered, pure and applied, old and new. See the inside back cover for submission instructions.

Correspondence concerning the *Bulletin* should be sent, in the first instance, by e-mail to the Editor at

`ims.bulletin@gmail.com`

and only if not possible in electronic form to the address

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Further information about the Irish Mathematical Society and its *Bulletin* can be obtained from the IMS webpage

`http://www.irishmathsoc.org/`

EDITORIAL

It is with great sadness that I mark the very recent and untimely death of Richard Timoney, a dear friend and a mainstay of the Irish mathematical community; bitter news to hear, and bitter tears to shed. A meeting to mark his sixty-fifth birthday was already arranged, to take place at TCD in May, and there will be other tributes, but for now I must note his long and efficient service as curator of the Society's website, which carries the digital version of the Bulletin. With typical thoroughness, and despite his illness, Richard very recently completed an obituary of his colleague Tim Murphy, which appears in this Bulletin, and arranged the continued care of the website (— Michael Mackey is now looking after this).

I have previously reported on the difficulties and expense involved in printing the Bulletin. The paper edition of Bulletin 81 was printed at UCD, thanks to arrangements made by Thomas Unger. At the same time, the IMS Committee was working on a plan for the future. A survey of the membership was conducted, and the results informed the deliberations of the Committee, which decided in December to cease the publication of a paper edition, with immediate effect. Accordingly, this issue, Bulletin 82, is published only on the Society's website. I would like to pay tribute particularly to Gordon Lessells of UL, who for many years has looked after the dissemination of the printed Bulletin to members and copyright libraries, and to Tony Waldron of MU, who looks after the exchanges. At one stage we had about 45 incoming exchange journals, and these were a valuable national resource, particularly in pre-internet times. The present number has dropped to 32, as exchange partners "went digital". It remains to be seen how this will develop.

To facilitate members who might wish to print the whole issue, the website will carry a pdf file of the whole Bulletin 82, in addition to the usual pdf files of the individual articles. As a further convenience (which may suit some Departments and Libraries), a printed and bound copy of this Bulletin may be ordered online on a print-on-demand basis[1]. Beginning with Bulletin 83, it is intended that

[1] Go to `www.lulu.com` and search for *Irish Mathematical Society Bulletin* or use `https://tinyurl.com/y8kgnqme`.

the page layout will be changed, reducing the page count without diminishing the content. It was not feasible to execute this change for Bulletin 82, since the content has been prepared for the current layout.

The November 2018 issue of the AMS Notices (volume 65, number 10) is largely devoted to the late Maryam Mirzakhani, and has accounts of her work by Ursula Hamenstädt, Scott Wolpert, Alex Wright and Anton Zorich. Given the lamentable state of university maths education in Ireland, it would not be unreasonable to advise a bright and ambitious undergraduate to master whatever mathematics is on offer in her school, and to think about open questions as they arise (especially the simple ones), but to concentrate her efforts on learning enough to understand what Hamenstädt has to say, and follow the path into Mathematics that Mirzakhani blazed. When asked how he knew so much, Abel is reported to have replied that he only read the masters. It remains true that the way to a commanding view of the frontiers of our ignorance is to follow the masters of our art. We live in a golden age of mathematics. More than half the research mathematicians who have ever practiced are alive today, and great progress is being made. However, viewed from the highest level, most of the workers are apprentices and journeymen. There are relatively few masters, and just a few dozen grand masters, none in Ireland. You have to find these people, understand their work, and take it from there.

Of course, members may disagree with this view, and sympathize more with that expressed by Beauzamy, in his article *Real Life Mathematics* in BIMS 48 (2002) 43-6: "Most current mathematical research, since the 60s, is devoted to fancy situations: it brings solutions which nobody understands to questions nobody asked."

Speaking of questions people ask, it seems a little odd that the global warming debate is regarded as over, despite the fact that there appears to be much analysis to be done. The ranks of the climate change deniers may well include many fools, but Ray Bates is a sober and competent Irish meteorologist, and deserves to be heard when he voices criticism of the scientific methodology behind the recent IPCC report.

Ben McKay at UCC passed on a request from the the International Mathematical Union that we bring the Heidelberg Laureate Forums to the attention of members. "At HLF all winners of the

Fields Medal, the Abel Prize, the ACM A.M. Turing Award, the Nevanlinna Prize, and the ACM Prize in Computing are invited to attend. In addition, young and talented computer scientists and mathematicians are invited to apply for participation. The previous HLFs have been an exceptional success. The HLF serves as a great platform for interaction between the masters in the fields of mathematics and computer science and young talents."

The next forum will take place in Heidelberg during September 22-27, 2019. See `http://www.heidelberg-laureate-forum.org`. Applications for participation at the 7th HLF are open in three categories: Undergraduates, PhD Candidates, and PostDocs. In Ireland the RIA and the IMS can nominate young researchers. The deadline for application is February 15, 2019.

Links for Postgraduate Study

The following are the links provided by Irish Schools for prospective research students in Mathematics:

DCU: `mailto://maths@dcu.ie`

DIT: `mailto://chris.hills@dit.ie`

NUIG: `mailto://james.cruickshank@nuigalway.ie`

MU: `mailto://mathsstatspg@mu.ie`

QUB: `http://www.qub.ac.uk/puremaths/Funded_PG_2016.html`

TCD: `http://www.maths.tcd.ie/postgraduate/`

UCC: `http://www.ucc.ie/en/matsci/postgraduate/`

UCD: `mailto://nuria.garcia@ucd.ie`

UL: `mailto://sarah.mitchell@ul.ie`

UU: `http://www.compeng.ulster.ac.uk/rgs/`

The remaining schools with Ph.D. programmes in Mathematics are invited to send their preferred link to the editor. All links are live, and hence may be accessed by a click, when read in a suitable pdf reader.

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Irish Mathematical Society Bulletin 82, Winter 2018

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NOTICES FROM THE SOCIETY

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Officers and Committee Members 2019

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	UL	Mr G. Lessells
Maynooth	NUI	Prof S. Buckley
Tallaght	IT	Dr C. Stack
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Applying for I.M.S. Membership

(1) The Irish Mathematical Society has reciprocity agreements with the American Mathematical Society, the Deutsche Mathematiker Vereinigung, the Irish Mathematics Teachers Association, the Moscow Mathematical Society, the New Zealand Mathematical Society and the Real Sociedad Matemática Española.

(2) The current subscription fees are given below:

Institutional member	€200
Ordinary member	€30
Student member	€15
DMV, I.M.T.A., NZMS or RSME reciprocity member	€15
AMS reciprocity member	$20

The subscription fees listed above should be paid in euro by means of a cheque drawn on a bank in the Irish Republic, a Eurocheque, or an international money-order.

(3) The subscription fee for ordinary membership can also be paid in a currency other than euro using a cheque drawn on a foreign bank according to the following schedule:

If paid in United States currency then the subscription fee is US$ 40.00.
If paid in sterling then the subscription is £30.00.
If paid in any other currency then the subscription fee is the amount in that currency equivalent to US$ 40.00.

The amounts given in the table above have been set for the current year to allow for bank charges and possible changes in exchange rates.

(4) Any member with a bank account in the Irish Republic may pay his or her subscription by a bank standing order using the form supplied by the Society.

(5) Any ordinary member who has reached the age of 65 years and has been a fully paid up member for the previous five years may pay at the student membership rate of subscription.

(6) Subscriptions normally fall due on 1 February each year.

(7) Cheques should be made payable to the Irish Mathematical Society. If a Eurocheque is used then the card number should be written on the back of the cheque.

(8) Any application for membership must be presented to the Committee of the I.M.S. before it can be accepted. This Committee meets twice each year.

(9) Please send the completed application form with one year's subscription to:

The Treasurer, IMS
School of Mathematics, Statistics and Applied Mathematics
National University of Ireland
Galway
Ireland

E-mail address: subscriptions.ims@gmail.com

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Timothy Gayleard Murphy

RICHARD M. TIMONEY

Tim Murphy, as he was known by students and colleagues alike, died on February 15th 2017. He was appointed a lecturer in (pure) Mathematics in Trinity College Dublin on 1st January 1965, and continued to give lectures and supervise projects until the first semester of 2016–17.

Despite his last name, Tim was English and born in London on June 26th, 1933 to scientific parents. His father Alfred was a metallurgist, became a professor in Birmingham and later Principal of Cranfield College in 1955, at a time when the College was seeking to make the transition from an RAF institution to university status. Tim's mother Helen Elaine Blanch Millar was from Jamaica and an organic chemist. Tim was sent to boarding school, Staunton in Somerset, at the early age of six and graduated with a first in Mathematics at St. John's College, Cambridge. He moved to St. Catherine's College for his PhD under the supervision of the great number theorist Louis Mordell. It is not easy to find out what Mordell was like as a supervisor, but Tim recounted recently to David Malone that his own interest in computing originated from working with Swinnerton-Dyer on code to check out number theoretic conjectures including some that led to the to the famous Birch, Swinnerton-Dyer conjecture. That would have been in the late 1950s or early in the 1960s where the computers would have been very early models. Certainly Cambridge had advantages over other universities in hosting the design and construction of some of the very earliest computers. Wikipedia gives the date 1937 for the start of a general computing service at Cambridge (presumably with human power originally rather than machines), and it was a unit within Mathematics — an arrangement espoused strongly as a philosophy by Tim over many years.

In fact Mordell is said to have formally retired in 1953, but presumably remained more or less active until he died in 1972. Mordell

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was a professor at Manchester for a long period, moving to Cambridge in 1945. As Tim was born in 1933 his first contact with Mordell must have been around the year of Mordell's retirement. While at Cambridge, Tim got to know Roger Penrose well (they shared rooms) and hatched a plan to write up Penrose's approach to tensor calculus, a plan that never came to fruition despite later efforts to employ MetaFont to typeset the notation. A major influence on Tim was a year (1962–63) he spent in Paris, where he was exposed to many mathematical ideas that seem to have fundamentally affected his interests for the rest of his life. Tim spent the academic year 1970–71 at Essex University.

Tim published very few mathematical papers and one must look elsewhere to understand the depth of the influence he had over his decades at TCD. He was an inspiring lecturer, at least for those who could get used to his style. I mean lecturing style here but it would be remiss of me not to comment on his wardrobe — an Arran sweater he wore in all weathers, decorated with a long red scarf in cold snaps. David Spearman tells me the scarf was one for the Lady Margaret Boat Club, the boat club of St. John's College, Cambridge.

In his lectures, I think he liked to declaim things in a controversial way at times and constructed his lecture courses so as to reflect his own view of the best way to do things, rather than the way that might be used most often by others. I think the course (or module in our modern parlance) that would have seen the largest interaction between Tim and his students would be the 111 Algebra course he gave for many years to first year Mathematics students. It had lecture notes printed on the kind of computer paper we used years ago (used by line printers, with blue horizontal lines and perforated edges where the tractor feed pulled the paper) and spent a huge proportion of its time on multiple choice tests. Tim wrote on the board in the copper-plate script that he also used to fill his spiral notebooks, a very distinctive and clear script. There were one or two lectures each week and then a quiz late on Friday afternoon, a computer graded quiz where the results were fitted to a bell curve. For a beginning abstract algebra course, it dealt with fairly difficult topics such as character tables.

Over the years, Tim gave quite a variety of modules in algebra and number theory to 3rd and 4th year students. Group representations was a regular one, every other year, and there were also modules shared with Michael Purser on either coding theory or cryptography (in alternate years). Tim's contribution was a somewhat complementary topic, either finite fields to go with coding theory, or information theory for the cryptography. But a really novel thing was a module called *Fermat's last theorem* delivered in 1993–94. Wiles first announced his proof on Wednesday 23 June 1993, but in fact a proof that was deemed correct had to wait until 1995. Versions of the module that Tim delivered later had less misleading titles.

It would be remiss of me not to mention Math Competitions. Whenever the (Dublin University) Mathematical Society wanted to select a team for an Intervarsity Mathematics competition, or to host one of them, they would rely on Tim to come up with the questions and to grade them. Usually Tim asked for suggestions from colleagues, but for many years he would get little help. In recent years, perhaps this changed and there were attempts to have an extra-curricular class so that students could practice problem solving. While Tim helped, he was no longer the only colleague doing the work.

In a much less mathematical vein, Tim also organised a module on *Practical Computing*, one lecture a week for a term plus a small group practical on the Unix system of the School of Mathematics. The lectures dealt briefly with many topics in computing — C, Java, web pages, LaTeX amongst them, but we turn now to Tim's interests in computing.

While it seems that Tim was always interested in computing, we begin with the UNIX system of the School that started in 1980 with a Digital pdp11/23 computer. Tim persuaded the leaders of the School (Brian Murdoch who was the Professor of Pure Mathematics and David Spearman, the Professor of Applied Mathematics) to use a windfall that came from the Bank of Ireland to buy the pdp. The windfall arose because the Banks were getting adverse publicity for their large profits, and it seems each department in TCD was to get a certain amount of money. Tim was very convinced that a Unix based computer was the way to go (an academic source code license was available from AT&T if one went through some paperwork). I think Digital was the leading manufacturer of mini computers, and the pdp was an affordable option. Besides it was the machine used by Kernighan & Ritchie to develop the C programming language, another of Tim's strongly held views being that C was the language to use (and it is the language for the UNIX operating system). Of course, by todays standards it was incredibly puny, and it absorbed our annual equipment budget to add some RAM (128K extra!) or a math coprocessor.

It was in those days quite a business to boot a pdp without any operating system, even though Tim had a complete system on tape he got from a contact at a UK university. Tim recruited the services of Brendan Lynch, who held a research position in Computer Science. Tim was good at inspiring students and others to put many many hours into the system, and Brendan was in technical charge for some years at the start.

In order for the machine to be usable, we needed a computer room to house terminals (type adm 3a with built in keyboards) and Tim was happy to donate his own office for this purpose. Serial cables were installed by students (and serial wires to staff offices came later). Indeed Tim rarely used whatever office he was assigned and often found a worthy alternative use for it (worthy in his view). In 1980, Tim still had rooms in College and operated between those and more or less public places like the tea room.

Tim believed in allowing students make almost all the day to day decisions about administering the pdp system. Many might regard it as a brave or rash way to do things, but it allowed generations of students to get experience administering a multi-user UNIX system, a deep experience that stood them in great stead for their future careers. They were typically very innovative in their approach as well. That method of running the system persisted until the early 2000s when the paranoia of the powers that be intervened. Amongst the other (non guru) Maths students who did not run the system, there were some who maybe knew enough to be involved in system administration, but many others learned a lot from using the system.

For many years Tim strongly advocated that there be a joint degree in Mathematics & Computing, one that would be much more theoretical and mathematical than the BA in Computer Science. He often referred to the theories of Dana Scott, for instance. However, Tim never could overcome the opposition of John G. Byrne

(1933–2016), the Professor of Computer Science. Byrne believed that Pascal was the appropriate language to use for introductory programming, for example, and not C. On Tim's suggestion, we did create and advertise a somewhat virtual 'Computing Option' within the Mathematics programme. It was very popular before the dot-com crash and was reliant on the fact that modules in computing for students of computer engineering (topics such as 68000 assembly language programming, databases, computer graphics) were available to Mathematics students.

A few years post the pdp's arrival, the BBC micro gained in popularity and Tim was attracted to them also, despite the fact that they used BBC basic and that was nothing like UNIX. Later, in 1989, members of staff in TCD collaborated with Apple Computers to run a scheme lending Apple MAC Plus computers to about 100 students. Tim was not formally a part of the project (which was called *Project Macintosh (**MAC**intoshes **INTO** **S**tudents **H**ands)* and the principals were Antony Unwin (of Statistics, Project Director), Lorna Harding (Sociology), Seamas O'Buachalla (Teacher Education), Myra O'Regan (Statistics) and James Wickham (Sociology). Their project was to investigate the effect on students of having (or ceasing to have) their own computer. But Tim was well enough known to them that the first year Mathematics students were one of the groups that got loans of the MACs for about 3 separate years.

In 1983 there was an announcement that "the 14th meeting of the European UNIX* Systems User Group will take place at Trinity College in Dublin with Timothy Murphy of the School of Mathematics acting as host and local organiser." It was quite a significant affair and illustrates Tim's international contacts.

An initiative that was greatly appreciated by David Simms (1933-2018), who ran our departmental research library, was Tim's idea for an online catalogue. It was initially software written by Tim based on a UNIX program called refer, but it was refined considerably by a student Simon Brown (whose father was in fact the TCD librarian at the time). Over time more has been done to it including a web interface by Sharon Murphy (no relation), but we are still using essentially the same system.

Tim was also very interested in Donald Knuth's TEX system, though he was also an early advocate of LATEX (designed by Leslie

Lamport). It seems TEX was initially developed on a pdp-10 in 1978, but a rewrite TEX82 was associated with the Pascal language and required a 32 bit computer (while out pdp11/23 and subsequent replacement pdp11/73 were 16 bit machines). Eventually we got an IBM 386 PC clone in the office and an attached Apple Laser printer to cater for LATEX (and TEX). Prior to that we had a Diablo printer, a fancy daisy wheel printer in its day, and [4] is early evidence of Tim writing a device driver for it. As with the BBC micros and MACs, we relied on serial lines and a programme called kermit to transfer LATEX source files from the pdp (where we usually used a line editor called `em`, a vast improvement on `ed`, to create the files). In those days, running a modestly long document through LATEX would take several minutes, and of course we would need to repeat the compilation to sort out compile time errors. Earlier LATEX was available on a computer Lab VAX, even harder to interact with.

Tim, an unlikely entrepreneur given his lifelong left-wing views and his membership of the Labour party in Dun Laoghaire, set up a campus company to typeset stuff using LATEX and I guess he diversified into being a distributor for AT&T Unix for IBM PCs. He was an early advocate for Linux and (in the latter 1990s) used to bulk buy Red Hat installation CDs, which he then offered to students for a nominal fee.

Another capitalist venture (I am not quite sure where the proceeds ended up) was a summer school for young students (called Summer School for Schools) that ran successfully for quite a few years on our system. It was an introduction to UNIX and C programming and largely run by students (on our terminals). Some of the participants were very young and perhaps there was a worry that some were too young, but it was a successful venture in the days before we ever thought of Garda vetting or the like. Another venture was a 'Student Summer Jobs Scheme' where Tim found interesting projects for students to work on

In fact as many papers [4, 5, 7, 6] about TEX survive as formal mathematical publications Tim wrote. I guess he was a regular at the annual meetings of TUG, the TEX User Group. He was very proud that he managed to have Donald Knuth invited as Donegall Lecturer in TCD in 1982. The Donegall lectureship is old (1668), funded by a benefaction to TCD, but unfortunately the benefaction produced income that declined in real terms over the centuries. It became a way to top up the salary of a professor of Mathematics, but even that stopped. Tim's colleague Trevor West managed to have the fund resurrected and re-purposed as a distinguished visiting lectureship. For some years there was a stellar line up of lecturers but that has also stopped again in this century.

For many years, Tim produced LaTeX notes on his advanced lecture courses and these show a remarkable appreciation of current mathematical trends, as I mentioned above. These notes are available on the web, though definitely not presented in a slick way.

Tim was a regular at the Colloquium until quite recently and would always have insightful questions for the speaker. He was also an incessant contributor to newsgroups including `sci.math` and `sci.math.research`, but I think there is no way to retrieve those contributions now.

Tim looked slimmer and trimmer than ever near the end of his life, but he was grief stricken by the passing of his daughter in autumn 2016 and soon afterwards he was diagnosed with cancer. He is survived by his wife Anne, brother Paul (who led the particle Physics group at Manchester from 1965 to 1991 and was awarded the Rutherford medal jointly with J. Thresher in 1980), two sons by a former marriage and 3 grandchildren.

References

[1] Timothy Murphy, *Corrections: "The dissection of a circle by chords" (Math. Gaz.* **56** *(1972), no. 396, 113–115)*, Math. Gaz. **56** (1972), no. 397, 235–236. MR 0500446

[2] ______, *The dissection of a circle by chords*, Math. Gaz. **56** (1972), no. 396, 113–115. MR 0500445

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[4] ______, *Diabolic TeX*, TUGboat **2** (1981), no. 3, 15–21.

[5] ______, *PostScript, QuickDraw, TeX*, TUGboat **12** (1991), no. 1, 64–65.

[6] ______, *JAVA and TeX*, TUGboat **20** (1999), no. 3, 307–312.

[7] ______, *Abstract: The Penrose notation: a LaTeX challenge*, TUGboat **21** (2000), no. 3, 291.

[8] Timothy Murphy and T. T. West, *Inner products in Banach spaces and supports of Radon measures*, Proc. Roy. Irish Acad. Sect. A **69** (1970), 55–61 (1970). MR 0259575

Richard Timoney was a colleague of Tim Murphy's from January 1980.

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President's Report 2018

Introduction and thanks

The new members of the IMS Committee for 2018 are Leo Creedon and Martin Mathieu, replacing outgoing members Bernd Kreussler and Mícheál Mac an Airchinnigh, both of whom I would like to thank for their dedicated work on the Committee.

I wish to record the Society's thanks to Anthony O'Farrell for his ongoing excellent work as editor of the IMS Bulletin. Thanks also go to the other members of the Bulletin team: Ian Short (problem page maintainer), Gordon Lessells (who oversaw printing and distribution), and the Editorial Board (Tom Carroll, Jim Cruickshank, Dana Mackey, Pauline Mellon, Ann O'Shea, Ian Short, and Thomas Unger).

Finally, I would also like to thank Richard Timoney for maintaining the Society's webpages.

IMS meetings and IMS-supported meetings

The Society's 2018 "September meeting" was held at University College Dublin (30–31 August). As usual, this functioned as both the main scientific meeting of the Society and its AGM. At the AGM, Pauline Mellon was elected as the incoming president, and Tom Carroll as the incoming vice-president, of the IMS. I wish them well when their terms start on 1 January 2019. Special thanks go to the organisers, Pauline Mellon, Chris Boyd, and Michael Mackey, for organising this event. The 2019 "September meeting" will be held in NUIG.

The Committee also met on 14 December in University College Cork. That meeting was followed immediately by the 2nd IMS Invited Lecture *Burnside Algebras of Finite Groups*, given by Götz Pfeiffer. It is my hope that this new tradition of annual invited lectures following the end-of-year Committee meetings will continue.

At the December Committee meeting, the recent survey of members was discussed. Supported by the opinions in that survey, a decision was made to increase membership rates by a modest amount. It was also decided that the IMS Bulletin should move to an electronic format in future.

The Society supported the following meetings held during 2018:

- Quantum Information Theory, UCC: February 15.
- Groups in Galway, NUIG: May 18–19.
- British and Irish Geometry Meeting 2018, QUB: June 1–2.
- International Conference on Complex Analysis, Potential Theory and Applications, UCD: June 11–15.
- Topics in Functional Analysis, NUIG: July 9–11.
- Women in Mathematics in Ireland, UCD: August 29.

Thanks go to the Treasurer, Götz Pfeiffer, for efficiently overseeing the application process, and for making the task of deciding on funding less onerous.

EMS Meetings

The annual European Mathematical Society Meeting of Presidents took place at Maynooth University on 14–15 April. The meeting included three event presentations (including one on the 2020 European Congress of Mathematicians in Slovenia) and seven national society presentations (including one by the IMS). It was a useful opportunity to discuss matters of mutual interest with other national mathematical society presidents and representatives. Various documents, including a report on the meeting, can be found at `http://archive.maths.nuim.ie/staff/sbuckley/meetings/18EMS/`

I also attended the EMS Council Meeting (Prague, 23–24 June) as the IMS delegate. A new president (Volker Mehrmann) and other officers were elected, and various items of EMS business were discussed.

Education Subcommittee

The Education Subcommittee discussed textbooks during 2018. Concern was raised about content errors in books, as well as misalignments between books and the syllabus. A system for vetting textbooks could be useful, and this matter will be discussed further.

The Subcommittee advised me with regard to the Teaching Council Subject Criteria Consultation Session which was to place in Maynooth on 14 November, and I subsequently reported to the IMS Committee. We chose Ronan Flatley to represent us at that session. Ronan kindly agreed to attend, and produced a report on the session. The Teaching Council had a mid-December deadline for feedback. On the basis of discussions within both the IMS Committee and the Education Subcommittee, I subsequently sent a letter to the Teaching Council indicating some concerns that we had.

I would like to thank the Subcommittee for its hard work throughout the year.

Stephen Buckley
December 2018

E-mail address: sbuckley@nuim.ie

Irish Doctorates Completed

The following are the names and thesis titles for PhD degrees in Mathematics completed at Irish universities in the period from the start of 2017 to March 2018. The information has been provided by department representatives, in response to an invitation. In future, departments are requested to send the information for each year to the end of March to the editor at the address below.

DCU:

Denis Patterson. Asymptotic Growth in Nonlinear Stochastic and Deterministic Functional Differential Equations.

DIT:

Eilis Kelly. Mathematical Modelling of Random Antibody Adsorption and Immunoassay Activity.
Paul O'Reilly. Mathematical Modelling of Optical Patterning in Photopolymer Systems.

UCD:

Alan Benson. Strategies for intractable Bayesian models.
Dimitra Salmanidou. Numerical modelling and statistical emulation of landslide induced tsunamis: the Rockall Bank slide complex.
Joseph Brennan. On the emergence of extreme ocean waves.
Riccardo Rastelli. Latent variable models for networks and finite mixture distributions.
Michael Fop. Advances in Model-based Clustering and Classification.
Lampros Bouranis. Advances in the Bayesian Analysis of statistical models with intractable normalising constants.
Nancy (Duong) Nguyen. Reweighting Approaches to Nonresponse in Household Surveys.

UL:

Catherine Timoney: Numerical Analysis of Singularly Perturbed Nonlinear Reaction Diffusion Equations.
Aoife Guerin: The complex system of problem solving providing the conditions to develop proficiency.

David O'Sullivan: Dynamics of Behaviour and Information Diffusion on Complex Networks: Analytical and Empirical Perspectives.

E-mail address: ims.bulletin@gmail.com

Minutes of the Annual General Meeting
held on August 31, 2018 at University College Dublin

Present: P. Barry, C. Boyd, S. Buckley, T. Carroll, L. Creedon, A. Cronin, R. Dark, M. Destrade, N. Dobbs, S. Dineen, E. Gill, B. Goldsmith, M. Golitsyna, J. Grannell, P. Greaney, T. Hüttemann, B. Kreußler, G. Lessells, R. Levene, P. Lynch, D. Mackey, M. Mackey, D. Malone, M. Mathieu, J.P. McCarthy (guest), P. Mellon, A. Mustaţă, A. O'Farrell, G. Pfeiffer, M. Rosbotham, H. Šmigoc, N. Snigireva, M. Venkova.

Apologies: J. Gleeson, C. Mac an Bhaird, E. Oldham, M. OReilly, C. Stack, A. Wood.

1 Minutes

Minutes of the last meeting were accepted.

2 Matters Arising

J. Grannell's document on the new applied mathematics programme at leaving certificate level has been forwarded to the NCCA's Mathematics Development Group (MDG). The MDG produced a draft before Christmas incorporating a number of our suggestions.
J. Grannell suggested writing to the NCCA to acknowledge this positive action. There have also been a number of meetings between J. Grannell, D. Wraith and J. Gleeson to provide PDST sessions on new parts of the course. Teachers seem to appreciate the work. Thanks were extended to the IMS Education Subcommittee and to J. Grannell, D. Wraith and J. Gleeson in particular.

3 Correspondence

The society had received an e-mail from A. Wood about arrangements for the Stokes centenary in 2019. S. Buckley noted that it would be good for the society to provide some support for events during the year. E. Gill said that he would be happy to help to coordinate and it could link in with programmes such as *Boole to School* and Hamilton events. A. Mustaţă asked if it would be possible to do a Stokes-specific version of Boole to School. P. Lynch noted he has suitable Stokes material. S. Buckley offered to coordinate between interested parties over the next few months.

4 Membership Applications

Membership applications had been approved for E. Kelly-Delahunt,

G. Slerin, J.P. McCarthy, P. Siegl, A. Perrotta, M. Bustamante, M. Jordan-Santana and N. Dobbs.

5 President's Report

- S. Buckley noted that L. Creedon and M. Mathieu had joined the committee at the start of 2018. He also thanked A. O'Farrell and the editorial board of the bulletin for their support during the year. G. Lessells and R. Timoney were also thanked for practical help with production of the bulletin.
- The committee met in QUB in December, and J. Gleeson acted as invited speaker. The next committee meeting will be held in UCC in December. The next AGM is expected to be held in NUIG.
- Six conferences have been supported by the society through the year: Quantum Information Theory (UCC, Feb 2018), Groups in Galway 2018 (NUIG, May 2018), British and Irish Geometry Meeting (BI-GeM, QUB, Jun 2018), International Conference on Complex Analysis, Potential Theory and Applications (ICCAPTA, UCD, Jun 2018), Topics in Functional Analysis (NUIG, Jul 2018), Women in Mathematics in Ireland (UCD, Aug 2018).
- The annual meeting of EMS presidents was held in Maynooth in April. S. Buckley (representing the IMS) and M. Mathieu (representing the LMS) attended the EMS council meeting in Prague. The next big EMS meeting will be held around the European Mathematical Congress in 2020.

  The EMS can support events and application forms are available on the EMS website. S. Buckley and M. Mathieu can provide information on the types of events that are suitable for funding.

6 Treasurer's Report

A report on 2017 finances were circulated. There was a small shortfall of around €600, as well as a banking error that was corrected. The main income is from membership and the main outgoings are for the bulletin, conference support, the main IMS meeting and our EMS subscription. Online access to accounts has been arranged.

The report was accepted subject to minor modifications. Thanks were extended to G. Pfeiffer.

7 Bulletin

A. O'Farrell thanked the many people involved in the editorial board and production of the journal. G. Lessells had been looking after printing in Limerick, but this arrangement was coming to an end, and the cost of printing was set to increase from the previous figure of around €550. This time the bulletin has been printed via `lulu.com` at a cost of around €870. An Irish printing operation in UCD has been identified by T. Unger who can print at a similar cost. Thanks were extended to T. Unger.

The committee had discussed the sustainability of producing printed copies of the bulletin, given printing and distribution costs. S. Buckley noted that the committee was going to conduct a survey of members to get feedback on these and other matters. B. Kreußler asked about current costs and A. O'Farrell noted that current costs were approximately €850 for printing and €300 for postage. There are also exchange copies and copyright library copies that need to be delivered. S. Dineen noted that physical copies are important, but members should be consulted. It might be possible to obtain sponsorship or spread costs by distributing copies to schools. P. Mellon agreed that there risks with electronic-only publications that can disappear from the web. C. Boyd asked if libraries can archive electronic journals.

G. Lessells noted that the bulletin and copies of the EMS booklet were available. Thanks were extended to T. O'Farrell and the team.

8 Educational Subcommittee

- The issue of the new applied mathematics programme had been covered in matters arising. There was a general impression that the NCCA was currently open to advice and input.
- The committee was currently discussing textbooks. Many books had errors in content and/or are not well aligned with the syllabus. A system for vetting textbooks could be useful, and a document will be produced.

  M. Mackey asked if the committee could recommend books. A. O'Farrell noted that this could be a substantial piece of work. B. Goldsmith noted it might also be legally complicated, and the NCCA might be better placed to address the issue with the society's support. A. O'Farrell observed that a clear definition of the syllabus is also an issue, it may be helpful for textbook authors to flesh out the syllabus to something more detailed that could be referenced by textbook authors. C. Boyd asked if the IMTA have a position, and suggested we could follow up with members with strong IMTA links. L. Creedon suggested it would be good to invite the IMTA president to the next meeting to strengthen links.

- Thanks were extended to the education subcommittee.

9 Elections

- The president and vice president positions were available for filling. P. Mellon was elected president and T. Carroll was elected vice president.
- J. Gleeson has reached the end of his term.

10 AOB

- P. Mellon noted that there will be a conference in May for R. Timoney's 65th birthday on Analysis, Geometry and Algebra.
- E. Gill noted that the 13th Maths Week will be run this year, centered on Hamilton Day, as usual. Get in touch if you plan to run events or have a venue.
- There will be a meeting for the 75th birthday of T. Laffey organised by the Irish Linear Algebra and Matrix Theory group.

IMS Annual Scientific Meeting 2018
University College Dublin

August 30th-31st

The 28th annual scientific meeting of the society was hosted by the School of Mathematics and Statistics at University College Dublin on the final Thursday and Friday in August. (This followed Women in Mathematics Day which was held on Wednesday 29th in UCD and also supported by the society.) The organisers were Chris Boyd, Michael Mackey and Pauline Mellon.

Opening welcome remarks were made by Professor Joe Carthy, principle of the UCD College of Science who encouraged members to continue their work in communicating the utility and beauty of mathematics to prospective students.

The scientific programme contained a mixture of 30 minute and 50 minute lectures and, observant of the membership, covered topics whose variety might rival the international congress. For example, we heard about blockchain protocols, connections between random matrices and enumerative geometry, Japanese lesson study, Q-reflexive Banach spaces and waves in soft tissue. (The full list of talks follows.)

Unannounced on the programme, the lecture of Professor Robin Harte on Thursday was followed by some words of tribute from our president marking the occasion of the speaker's 80th birthday as well as his many mathematical contributions. As befits such occasions, the assembly partook of chocolate cake.

Friday morning's session was comprised of students and early stage researchers where the standard of work and presentation was lauded by the members. Friday's schedule also included the society's AGM which is reported on separately in these pages.

The web page for the meeting, which includes abstracts of presented talks, is archived at

https://maths.ucd.ie/ims2018

The organisers are grateful to all who participated in the meeting but especially to our speakers whose lectures were given in the following sequence.

Thursday 30th August

Sarah Mitchell (UL) - *Numerical challenges in applications of Stefan problems.*

Emanuele Ragnoli (IBM Research) - *DAG based blockchain protocols. Early days, challenges and opportunities.*

Nikos Georgiou (WIT) - *A New Geometric Structure on Tangent Bundles.*

J.P. McCarthy (CIT) - *The Diaconis-Shahshahani Upper Bound Lemma for Random Walks on Finite Quantum Groups.*

Robin Harte - *Cofactor Matrix Theory.*

Thomas Hüttemann (QUB) - *Finite domination - from topology to graded algebra.*

Michel Destrade (NUI Galway) - *Waves and Wrinkles in Soft Solids.*

Friday 31st August

Mayya Golitsyna (UCD) - *Universal Taylor Series.*

Paul Greaney (NUI Galway) - *Instabilities in Thin Dielectric Elastomers.*

Eduardo Mota Sánchez (UCC) - *Heun's equations and CMC surfaces.*

Milena Venkova (DIT) - *A new example of a Q-reflexive space.*

Aoibhinn Ní Shúilleabháin (UCD) - *Developing Mathematical Knowledge for Teaching through Lesson Study.*

Nina Snigireva (NUI Galway) - *A tale of two norms.*

Neill O Connell (UCD) - *Some new perspectives on moments of random matrices.*

Report by M. Mackey (email: mackey@maths.ucd.ie)

Reports of Sponsored Meetings

Women in Mathematics Day Ireland 2018
29 August 2018, O'Brien Centre for Science, UCD

Organisers

School of Mathematics and Statistics, University College Dublin

Co-Chairs: Isabella Gollini, Aoibhinn Ní Shúilleabháin
Committee members: Michelle Carey, Vasiliki Dimitrakopoulou, Mark Dukes, Claire Gormley, Gabrielle Kelly, Pauline Mellon, Neil O'Connell, Adamaria Perrotta, Michael Salter-Townshend, John Sheekey, Helena Smigoc.

The Women in Mathematics Day Ireland (WIMDI) has become an annual conference since its inauguration in 2010. The conference is a multi-faceted event that includes plenary talks, academic and industrial research talks, poster sessions for students, and the centrepiece of this year's event has been the celebration and acknowledgement of Sheila Tinney's work in the mathematical sciences and marks the centenary of her birth.

Dr. Sheila Tinney was the first Irish woman to receive a PhD in Mathematics, was the first female fellow of DIAS (Dublin Institute of Advanced Studies), was one of the first four women to be admitted to the Royal Irish Academy and also lectured at UCD.

This year's event attracted almost 150 people from all over Ireland, and the audience at the event included girls from St Mark's Community School, students, lecturers and general public from around the country.

The day was opened by Minister for Higher Education Mary Mitchell O'Connor and the UCD Vice-President for Research, Innovation & Impact Prof. Orla Feely.

In celebration of the centenary of Sheila Tinney's birth, an Irish Scientist plaque was unveiled in her honour. The plaque was sponsored by the UCD College of Science and is now hanging at the entrance to the School of Mathematics & Statistics. As an additional acknowledgement of this special celebration, the national Women in Technology & Science (WITS) organisation presented the UCD School of Mathematics & Statistics with a print of the Vera Klute portrait of Sheila Tinney. Celebrations of Sheila Tinney continued with presentations from her son Hugh and daughter Ethna.

The day followed with scientific talks made to be addressed to the very diverse audience.

The second half of the morning contained two scientific and inspirational talks: Dr Doireann O'Kiely, a UCD graduate and now a postdoc in the University of Oxford spoke on her work in thin films showing how mathematical modelling enables the exploration of practical problems and Prof Sally McClean (Ulster University) gave the keynote entitled "The working model: making an impact with mathematics".

Nine students presented at the poster session which took place during the lunch break. This provided an important opportunity for the students to engage with peers, meet leaders in academia and industry and communicate their work. It also provided insight to post-primary and undergraduate attendees on potential paths for their studies in mathematical sciences. WITS sponsored a price for the best poster.

After lunch there were three more talks by Dr Sandra Collins (Director of the National Library Ireland) who spoke about how mathematics has enabled to develop a most interesting career, Prof Ailish Hannigan (University of Limerick) who spoke on her work on participant modelling in health care and Dr Anca Mustata (University College Cork) who described her research on algebraic geometry.

The second part of the afternoon session was more centred on future generations and included presentations on the Girls Maths Olympiads and two talks on initiatives about how to improve gender equality in mathematics: Dr Romina Gaburro (University of Limerick) talked about the European Women in Mathematics network and Dr Rochelle Fritch presented the SFI Gender Strategy 2016-2020.

The programme was as follows:

09:00	Registration & Tea/Coffee
09:30	**Opening Address** • Minister Mary Mitchell O'Connor (Minister of State for Higher Education) • Prof. Orla Feely (UCD Vice-President for Research, Innovation & Impact) • Unveiling of plaque for Sheila Tinney
09:50	**Celebration of Sheila Tinney** Chair: Aoibhinn Ní Shúilleabháin • Hugh Tinney and Ethna Tinney • Presentation of portrait of Sheila Tinney - Dr Marion Palmer of WITS • Dr. Doireann O'Kiely (Postdoctoral Research Assistant at Oxford)
11:00	Tea/Coffee
11:30	**Keynote lecture** Chair: Gabrielle Kelly • Prof. Sally McClean (Professor of Mathematics, Ulster University)
12:30	Lunch & **Poster session**
13:50	**Distinguished Women in the Mathematical Sciences** Chair: Claire Gormley • Dr. Sandra Collins (Director of the National Library) • Prof. Ailish Hannigan (Associate Professor of Biomedical Statistics at University of Limerick)
15.00	**Best Poster Award**
15.10	**Distinguished Women in the Mathematical Sciences** Chair: Mark Dukes • Dr. Anca Mustata (Lecturer in Mathematics at University College Cork)
15.45	**Girls Mathematics Olympiad** Chair: Pauline Mellon • Mayya Golitsyna (Deputy Leader of the Irish team for the 2018 European Girls' Mathematical Olympiad) • Anna Mustata (International Mathematical Olympiad & European Girls Mathematical Olympiad 2017) • Laura Cosgrave (European Girls Mathematical Olympiad 2018) • Tianyiwa Xie (European Girls Mathematical Olympiad 2018) • Yixin Huang (European Girls Mathematical Olympiad 2018)
16.10	**Initiatives on improving gender equality in mathematics and closing remarks** Chair: Isabella Gollini • Dr. Romina Gaburro - regional coordinator for Ireland of the European Women in Mathematics (EWM) • Dr. Rochelle Fritch - SFI Gender Strategy
16.30	Drinks reception

Poster Session

Presenter	**Other authors Supervisors**	**Affiliation**	**Title**
Nisreen Alokbi	Graham Ellis	NUI Galway	Mapper algorithm and groupoid methods for data analysis
Faiza Alssaedi	Niall Madden	NUI Galway	Numerical solution of a complex-valued singularly perturbed differential equation
Hannah Conroy Broderick	Yipin Su Weiqiu Chen Michel Destrade	NUI Galway	Wrinkling in soft dielectric plates
Leanne Durkan	Niels Warburton Adrian Ottewill	UCD	Gravitational Waves
Róisín Hill	Niall Madden	NUI Galway	Balanced norms and mesh generation for singularly perturbed reaction-diffusion problems
Emma Howard	Maria Meehan Andrew Parnell	UCD	Identifying the reasons behind students' engagement patterns
Jennifer Kelly		TCD	Whole school 'Buy in' to numeracy: Developing a new cross curricular DEIS Numeracy policy on L2LP content through lesson study methodology accessible to all.
Samyukta Venkataramanan	Stephen O'Sullivan	DIT	Interest Rate Calibration and Parameter Estimation of Affine Term Structure Models
Jinbo Zhao	Michael Salter-Townshend Adrian O'Hagan	UCD	Morbidity Risk Distributions of Common Life-Shortening Conditions Based on Genetic Data

Report by Isabella Gollini, University College, Dublin
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British and Irish Geometry Meeting 1-2 June 2018, Queen's University, Belfast

The British and Irish Geometry Meeting was held at Queen's University Belfast on 01–02 June 2018, organised by Brian McMaster and Thomas Hüttemann. The international research conference incorporated the annual Irish Geometry Conference this year, with participants and speakers from Ireland, the UK and Germany. The presentations covered the wide range of geometry research of the lively research communities in Ireland, the UK and Europe, and highlighted connections to related areas such as group theory and homotopy theory. Titles and abstracts of the talks are listed below.

Baumeister, Barbara (Universität Bielefeld, Germany): *The smallest non-abelian quotient of* $\mathrm{Aut}(F_n)$

The non-abelian finite simple group $L_n(2)$ is a quotient of $\mathrm{Aut}(F_n)$ (factor out F_n' and then reduce modulo 2). In the talk I will confirm the conjecture by Mecchia-Zimmermann that this is the smallest non-abelian finite quotient of $\mathrm{Aut}(F_n)$. On the way some other nice and new results will appear.

This is joint work with Dawid Kielak and Emilio Pierro.

Berndt, Jürgen (King's College London, UK): *Symmetries in Riemannian geometry*

Symmetry is one of the fundamental concepts in geometry. In the talk I plan to give a survey about some old and new results in Riemannian geometry involving continuous symmetries. In the first part of the talk I will motivate some concepts involving symmetry. This will lead us to homogeneous spaces and symmetric spaces, which were studied thoroughly by Felix Klein and Élie Cartan respectively. The modern and quite general question I plan to discuss is: What spaces are close to homogeneous spaces and symmetric spaces, and why might they be of interest?

Brady, Tom (Dublin City University, Ireland): *Triangulating the permutahedron*

For an Artin group $A(W)$ of finite type W, we construct a homotopy equivalence from the $A(W)$ classifying space of Salvetti to the one defined by noncrossing partitions. The construction involves the type-W associahedron. This is joint work with Emanuele Delucchi and Colum Watt.

Burns, John (NUI Galway, Ireland): *Graded Lie Algebras — their representations and applications*

Let $M = G/P$ be a rational homogeneous manifold, with P a maximal parabolic subgroup of a complex simple Lie group G. Viewing the Lie algebra of G as a graded Lie algebra in a natural way, we use some simple representation theory to give uniform (for all complex simple G) formulae for the dimension of M, the dimensions of the irreducible factors of the restriction of the isotropy representation to a Levi subgroup of P and the nef values of homogeneous line bundles on M. We also give a selection of applications of our results. This is joint work with Adib Makrooni.

Heller, Lynn (Universität Hannover, Germany): *Recent progress in integrable surface theory*

I consider surfaces in 3-space which are critical with respect to certain geometric variational problems, such as CMC and minimal surfaces and (constrained) Willmore surfaces. In this talk I want to give an overview on recent results on the construction of new examples of higher genus CMC surfaces and on the identification of constrained Willmore minimizers in the class of conformal tori. Moreover, by viewing minimal surfaces in different space forms within the constrained Willmore integrable system, counterexamples to a question of Simpson are constructed. This suggests a deeper connection between Willmore surfaces, *i.e.*, rank 4 harmonic maps theory, and the rank 2 theory of Hitchin's self-duality equations.

This talk is based on joint work with Cheikh Birahim Ndiaye, Sebastian Heller and Nicholas Schmitt.

Kang, Sungkyung (University of Oxford, UK): *A transverse knot invariant from Z/2-equivariant Heegaard Floer cohomology*

A $\mathbb{Z}/2$-equivariant Heegaard Floer cohomology of based double coverings of S^3 along a based knot, defined by Lipshitz, Hendricks and Sarkar, is a well-defined isomorphism class of $F_2[\theta]$-modules. In this talk, we will see why this invariant is a natural invariant, and is functorial under based cobordisms. Then we will observe that, given a based transverse knot K in the standard contact S^3, we have a well-defined element in the $\mathbb{Z}/2$-equivariant Heegaard Floer cohomology, which depends only on the transverse isotopy class of K, and this element is functorial under certain class of symplectic cobordisms.

Kedra, Jarek (University of Aberdeen, UK): *On qualitative counting of closed geodesics*

Let (X, d) be a geodesic metric space, *e.g.*, a complete Riemannian manifold. We consider closed geodesics passing through the basepoint $*$ in X and ask the following questions:

(1) Do they generate the fundamental group of X?
(2) If yes, then how fast?

We measure the speed of generation as follows. If there exists a number $C > 0$ such that every element of the fundamental group of X is a concatenation of at most C closed geodesics then we say that this is fast generation. On the other hand, if no such number C exists then the generation is slow.

I will present various examples and show how to answer the above questions in certain cases. This is joint work with Michał Marcinkowski.

Montgomery, Tasha (Queen's University Belfast, UK): *On the projective line associated to a strongly $\mathbb{Z}$-graded ring*

It is known that the K-theory of the projective line over an arbitrary commutative ring splits into two copies of the K-theory of the ground ring. This was generalised, by Bass and Quillen, to noncommutative rings. My aim for this talk is to give a further generalisation, by considering a projective line associated to a graded ring. The process, perhaps surprisingly, works much like in the "classical" case, however new phenomena are quickly encountered. For example, the familiar family of twisting sheaves from algebraic geometry now depends on a two-parameter construction as opposed to just one. This work is part of my ongoing PhD thesis project under the supervision of Dr. Thomas Hüttemann.

Pausinger, Florian (Queen's University Belfast, UK): *On lattices and their shortest vectors*

The hexagonal lattice gives the highest density circle packing among all lattices in the plane. In this talk I first recall the basic notions about lattices in the plane, before I construct a sequence of lattices with integer bases that approximate the hexagonal lattice. The construction uses elementary number theory and is based on particular palindromic continued fraction expansions. As an application I obtain lattices modulo N with longest possible shortest distances.

Rees, Mary (University of Liverpool, UK): *An example of puzzles and parapuzzles in complex dynamics*

Topologically, a puzzle (in complex dynamics) is a sequence of successively larger finite graphs on the Riemann sphere. Dynamics, and the iterative definition of the graphs is given by a holomorphic map f, which, for present purposes, we will take to be a rational map of the Riemann sphere. Then the first graph in the sequence, say G_0, satisfies $G_0 \subset f^{-1}(G_0)$. We then define $G_n = f^{-n}(G_0)$, so that $G_n \subset G_{n+1}$ for all n. Puzzles tend to be locally persistent. For instance if $f = f_0$ is in a family of maps f_λ parametrised by an open subset Λ of the complex plane with $0 \in \Lambda$ then it often happens (and can be proved) that G_0 can be isotoped to a graph $G_0(\lambda)$ with $G_0(\lambda) \subset f_\lambda^{-1}(G_0(\lambda))$, at least for λ near 0. We can then define $G_n(\lambda) = f_\lambda^{-n}(G_0(\lambda))$. It is then not usually true that the graphs $G - n(\lambda)$ are all isotopic. But the way in which the graphs change can often be recorded in a parapuzzle.

The most famous puzzles and parapuzzles are the *Yoccoz* puzzles and parapuzzles, so-called after J-C Yoccoz made important advances in a conjecture called MLC using them, in the 1980's. The aim is to look at these famous examples briefly and discuss how the ideas can apply to other situations and more generally.

Shakir, Qays (NUI Galway, Ireland): *Contacts of Circular Arcs Representation of Certain Torus Graphs*

We will discuss representations of surface graphs as contact graphs of configurations of circular arcs. In this representation, vertices of the graphs are represented by circular arcs in surfaces of constant curvature while their edges are represented by the contacts of circular arcs. We first review some previously known results for contact representations in the plane. Then we show that every (2,2)-tight torus graph can be represented by a circular arc configuration in the flat torus. This work forms part of a joint project with James Cruickshank, Derek Kitson and Stephen Power.

Sugrue, Danny (Queen's University Belfast, UK): *Rational G-Mackey functors for G profinite*

Rational Mackey functors for a compact topological group G are a useful tool for modelling rational G-equivariant cohomology theories. Having a better understanding of Mackey functors will enhance our understanding of G-cohomology theories and G-equivariant homotopy theory in general. In the compact Lie group case, rational Mackey functors have been studied extensively by John Greenlees (and others). In this talk we will discuss what can be shown in the case where G is profinite (an inverse limit of finite groups).

Weiss, Ittay (University of Portsmouth): *A metric formalism for topology with a view to persistence theory*

Topological Data Analysis (TDA) employs topological techniques to identify geometric features in data involving, for instance, clustering problems and the persistence of phenomena to distinguish between relevant information and noise. The motivating philosophy behind topological approaches to geometric problems involves the inherent blindness of topology to certain metric issues such as dimensionality (of data). This apparent clash of philosophies embodied by TDA is present at the outset: the techniques of TDA are topological but require a metric. Based on work of Kopperman and Flagg from around 1990 it will be shown that there is a suitable generalisation of the concept metric space giving rise to a genuinely metrically flavoured formalism equivalent to topology. This metric formalism for topology will be explored with a heightened emphasis on metric geometrical aspects.

Report by Thomas Hüttemann, Queen's University Belfast
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Irish Mathematics Learning Support Network 11th Annual Workshop
15 December 2018, O'Brien Centre for Science, UCD

The 11th Annual Workshop of the Irish Maths Learning Support Network (IMLSN) took place on December 15 2017 in North West Regional College, Derry/ Londonderry.

The theme of the workshop was *Supporting students, raising standards in maths at secondary and Higher Education level* and delegates included second level teachers, tutors and lecturers involved in mathematics and statistics support in further and higher education and more generally in third level mathematics education.

The keynote speakers were Ciarán Mac an Bhaird (Maynooth University), Chris McCallion (Letterkenny IT) and Jonathan Cole

(Queens University). In his talk *Evaluating Maths Learning Support Provision: What is the point?*, Ciarán Mac an Bhaird discussed the issues involved in gathering and analysing usage / feedback data and its usefulness in evaluating the effectiveness of Maths Learning Support Services. He focused on the vast experience of the Maynooth MLC in this regard.

Open Source Freeware as a Tool for Learning Support was the title of both a paper and practical demonstration given by Chris McCallion. They related to a teaching project underway as part of an Access Programme at LYIT, in which open source and freeware products are being used to provide both student learning support tools and assessment tools.

The paper *Formative assessment in mathematics using log books, peer assessment and reflection*, presented by Jonathan Cole considered the effectiveness of incorporating log books, with weekly worksheets, reflections and peer-feedback. These were introduced as part of the redesign of a core mathematics module for first year Engineering students. The impact of which was represented by improvements in the student experience, exam performance, preparedness for second year and overall confidence in their academic ability.

Over the course of the day there were three additional talks which detailed the impact of maths learning support services on student progression at UL (Olivia Fitzmaurice, Aoife Guerin and Richard Walsh UL), highlighted the important socio-cultural role played by maths support centres (Kirsten Pfeiffer UCG) and outlined the observations of the first full-time maths learning support tutor in Ireland (Peter Mulligan MU).

In addition the afternoon session included three TEL related parallel workshops. These practical sessions focused on different technologies and how they can be used to the enhance teaching and learning of mathematics. They were: *Open Source Freeware as a Tool for Learning Support* (Chris McCallion LYIT); *Enhanced Technologies Interactive Classroom Delivery* (Martin Peoples NWRC) and *Developing Critical Thinking and Problem Solving Skills for Mathematics* (Franz Schlindwein, Founder of Qubizm Ltd).

The workshop provided delegates with an opportunity to share experiences, discuss challenges and find potential solutions to issues encountered in maths support provision. The organisers are grateful for financial support from the Irish Mathematical Society.

Report by Fiona Lawless, North West Regional College, Derry/Londonderry
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International Conference on Potential Theory, Complex Analysis and Applications 11-25 June 2018, UCD Dublin

International Conference on Complex Analysis, Potential Theory and Applications" was held at University College Dublin on 11-15 June, 2018. It attracted 76 participants from 22 countries, and featured 18 keynote talks by international experts from Ireland, Canada, Sweden, Norway, USA, Spain, Greece, Germany, Czech Republic and Japan. In parallel afternoon sessions 39 further talks covered a wide range of themes in Complex Analysis and Potential theory. The conference marked the 60th anniversary of Professor Stephen J. Gardiner MRIA, who has contributed extensively to the fields of Potential Analysis and Complex and Harmonic Approximation.

The following list of plenary speakers and their talks provide a flavour of the conference:

1. Hiroaki Aikawa (Hokkaido University, Japan)
   Global integrability of supertemperatures

2. Catherine Bénéteau (University of South Florida, USA)
   Optimal polynomial approximants: Zeros and Limit Points
3. Jana Björn (Lindköping University, Sweden)
   Fine potential theory via analysis on metric spaces
4. Tom Carroll (University College Cork, Ireland)
   The sharp constant in the Sobolev-Poincaré inequality for a region
5. Wolfhard Hansen (University of Bielefeld, Germany)
   Nearly hyperharmonic functions and Jensen measures
6. Dmitry Khavinson (University of South Florida, USA)
   What is an "inner function"?
7. Erik Lundberg (Florida Atlantic University, USA)
   Polynomial and entire solutions to the Dirichlet problem
8. Myrto Manolaki (University of South Florida, USA)
   Behaviour of optimal polynomial approximants on the unit circle
9. Jürgen Müller (University of Trier, Germany)
   Generic boundary behaviour of Taylor series
10. Vassili Nestoridis (University of Athens, Greece)
    From universality to generic non-extendability and totally unbounded functions in new localized function spaces
11. Ivan Netuka (Charles University Prague, Czech Republic)
    Stephen J. Gardiner's contribution to potential theory
12. Joaquim Ortega-Cerda (University of Barcelona, Spain)
    Chebychev quadrature formulas in algebraic manifolds
13. Jordi Pau (University of Barcelona, Spain)
    Weak factorization of Bergman and Hardy spaces
14. Thomas Ransford (Université Laval, Canada)
    A uniform boundedness principle in pluripotential theory
15. Edward Saff (Vanderbilt University, USA)
    Energy Bounds for Minimizing Riesz and Gauss Configurations
16. Kristian Seip (Norwegian University of Science and Technology, Trondheim, Norway)
    Hardy and BMO spaces of Dirichlet series
17. Henrik Shahgholian (Royal Institute of Technology Stockholm, Sweden)
    Free boundaries on Lattice, and their scaling limits
18. Tomas Sjödin (Lindköping University, Sweden)

Some applications of Partial Balayage

The scientific programme and further details can be found at https://maths.ucd.ie/iccapta.

The organizers of the conference, Dr Marius Ghergu and Dr Hermann Render are grateful to UCD, SFI and the Irish Mathematical Society for financial support of the conference.

Report by Hermann Render, University College, Dublin
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Quantum Information Theory
15 February 2018, University College Cork

A day of talks on aspects of quantum information theory was held at UCC on 15 February 2018, jointly organised between the School of Mathematical Sciences and the Department of Physics. There were four talks, each of one hour's duration, by a range of speakers from across Europe, to an audience with from a wide variety of backgrounds. The previous evening Reinhard Werner delivered the 2018 Boole Lecture, entitled *The Simple Argument that Changed the World View of Physics*. The speakers, titles and abstracts for the workshop were as follows:

Reinhard Werner (Leibniz University Hanover, Germany)
An overview of measurement uncertainty

I will review some recent results on measurement uncertainty relations, i.e., quantitative obstructions to the joint measurability of two or more observables, which are complimentary to the usual preparation uncertainties which bar the existence of preparations which are simultaneously sharp for two observables. The basic definition of measurement uncertainty uses transport distance, and Kantorovich duality to turn the problem of determining optimal bounds into a semidefinite program. For projection-valued observables an alternative definition based on the idea of calibration is available. I then sketch a proof that the preparation uncertainty bounds are lower than the calibration measurement bounds are lower than the measurement uncertainty bounds, but that all these coincide in the case of Fourier-related pairs of observables.

Hans Maassen (Radboud University Nijmegen, Netherlands)
Ergodic behaviour of measurement sequences

We consider a general measurement on a finite quantum system, repeated infinitely often. We show that observation of the asymptotic or 'macroscopic' behaviour of the record amounts to a well-defined von Neumann (or "projective") measurement on the system. The asymptotic behaviour can be viewed as establishing itself in the course of the observation, or as being revealed. This phenomenon was known in the 'non-demolition' case and has been named by Fröhlich et al. as 'the emergence of facts in quantum mechanics'.

Rupert Levene (University College Dublin)
Complexity and capacity bounds for quantum channels

We generalise some graph parameters to non-commutative graphs (a.k.a. operator systems of matrices) and quantum channels. In particular, we introduce the quantum complexity of a non-commutative graph, generalising the minimum semidefinite rank. These parameters give upper bounds on the Shannon zero-error capacity of a quantum channel which can beat the best general upper bound in the literature, namely the quantum Lovász theta number.

This is joint work with Vern Paulsen (Waterloo) and Ivan Todorov (Belfast).

Martin Lindsay (Lancaster University, UK)
Stochastic dilation of minimal quantum dynamical semigroups

In this talk I shall describe a new approach to the realisation of quantum dynamical semigroups as Markov semigroups associated with quantum stochastic flows. Intrinsic limitations of the approach via quantum stochastic differential equations are overcome, by relying upon some holomorphy at the Hilbert space level.

The organisers are very grateful for the support received from the Boole Centre for Research in Informatics, the Irish Mathematical Society, the School of Mathematical Sciences and the Department of Physics.

Report by Stephen Wills, University College, Cork
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Topics in Functional Analysis 2018
July 9–11, 2018, NUI Galway

A three day conference entitled *'Topics in Functional Analysis'* was held in NUI Galway from the 9th to the 11th of July 2018 in celebration of the 65th birthday of Ray Ryan. The meeting attracted participants from Argentina, Brazil, Finland, Germany, Ireland, Japan, Korea, Spain and the USA and featured 18 talks. Topics covered included: mappings between spaces of analytic functions, the Bishop-Phelps theorem, Dirichlet series and geometry of Banach spaces. The speakers and titles were:

Pilar Rueda, *Weighted spaces of Lipschitz functions.*

Luiza Moraes, *On the Continuity of the Composition Operation in* $\mathcal{H}_b(E, E)$.

Barry Turett, *Renormings of classical Banach spaces in metric fixed point theory.*

Yun Sung Choi, $\sigma_a(D) - \sigma_\mu(D)$ *for a general Dirichlet series* D.

Clifford Gilmore, *Dynamics of Derivations.*

Sun Kwang Kim, *The Bishop-Phelps-Bollobas point property.*

Anna Kaminska, *Banach envelopes.*

Tatsuhiro Honda, *Bloch functions on the homogeneous unit ball in a complex Banach space.*

Mikael Lindstrm, *Volterra operators mapping between Banach spaces of analytic functions.*

Pablo Galindo, *Interpolating sequences for weighted spaces of analytic functions on the unit Ball of a Hilbert space.*

Sean Dineen, *A polynomial miscellany.*

Dirk Werner, *Equivalent norms with an extremely nonlineable set of norm attaining functionals.*

Milena Venkova, *Polynomials on Tree Spaces.*

Han Ju Lee, *On the pointwise Bishop-Phelps-Bollobas property.*

Richard Aron, *Gleason parts on various disc algebras.*

Padraig Kirwan, *20 years of Extendibility of Homogeneous Polynomials.*

Anthony Brown, *Non-homogeneous tensor products and norms of projections between spaces of polynomials.*

Ignacio Zalduendo, *On the measure of polynomials attaining local maxima on a vertex.*

The meeting highlighted the contribution made by Ray to mathematics in Ireland and abroad through his research, exposition and mentoring.

The organizers, C. Boyd, M. Mackey, R. Quinlan and N. Snigireva would like to thank the Irish Mathematical Society for its financial support for this conference.

Report by Christopher Boyd, University College, Cork
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A duplication theorem for the Hermite polynomials

HORST ALZER

Abstract. The generalized Hermite polynomials $H_{n,p}(z)$ are generated by

$$\exp(pzt - t^p) = \sum_{n=0}^{\infty} \frac{H_{n,p}(z)}{n!} t^n \quad (p \in \mathbb{N}).$$

We prove that the formula

$$H_{n,p}(az) = n! \sum_{k=0}^{[n/p]} \frac{(a^p - 1)^k a^{n-pk}}{k!(n-pk)!} H_{n-pk,p}(z)$$

holds for all integers $n \geq 0$, $p \geq 1$ and $a, z \in \mathbb{C}$. The special case $p = a = 2$ leads to the following duplication theorem for the classical Hermite polynomials:

$$H_n(2z) = n!\, 2^n \sum_{k=0}^{[n/2]} \left(\frac{3}{4}\right)^k \frac{H_{n-2k}(z)}{k!\,(n-2k)!}.$$

The classical Hermite polynomials $H_n(z)$ $(n = 0, 1, 2, ...;\ z \in \mathbb{C})$, named after the French mathematician Charles Hermite (1822 - 1901), are generated by

$$\exp(2zt - t^2) = \sum_{n=0}^{\infty} \frac{H_n(z)}{n!} t^n.$$

They can be written explicitly as

$$H_n(z) = n! \sum_{k=0}^{[n/2]} (-1)^k \frac{2^{n-2k}}{k!(n-2k)!} z^{n-2k}$$

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or by using the Rodrigues formula as

$$H_n(z) = (-1)^n e^{z^2} \frac{d^n}{dz^n} e^{-z^2}.$$

In particular, H_n is a polynomial of degree n with leading coefficient 2^n. The first few polynomials are

$$H_0(z) = 1, \quad H_1(z) = 2z, \quad H_2(z) = 4z^2 - 2,$$

$$H_3(z) = 8z^3 - 12z, \quad H_4(z) = 16z^4 - 48z^2 + 12.$$

We have the symmetry relation

$$H_n(-z) = (-1)^n H_n(z),$$

the recurrence relations

$$H_{n+1}(z) = 2zH_n(z) - 2nH_{n-1}(z), \quad H_n'(z) = 2nH_{n-1}(z)$$

and the pseudo-addition formula

$$H_n(x+y) = 2^{-n/2} \sum_{k=0}^{n} \binom{n}{k} H_k(\sqrt{2}x) H_{n-k}(\sqrt{2}y).$$

In the theory of differential equations H_n appears as a solution of the second-order linear differential equation

$$y'' - 2xy' + 2ny = 0.$$

There is a large body of literature on these functions. Indeed, since the Hermite polynomials have remarkable applications in the theory of special functions, probability theory, physics and other fields, they have attracted the attention of numerous researchers. Their main properties are collected, for example, in [1, chapter 22], [2, chapter 13.1], [5, chapter 1].

In this note, we present a duplication theorem for the Hermite polynomials which we could not locate in the literature.

Theorem 1. *For all nonnegative integers n and complex numbers z we have*

$$H_n(2z) = n!\, 2^n \sum_{k=0}^{[n/2]} \left(\frac{3}{4}\right)^k \frac{H_{n-2k}(z)}{k!\,(n-2k)!}. \tag{1}$$

Actually, we prove a bit more. We offer an identity satisfied by the generalized Hermite polynomials $H_{n,p}(z)$ which are given by

$$\exp(pzt - t^p) = \sum_{n=0}^{\infty} \frac{H_{n,p}(z)}{n!} t^n \quad (p \in \mathbb{N}). \tag{2}$$

Obviously, $H_{n,2} = H_n$. We have the explicit representation

$$H_{n,p}(z) = n! \sum_{k=0}^{[n/p]} (-1)^k \frac{p^{n-pk}}{k!\,(n-pk)!} z^{n-pk}.$$

See [3] and [4] for more information on these functions.

We show that the following extension of (1) is valid.

Theorem 2. *For all integers $n \geq 0$, $p \geq 1$ and complex numbers a, z we have*

$$H_{n,p}(az) = n! \sum_{k=0}^{[n/p]} \frac{(a^p - 1)^k a^{n-pk}}{k!(n-pk)!} H_{n-pk,p}(z). \tag{3}$$

Proof We have

$$\exp(p \cdot az \cdot t - t^p) = \exp\big(p \cdot z \cdot at - (at)^p\big) \cdot \exp\big((a^p - 1)\, t^p\big). \tag{4}$$

From (2) and (4) we obtain

$$\sum_{n=0}^{\infty} \frac{H_{n,p}(az)}{n!} t^n = \sum_{n=0}^{\infty} \frac{a^n H_{n,p}(z)}{n!} t^n \cdot \sum_{n=0}^{\infty} \frac{(a^p - 1)^n}{n!} t^{pn}. \tag{5}$$

Let

$$u_n = u_{n,p}(a, z) = \frac{a^n H_{n,p}(z)}{n!}, \quad v_n = v_{n,p}(a) = \frac{(a^p - 1)^n}{n!}$$

and

$$\delta_n = \delta_{n,p} = \begin{cases} v_{n/p}, & \text{if } p|n, \\ 0, & \text{otherwise.} \end{cases}$$

We have

$$\sum_{n=0}^{\infty} v_n t^{pn} = \sum_{n=0}^{\infty} \delta_n t^n. \tag{6}$$

Applying (6) yields

$$\begin{aligned}\sum_{n=0}^{\infty} u_n t^n \cdot \sum_{n=0}^{\infty} v_n t^{pn} &= \sum_{n=0}^{\infty} \sum_{\nu=0}^{n} u_{n-\nu} \delta_\nu t^n \\ &= \sum_{n=0}^{\infty} \sum_{k=0}^{[n/p]} u_{n-pk} \delta_{pk} t^n \\ &= \sum_{n=0}^{\infty} \sum_{k=0}^{[n/p]} u_{n-pk} v_k t^n. \qquad (7)\end{aligned}$$

A comparison of the coefficients of the power series given in (5) and (7) reveals that

$$\frac{H_{n,p}(az)}{n!} = \sum_{k=0}^{[n/p]} u_{n-pk} v_k = \sum_{k=0}^{[n/p]} \frac{a^{n-pk} H_{n-pk,p}(z)}{(n-pk)!} \frac{(a^p-1)^k}{k!}$$

which is (3).

Remark 1. If we set $p = a = 2$ in (3), then we obtain the duplication formula (1).

Remark 2. The referee wrote: "The choice $a = i$ is also interesting when $p = 2$." In this case we get from (3)

$$H_n(iz) = i^n n! \sum_{k=0}^{[n/2]} \frac{2^k}{k!(n-2k)!} H_{n-2k}(z).$$

Remark 3. From (3) we conclude that for $n \geq 0$, $p \geq 1$ and $a, z \in \mathbb{C} \setminus \{0\}$ we have

$$a^n \sum_{k=0}^{[n/p]} \Big(1 - \frac{1}{a^p}\Big)^k \frac{H_{n-pk,p}(z)}{k!(n-pk)!} = z^n \sum_{k=0}^{[n/p]} \Big(1 - \frac{1}{z^p}\Big)^k \frac{H_{n-pk,p}(a)}{k!(n-pk)!}.$$

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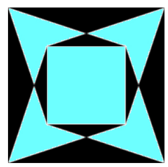

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André Darré, an early Maynooth teacher of mathematics

ROD GOW

Abstract. André Darré was an exiled French priest who worked at Maynooth from 1797 until 1813, when he returned to France. He taught natural philosophy from 1801 and he was responsible for providing his students with rudimentary instruction in mathematics. His main contribution to education in Ireland is the writing and publication of his book "Elements of geometry, with both plane and spherical trigonometry" (Dublin, 1813). This article explores Darré's career in Ireland, and his attempts to regain his home in the south-west of France. Our main source is letters written by Darré to a fellow exiled priest, which were published in 1910, but are probably not well known. The letters shed interesting light on the lives of French exiles in Dublin following the disruptions caused by the French Revolution.

The Royal College of St Patrick, Maynooth, was established by an act of the Irish Parliament in June 1795. In the aftermath of the French Revolution, several exiled French priests found refuge in Ireland and were appointed to academic positions at the college during its early years. One such émigré was André Darré, who taught Natural Philosophy from 1801 and provided his students with a textbook on geometry in 1813, shortly before his return to his home in Auch in the south-west of France. In this essay, we will provide some details about Darré's life in Ireland, his work at Maynooth, and his subsequent departure for France. Darré is mentioned with approval in *Maynooth College: its Centenary History*, written by the Reverend John Healy in 1895 to celebrate the centenary of the foundation, [9]. Likewise, Patrick Corish in his bi-centenary history *Maynooth College 1795-1995* makes occasional reference to Darré, [5]. We will use letters written by Darré to a fellow exiled priest to give a picture of his life among a group of French émigrés led by the de Basterot family, who resided in North Cumberland Street in Dublin, and of

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his activities during the years of enforced separation from his family and friends in Auch. Not the least aspect of this story is his burning desire to end his exile, despite enjoying what appears to have been a fulfilling and successful life in Maynooth.

Darré

(Image courtesy of St Patrick's College, Maynooth)

Let us first account for the presence of French priests at the foundation of Maynooth College. We have used [10] for background information on the history of France in the late 18th century. In the wake of the French Revolution in 1789, governments in France were antagonistic to the Catholic Church, and to the Christian religion in general. By 1791, clergy in France were required to swear an oath of loyalty to what was known as the Civil Constitution of the Clergy. The provisions of the Civil Constitution required a considerable reduction in the number of bishoprics, popular election of bishops and priests, and severance of the ties that bound the Church to Rome. (In 1801, a Concordat was signed between France and the Papacy, and subsequently there was a gradual return to the status quo ante in relations between the state and the Catholic Church in France.)

Papal authority was necessary to make the oath, but in April 1792, Pope Pius VI denounced the Civil Constitution. This led to a split in the French church, as many clergy, including seven bishops, agreed to sign, whereas many others refused. Those clergy who refused to take the oath were called *non-juring* or *refractory* priests. Legislation of August 1792 stipulated that refractory priests must either leave France or face deportation to French Guiana. Indeed, denunciation by twenty or more citizens was enough to precipitate deportation, a process which became known as the *dry guillotine*, since it usually resulted in the rapid death of its victims. The future prospects of refractory priests certainly looked bleak when over 200 of them were killed in a massacre of September 1792 during a wave of mob rule in Paris, prompted by fears of foreign invasions of France. Numerous refractory priests chose to emigrate from France, and André Darré was one of their number.

For us, Darré's most interesting and valuable contribution to education in Ireland is the writing and publication of a textbook entitled *Elements of Geometry, with both Plane and Spherical Trigonometry. Designed for the use of the Students at the R. C. College, Maynooth.* The work was printed and published in Dublin in 1813 by H. Fitzpatrick of 4, Capel Street, printer and bookseller to the college.

Let me explain my special interest in Darré and his book. In April 2017, I attended the Huxley Lecture on the History of Mathematics, given in Maynooth, and went on a guided tour of the Russell Library to see an exhibition of mathematical texts. One of the books on display was the Maynooth copy of Darré's *Elements of Geometry*. I wondered subsequently how easy it might be to obtain my own copy of this book. I checked the online bookselling site Abebooks but, not surprisingly, initially found nothing. A few days later, I checked again and, to my great amazement, discovered a French bookseller offering what appeared to be a very attractive example of the desired work. I had no hesitation in seizing what was likely to be the only opportunity to purchase this rare survival from the early days of Maynooth.

The book is attractively bound in full contemporary calf, with floral motifs in gold around the edges of the front and back boards. A binder's ticket on the front pastedown reveals that the book was bound by a certain Davet, located at the bookseller Delcros, in the rue Camarade, near the Collège, in the town of Auch. This information suggested a link to Darré himself, since he had been employed at the Collège royal in Auch before he left France, and he returned there in 1813. Probably, Darré brought this copy of his book with him and had it bound in an elegant manner in Auch, where much of his life was spent.

Next to the binder's ticket, a bookplate indicates that this book was formerly in the library of Monsieur Jean Barada, a specialist in military, revolutionary and Napoleonic history. A Google search quickly led to Jean Pierre Barada, a politician born in Auch in 1789. While he may not have previously owned the book, the connection to Auch fits well with our theme. Further online searching showed that a certain Jean Barada was writing in 1926 in the historical and military review *Carnet de la sabretache* on the subject of letters written by an enlisted man of the Revolutionary era (sabretache is an item worn by a cavalry officer). This Jean Barada is a promising candidate to be identical with the Jean Barada named on the bookplate.

A previous owner of the book (presumably Jean Barada) has researched Darré's life and left a few relevant documents in the book for later readers to find and perhaps investigate. We will write more about this in the course of this article. Most significant for us was a thin sliver of paper, bearing writing difficult to decipher, but making reference to the French journal *Revue de Gascogne* for 1910. (The *Revue de Gascogne* was published in Auch, and its yearly volumes relate much history pertinent to the town and its institutions, especially churches and the clergy. Perusal of these journals may throw further light on Darré.) Luckily, the *Revue de Gascogne* is available online in scanned form and I discovered the relevant article is *Lettres d'un prêtre auscitain réfugié en Angleterre*, written by P. Gabent (who may have been a priest), [7]. The prêtre in question is André Darré, and thus the title of the paper is inappropriate, as Darré was a refugee in Ireland, and only briefly resided in England.

As the title suggests, the journal article concerning Darré is based on letters, nine letters to be precise, that Darré wrote to his friend from Auch, the abbé Lubis. Lubis was a fellow priest and also exiled, in his case, in Spain. Lubis had succeeded in returning to Auch by 1804 and in the later letters, Darré relates his intense desire to come home to Auch and the vicissitudes that obstructed his return. The first letter is from Dublin, dated 17 October 1793, and addressed from 5 North Cumberland Street, Dublin (North Cumberland Street is not far from the Rotunda Hospital in north Dublin). The last letter is from Paris, dated 15 June 1813, and tells of Darré's imminent departure for Bordeaux, and then onwards to Auch.

The letters tell us something of the life of an exile in Ireland, and the first few relate news about events in France and the possibility of invasion by the allied armies of Britain, Austria, Prussia and Russia. There is also news of the extraordinary success enjoyed by the revolutionary armies of France. Darré considered the possibility of a French invasion of England to be slight, given British naval power, and he was correct in this assessment.

The last letters are more concerned with life back in Auch and the steps Darré took to repatriate himself. Indeed, for more than ten years, he devised plans to return home. It has to be said that not much information is conveyed about Darré's work at Maynooth, although one letter of 1813 does mention the geometry book. We find that Darré was well connected socially, mixing with such dignitaries as the second duke of Leinster, as well as a future prime minister of France and, apparently, the duke of Wellington.

Corish describes Darré's involvement in ending an armed uprising that occurred around Maynooth in July 1803, [5], p.45. We will be able to provide further information about why Darré might have been involved in this incident on the basis of a letter he wrote in 1804.

1. Brief biography of Darré

A short chronology of key events in Darré's life is given between pages 272 and 273 of [7]. While there are occasional inaccuracies in the information given, we will take certain dates as likely to be correct, since they are in general agreement with what we have found in the documents stored in the copy of the geometry book.

We read that Darré was born in the small town of Montaut on 5 February 1750. Montaut is situated in the Gers department, in the Occitanie region of south-western France. It is 31 km from Auch, the capital of the department. Auch itself is almost due west of Toulouse and was the seat of a catholic archdiocese until the French Revolution. Darré studied philosophy and theology at the university of Toulouse. He received the tonsure (entered the clerical order) at Toulouse on 16 March 1771 and was ordained a priest at Auch on 17 December 1774.

He was appointed parish priest of Saint-Cricq, near Auch, on 11 October 1790, vacating the position at the time of his exile in 1792 or 1793. He was also a teacher of philosophy at the Collège royal d'Auch, but we are not certain when he was first appointed. Following his return to Auch in 1813, he taught at the seminary in the town and was appointed a titular canon at the cathedral in October 1823. (He probably taught at the seminary as well before he left France.) He died at Auch on 30 January 1833, a few days short of his 83rd birthday.

We do not have any details about how Darré found refuge in Ireland or even when he embarked on his exile. The exodus of refugees from France reached its greatest intensity in the autumn of 1792, not least because of the fear of death, imprisonment or deportation, and we surmise that Darré departed his native land during this time of frightening persecution. In his letter of 17 October 1793, he relates that the previous winter (1792-93), he earned 50 gold louis by giving French lessons and he hopes to resume his teaching activities next winter. In the same letter, Darré writes that he has spent the month of August 1793 at the home of a rich lord, in the centre of the Irish countryside, but his host died a week after Darré's departure from his castle. Certainly, therefore, Darré was established in Ireland by the summer of 1793.

The Trustees of Maynooth College appointed him professor of logic, metaphysics and ethics at their meeting to decide staff appointments on 27 June 1795 (two days after the college president had been appointed). Healy states however that Darré did not begin his professorial duties until 1 May 1797, [9], p.200. We do not know why Darré took up his chair late, but the delay may relate to his naturalization, as we shall describe in the following section.

The Trustees had intended to establish a chair of mathematics at the foundation of the college, but no appointment was ever made to this chair. Instead, the Natural Philosophy course included topics in elementary mathematics, such as algebra, geometry and conic sections. The first professor of Natural Philosophy was another exiled French priest, Pierre-Justin Delort. He began his lectures as soon as he took up his position, on 6 October 1795, although his initial class consisted of only three students. In February 1801, Delort took leave of absence from the college for an approved period of six months, and Darré assumed Delort's teaching duties. Delort neglected to return to Maynooth, despite entreaties from the Trustees, and on 1 October 1802, Darré was formally appointed to the vacant professorship.

House of Commons reports of 1808 indicate that Darré received a salary of 85 pounds per annum, together with commons, lodging and the cost of coal and candles. The president of the college, Dr Byrne, by contrast, received 227 pounds and ten shillings, with additional advantages, such as the use of a servant.

2. Naturalization in 1796 and the de Basterot family

We came across an unexpected source concerning André Darré in the course of internet searching, namely, *Letters of denization and acts of naturalization for aliens in England and Ireland, 1701-1800*, [12]. In Appendix V, p.239, we find that on 8 July 1796, Darré was naturalized as an Irish (or British) citizen in the Court of Chancery, Dublin. The entry reads: *Andrew Darré, a French emigrant priest, native of the city of Montant [sic, Montaut] in the province of Gascony, France, formerly Professor of Philosophy in the Royal College of the city of Auch, and a parish priest in the same city, but now of North Cumberland Street in the city of Dublin.*

In addition to Darré, there are records of the naturalization of former French citizens, resident either in North Cumberland Street, or in nearby streets, and two of these are relevant to Darré. On 8 July 1796 there appears *Bartholomew de Basterot, Knight, and late a member of the Parliament of Bordeaux, France, and one of the Councillors of Lewis the Sixteenth, late King of France, but now of North Cumberland Street, Dublin.* A few days later, on 19 July 1796, we see the name of *James de Basterot, of North Cumberland Street, Dublin, Esq.* Somewhat curiously, Bartholomew de Basterot is listed as having already been naturalized on 1 July 1795. The reason for this duplication is unclear, unless a mistake in the legal process had occurred.

The parliament (*parlement*) of Bordeaux, mentioned above, was a court of final appeal in the French judicial system. There were thirteen such parlements in France, with the most important in Paris. The parlements were abolished in 1790.

The de Basterot name is important for our account, as it occurs in a letter of 1 March 1794 that Darré wrote to the abbé Lubis. Darré recounts that he enjoys the company of a circle of French people in Dublin, and resides in the home of a French family. Unfortunately, his hostess, Madame de Basterot has just died at the age of 27, to Darré's great consternation and sadness. She was the second wife of Bartholomew (Barthélémy) de Basterot, and James (Jacques) de Basterot was the son of Bartholomew and his first wife, Frances, née French.

The de Basterots were part of a wealthy family resident in Bordeaux, who had left France in 1792. They have been well researched by genealogists and historians interested in Co. Galway landed gentry, such as the Frenchs and O'Briens, who were related by marriage to the de Basterots. We shall use the genealogical sources [3] and [8], as well as [2], to tell how the de Basterots came to be in Ireland, and how their life in Ireland evolved. Their story will help to illuminate Darré's story, and provide details of the milieu in which he moved.

An indispensable source for our exploration of the de Basterot family is the memoir *Souvenir d'enfance et de jeunesse*, [2], published in 1896 by Count Florimond de Basterot (we shall abbreviate this name to FdB). FdB (1836-1904) was the great grandson of Bartholomew de Basterot and grandson of James de Basterot. More details about him will emerge as we pursue our course, but we want first to explain the presence of his ancestors in Dublin in the 1790's.

Chapter 1 of FdB's memoir provides us with valuable information that fleshes out Darré's bare bones account of life in Dublin in the 1790's. We did not find perfect accord with the facts related, but then FdB wrote one hundred years after Darré. Darré's name is not mentioned in the memoir but we are able to form some idea of the nature of the society in which Darré lived between 1793 and 1796.

The story begins with a certain James French, a wealthy Irish gentleman who owned extensive estates around Duras (also written as Dooras or Durus), near Kinvara, in Co. Galway. French was born in 1716 and, as a wealthy Catholic living during the penal times, he was aware of the precariousness of his position in Ireland. He chose to live as little as possible in his homeland, preferring to reside with his family in Bordeaux. His daughter Frances (Fanny) married Bartholomew de Basterot in 1770. Bartholomew was the son Jean Baptiste de Basterot (1710-1786), who was president of the parlement at Bordeaux for almost fifty years. Jean Baptiste owned various mansions and chateaux, as well as estates in the winegrowing region of Médoc. Frances died in 1775 or 1776, leaving Bartholomew with one child, his son James.

Before the death of his father, Bartholomew made a second marriage, this time to Marguerite Ursule de Sans, a rich orphan, born in Saint-Domingue to parents from Bordeaux. Saint-Domingue was the French name of the western half of the island of Hispaniola, and is now the country of Haiti. It was a French colony during the 18th century, where a black slave labour force produced much of France's sugar and coffee. Fortunes were made in Bordeaux by trade in slaves and the importation of goods from Saint-Domingue. A slave revolt broke out in Saint-Domingue in August 1791, and the country was the scene of much military conflict between slaves and the armies of Britain, Spain and France, fighting each other. The independent country of Haiti emerged in 1804.

James French is reported to have settled the large sum of 500,000 francs on his daughter as her dowry. At the time of his death in 1786 only an insignificant part of the dowry had been made over to the de Basterots, but Bartholomew and his son believed that they were still entitled to this money, despite the death of Frances. With the outbreak of the Revolution, which made life dangerous for aristocrats, the de Basterots decided to come to Ireland to press a legal claim to the Duras estates of the deceased James French. They sailed from Pauillac (a small town in the Médoc region) in an English vessel, perhaps in the summer of 1792. They only just left Bordeaux in time, for soon after their departure, many former members of the Bordeaux parlement were imprisoned and subsequently executed. (In July 1792, non-juring priests in Bordeaux were facing summary death at the hands of unruly mobs, but the worst of the revolutionary violence in Bordeaux occurred in 1793 and 1794.) The slave revolt in Saint-Domingue would have rendered the estates there worthless, and this might well have encouraged the voyage to Ireland. Subsequently, as fugitives from France, their estates around Bordeaux would also have been forfeited until the time of the Bourbon restoration, and even then, only partial compensation was made available for estates sold to others.

FdB's memoir converges briefly with Darré's account in his letter of 1 March 1794 and we are able to make a good connection between

Darré and the de Basterot family. Darré informs his correspondent that Madame de Basterot's husband is engaged in a law suit, worth a considerable sum of money to his family, and the pursuance of his claim, more so than the Revolution, has brought him to Ireland. Darré observes that justice in Ireland, while ten times slower and thirty times more expensive than it was in France under the regime of the parlements, will eventually grant him all or part of what he is seeking, as he has an incontestable right in the eyes of the judges themselves. Nonetheless, it may happen that Monsieur de Basterot has to go to Saint-Domingue, where his wife had valuable residences, whose rents he now enjoys, to restore the residences if they have been devastated (during the revolt) and to administer them. Darré even confides that he may accompany de Basterot on his mission, although this event is by no means certain and can not take place for at least a year. We would comment here that such a plan seems highly implausible, as Saint-Domingue was surely in a state of chaos by this time and extremely dangerous for outsiders.

Returning to the memoir, FdB writes that his great grandfather established himself in a house with a big garden located in the north suburbs of Dublin. This is probably 5 North Cumberland Street, although the address is not given. He seems to have integrated into Dublin life quickly, as he was given a commission in the Rotunda Division, part of the Dublin Militia. He was well received by aristocratic society, which was very favourable to exiles, and became friendly with members of the Beresford family, whose chief was the marquis of Waterford. Bartholomew's house became a centre for émigrés less fortunate than himself. One such émigré was the abbé de Broglie, who is said to have been Bartholomew's commensal for more than a year (commensal means that they shared meals–we do not know if they lived in the same house). The de Broglie family is an aristocratic one in France and FdB wonders about the relationship between this abbé de Broglie and the current duke. It is a pity for relating this history that it is not the abbé Darré mentioned, but we do at least see the presence of well connected French priests in the Dublin society in which Darré and the de Basterots found themselves.

We observed two discrepancies in the accounts of Darré and Florimond de Basterot. FdB says that Marguerite de Basterot was delicate and could not stand the rainy Dublin climate, and that she died in 1795. We know from Darré's letter that she died in early 1794. Darré also states that her sister was an Ursuline nun who lived with him and his companions. FdB on the other hand says it was Batholomew's sister who was an Ursuline nun, called Félicité. We are more inclined to believe this version, as FdB gives further details, such as the fact that she had become deranged on being expelled from her convent in France, and that she lived with her brother for several years at the Duras estate, where her main company took the form of caged bullfinches and goldfinches.

Both sources agree that the legal proceedings against the French estate at Duras were expensive and slow moving. The matter was not settled in the de Basterot favour until 1796. Indeed, such were the costs that a substantial portion of the Galway estate had to be sold, to Robert Gregory of Coole Park, and Mark Lynch of Galway. FdB mentions that he had in his possession at Duras a single lawyer's account, which amounted to more than 30,000 francs, so Darré was correct in his assessment of excessive fees. FdB also refers to his great grandfather's naturalization, necessary since foreigners would otherwise have to wait several years before they could hold property in Ireland.

It is not pertinent, as far as we know, to Darré's life story, but the later history of the de Basterot family is worthy of a brief description here. Following the successful action in the courts of law, James de Basterot lived in the remaining part of the Duras estate with his wife, Adelaide, née O'Brien, who came from Fairfield, a house near Aughrim, in Co. Galway. They had one son, also Bartholomew de Basterot (1800-87). He and his mother were forced to sell much more of the estate in 1850, after the death of James in 1849. Bartholomew de Basterot spent most of his life outside Ireland and died in Italy. He served in the French diplomatic corps, and was ennobled as a baron. He is known for his book on chess, *Traité élémentaire du jeu des échecs* (Paris, 1852). He married a French woman of noble birth, and had a son Count Florimond de Basterot, whom we have already introduced. Born in Paris, Florimond was a noted traveller in the Americas, and wrote an account, *De Québec à Lima*, in 1860. He was an aesthete and well connected in literary circles, both in France and Ireland. He resided part of each year in Duras, and died there. It was at Duras House in 1897 that discussions took place to start an Irish national theatre. The participants included Augusta, Lady Gregory, whose home was the nearby Coole Park, and William Butler Yeats, who often stayed with her at Coole.

Further light on the de Basterot family and their connection to Co. Galway is shed by Jerome Fahey in [6]. There is an anonymous oil portrait in the National Gallery of Ireland, entitled *Monsieur le Comte de Basterot*, not currently on display. In terms of the style of dress and hair, it is dated to 1815-25, and it portrays a man in his thirties or early forties. On this basis, it is thought to represent James de Basterot (although we are not sure if he was entitled to be called comte). The painting was presented in 1978 by Mrs J. Smith, and thus is not obviously related by descent to the de Basterot family. A reproduction of the portrait, which is not available on the gallery's webpage, is found in [3]. Interestingly enough, James is described by Fahey as a talented artist, but we are unable to say if it is a self portrait.

3. Darré's letters of 1793-1813

We shall now summarize what we think are interesting or relevant parts of Darré's letters to the abbé Lubis. As there is far too much material to describe in an article of this nature, where the emphasis is mainly on Maynooth College and Darré's life in Ireland, we have chosen to concentrate on a few topics.

The letter of 17 October 1793 informs us that he enjoys reasonably good health, and has agreeable company. His French lessons provide for his basic needs, although during the summer, the rich people, among whom he finds his clients, are away on holiday in the country (and presumably his earning prospects are negligible). He remarks that his expenses are so dear that, if he had to pay for his lodgings and food, the 50 gold louis he had earned would have been too little for even a very reduced upkeep.

As a postscript to the letter, he gives the frightful news that he has just been informed of the condemnation and execution of the Queen of France. Now, as Marie Antoinette was executed on 16 October 1793, and the letter is dated 17 October 1793 (although the postscript might be slightly later), we see that international news travelled fast, even under the restrictions placed on countries at war. Furthermore, Darré has learned that people in France who receive letters from enemy countries are liable to arrest, and he therefore intends not to write to his family and friends in France, for fear of placing them in danger.

The second letter is dated 1 March 1794, and it is the letter that we have already quoted, as it makes the connection between Darré and the de Basterot family. It deals with such topics as the activities (and inactivities) of the allied armies raised against France, the existence of Irish supporters of the Jacobins and their sometimes overzealous support of democratic principles, conditions in France, with the possibility of famine, the constitution of the French army, rumours of a French invasion of England or Ireland, together with some news of his own life. He notes that there is little financial support in Ireland for émigrés. We comment that this contrasts with the case in England, since the government there provided accommodation for exiled clergy and gave an allowance of two pounds a week for priests, and ten pounds for bishops. This largesse was partly prompted by the declaration of war between Britain and France in February 1793. See [11], p.34.

Darré expects his own situation will improve as he becomes better acquainted with the language of the country, which he only knows imperfectly and can hardly speak or follow when spoken. He also describes a good meal he enjoyed in the company of three exiled French nuns, one of whom is the Ursuline, Félicité. He joined them in a game of whist afterwards. He asks plaintively whether a little recreation is forbidden to unfortunate exiles.

The third letter in the sequence is dated 13 August 1794. Darré is replying to a letter sent by Lubis from Spain on 12 June, which he had received on 7 August. Despite the prevalence of warfare, it was still possible to send and receive mail from abroad. There is no news of Darré's life in the letter. Instead, there is sadness expressed about the fate of exiled priests and despair about the success achieved by the French revolutionary armies in their battles against the European powers, who have not acquitted themselves well. There is a brief mention of the all-powerful Robespierre, who, proud of the military victories, refuses to make peace, even though the exhausted people want it. (Robespierre was in fact executed on 28 July 1794, a few days before this letter was written.) Darré takes some comfort from news that in his part of France, there is a tranquillity compared with elsewhere and fewer political prisoners than in other towns of the kingdom.

No further letters written by Darré were known to the author Gabent from 1794 until 1803. By 1803, order had been restored to France and exiles were hoping to return to their homeland. On 29 June 1803, Darré wrote to Mademoiselle Lubis, sister of his correspondent, living in the rue du Caillau, in Auch. The letter is not published but we are informed that Darré expresses a strong desire to return home, where his parents and brothers and sisters are still living. For some reason that he does not make known, he is unable to leave Ireland at that time.

Darré's fourth published letter is dated 1 November 1804 and it does provide us with more news of his life in Ireland. He congratulates the abbé Lubis on his return to Auch, where he is reunited with his sister, Mademoiselle Lubis. Darré had left a quantity of books in Auch with a certain Doctor Cortade, who has just died. Darré asks Lubis to visit Madame Cortade, offer his condolences, and request that the books be consigned to him. He also informs Lubis that he is making provisions for the financial well being of his (Darré's) sister, based on the money he has earned and saved in Ireland.

He writes that his holidays have just finished. He has passed more than six weeks with two young gentlemen who invited him to accompany them and have paid every expense. They travelled more than 300 leagues in a carriage to visit the towns and curiosities of the south of Ireland (a league measures about 3.5 miles). Three years ago, he visited the most beautiful part of the north of Ireland. Two years earlier, he made a journey to London and passed through all of England. These two other trips were made at his own expense and were very dear, but also indispensable. The trip to the north was for the good of his health and that to England was made with the intention of passing on to France. Darré did not make the voyage to France, as the British minister advised him to delay further, and he regrets now having taken his advice. If we understand this correctly, Darré is referring to the time when the Treaty of Amiens was in operation, and many British people travelled to France for sightseeing opportunities. We wonder why Darré hesitated. Certainly, some French émigrés were able to repatriate themselves during this interlude in hostilities.

There is some interesting news about Darré's life in Maynooth in this letter. Darré confides to the abbé Lubis that just a few days ago, he lost a good friend, the most important that he had, the duke of Leinster, whom he describes as the premier nobleman in the country. Darré states that he was intimately attached to the duke and his family. His castle is only 25 minutes away from the college and Darré dined there at least once a week. The duke has left an immense fortune in property. Darré joined the long funeral cortege that led to the duke's burial place, on his lands seven or eight leagues from the college. (We remark that there is a portrait of William Fitzgerald, 2nd duke of Leinster (the duke in question) in the National Gallery of Ireland.)

We add a few pertinent comments of our own here. The duke of Leinster had died on 20 October 1804, 11 or 12 days before this letter was written. He was a good friend of Maynooth College, which was built on land leased from him. Healy relates that Dr Delahogue, a French-born professor of dogmatic divinity at Maynooth, was also a great favourite with the Leinster family and frequently an honoured guest at Carton House, where he must surely have met Darré, [9], p.193.

The duke and Darré figure in an incident that occurred in 1803. An armed uprising was planned to take place around Maynooth on 23 July, perhaps leading to further action in Dublin. It is said that the duke was informed of the likelihood of a rebellion and informed the authorities in Dublin Castle. The position of the rebels in Maynooth rapidly became hopeless and they tried to avail of an offer from the duke to surrender and disperse without repercussions. Darré took part in negotiations that led to the surrender of the rebels on 25 July. This involvement may be related to his friendship with the duke, who was well known for his liberal politics and sympathy for nationalist causes. Attempts were made to implicate the college in the uprising, not least because of Darré's intervention, but rumours of collusion were probably entertained mainly by people who disapproved of the existence of the college in the first place. One can read more about the insurrection in [4], but there is no more information about Darré's part in the surrender beyond what we have related above.

Darré's fifth letter of the series is dated 5 February 1805 and written from Maynooth. He writes that there have recently been peace proposals between the warring nations, and if these have a happy conclusion, he is irrevocably decided to return to his home country, and even if the war continues, he is still determined to make his way to Gascony. The other main topic of the letter is discussion of the difficulties of recovering his books from Madame Cortade and how to circumvent the problems that have arisen.

The sixth letter is dated 27 July 1807. It seems that there have been problems having letters delivered to France, and Darré has entrusted a packet of letters to a friend who is travelling to Bordeaux on a neutral ship. Concerning his intended return to France, Darré hopes that peace with Britain will follow the armistices made by the French with Prussia and Russia, and then he will be able to travel. He will not embark during the winter because of the danger of storms at sea and his tendency to be sea sick. He will leave for certain at the beginning of the spring, provided he can find a port where he can disembark.

He has recently met in Dublin Madame the countess MacCarthy, from Toulouse, whom he knew before his exile, and she has brought him news of friends in France. She has travelled to Ireland also in pursuance of a lawsuit. Darré has just started two months of holidays at the beginning of July. He has spent most of his time staying with friends in Dublin; he sees this as a small advantage in a country where everything is priced exorbitantly.

The next letter was written on 12 January 1810. Darré laments that he has tried a hundred ways to return to France, each time without success. He will now try a new method, the only one that can succeed in the circumstances. He is writing to General Dessolles, and is sending Lubis an unsealed copy of the letter, which he asks him to read. Darré would have written directly to the general but he has read in the newspapers that Dessolles is leading a division of the French army in Spain. (We are not sure what Darré intended Lubis should do with the letter, perhaps merely to comment, as the original letter had not yet been sent.)

We will now try to explain the motivation for this proposed course of action and provide a context for General Dessolles. Jean-Joseph, first marquis Dessolles, was born in Auch in 1767, and his family, especially his mother, were personally known to Darré. Dessolles was a distinguished soldier and politician, serving as prime minister of France for most of the year 1819 (one of the main streets in Auch is named in his honour). The family was aristocratic, forming part of the *noblesse de robe*, a more recent subdivision of the French nobility. The family name was originally de Solles, but took the more bourgeois form Dessolles after the Revolution. The general served with the army in Italy, and, as indicated in Darré's letter, led a division in Spain between 1808 and 1810, returning to France in February 1811. Thus Darré was probably correct in not trying to send the general a copy of his letter when it was written. Darré proposes instead to write to the general's mother, hoping that she will pass on his requests to her son. His letter to Lubis contains a version of the letter he intends to send to Madame Dessolles, although he admits that he does not even know if she is still alive.

In the letter, Darré implores her to bring to an end his eighteen unhappy and painful years of exile. To this end, he begs that she intervene with her son to seek a passport from the French government. Darré is enclosing the necessary documents and addresses to obtain the desired passport. He remarks that a passport of the type he requires can be obtained, as Madame MacCarthy of Toulouse, whom we mentioned above, has just left Ireland with such a document. Indeed, Madame MacCarthy has promised to help him obtain the desired passport but Darré admits that he has more confidence in obtaining it through the efforts of Madame Dessolles and her son.

The author, Gabent, of the article has no idea whether Darré's letters to General Dessolles and his mother ever reached their destination, but, in any case, the passport was not forthcoming and Darré was unable to quit the mists of Ireland for another three years. Gabent is of the opinion that the disappointments of the failure of all his efforts led to a crisis in Darré's health, and he was seriously ill in 1812.

Miraculously, on 27 January 1813, Darré was able to write that he had finally received the eagerly sought-after passport. In the eighth letter of the sequence, he describes how he is recuperating from an illness that brought him to the edge of the grave, and states that his brother will provide Lubis with the details. By order of his doctors, Darré is restricting himself to moderate occupations of the mind. In addition to writing letters to France, he is replying to all the expressions of concern on the subject of his recovery, and has even taken up again the exercises for his teaching class. Of especial interest to us, he tells his correspondent that he has undertaken the printing of a small volume of mathematics, in the English language, the manuscript of which he had prepared a long time before the illness.

We remark here that, in view of his imminent departure for France, Darré's declaration in the published preface of his book, that he has *the intention of enlarging it, at some future period, and making it a complete Elementary book on Mathematics, by the addition of the Elements of the various branches of the Arithmetical and Algebraical Calculation, and the Conic Sections* seems less than sincere. He continues, just as unconvincingly, that *My treatises on the different branches of physics, which I have composed for the use of, and adapted to the established course of Studies in the College, shall also be published in succession.* This remark, which we have transcribed as written, and which does not make complete sense, suggests that Darré may have made use of lecture notes already in his possession from his teaching days in France.

Continuing with the letter, Darré intends to leave towards the middle of the spring, and would like to travel directly to Bordeaux. He fears however that he will have to traverse the whole of England, cross to Morlaix (in Brittany), and then cross all of France (in fact, he sailed to Le Havre). He ends by anticipating the happiness of embracing his dear friend once more.

Healy records in [9] that Darré was granted a year's leave of absence from the college by the Trustees on 3 February 1813, in view of his fragile state of health and his need to rest and recover. They resolved to give him a year's salary, paid in advance. The Trustees may have been surprised to learn that Darré almost certainly had no intention of returning, although, equally, they may have guessed what his true intentions were.

The ninth and final letter is written from Paris on 15 June 1813. Darré has already written from Le Havre to his brother in Auch and expects to arrive shortly. He will travel by stagecoach to Bordeaux, partly because he has business there, and partly because he wishes to continue his voyage with three gentlemen from Béarn (in the Pyrenees), who have accompanied him from London.

Darré goes on to relate that he has spent a couple of days with General Dessolles, who has a pleasant country house a few leagues from Paris. The general was walking in his gardens and recognized Darré immediately. He has also visited some of the principal features of Paris, but has seen enough, as he is anxious to return to Auch.

This concludes our story of André Darré, based on his own words, but, in the next section, we present a little more about what happened to him following his return to Auch.

4. Darré's return to Auch

We mentioned in the opening section that various documents had been placed in the copy of Darré's textbook that we purchased. The most significant of these is entitled *Bulletin de renseignemens. Académie de Cahors*. Bulletin de renseignemens might translate as *newsletter*. Cahors is a town in the south west of France, north of Toulouse, and is the capital of the Lot department.

We do not know the significance of the Académie de Cahors in Darré's life story. The document itself gives a very brief synopsis of his life, both before 1789 and at the present. It is not entirely certain when the document was compiled, but as Darré's age is given as 66 years, the year 1816 is most likely. A date of 1814 has been written in crayon on the recto of the document, but not in a contemporary hand, and we speculate that this date is incorrect.

The document affirms that Darré was born in Montaut. It also states that he entered into public instruction (teaching) in 1777. The verso bears writing in ink by a later hand. It seems to have been compiled on the basis of a study of the archives relating to police activities in the Gers department in 1813. We again speculate that it may have been written by an interested party such as Jean Barada. It appears that Darré was a person under some suspicion by the state authorities, on account of his long absence abroad.

We read that on 23 July 1813, the Duc de Rovigo, Minister of Police in the French government, has requested details about André Darré from the prefect of Gers. The Minister notes that Darré was born in Montaut, was a priest at Saint-Cricq, and a teacher of philosophy at Auch. In accordance with what we have already narrated, he notes that Darré disembarked at Le Havre in June 1813 and was authorized to return to his family. The Minister invites the prefect to monitor Darré's conduct because of his long stay abroad.

The prefect replied on 30 July that Darré is not on the list of émigrés, that he has professed philosophy at a college in Ireland, and that he is one of the most estimable priests in the diocese. (It is possible that there were official lists of undesirable émigrés, whose return to France was not encouraged.) Furthermore, Darré lives in Auch, with Monsieur Lubis. (The address given seems to be au Caillaou; this bears some resemblance to the Rue du Caillau, where Lubis's sister was residing in 1803, according to the unpublished 1803 letter. Possibly, both spellings are wrong.) Thus, there is further confirmation that Darré was reunited with his faithful correspondent.

It may be of interest to know that the Duc de Rovigo was René Savary (1774-1833), who served as Minister of Police from 1810 to 1814. He was a successful soldier and diplomat, and a loyal supporter of Napoleon, who ennobled him in 1808.

Gabent's article contains a further story concerning Darré's resumed life in Auch. This story is somewhat imprecise and apocryphal, but may contain a core of truth. The (future) duke of Wellington led a coalition of British, Portuguese and Spanish armies from Spain into the south of France in the autumn of 1813. They had reached Toulouse by April 1814, but the operations were curtailed soon after by the surrender of Napoleon, who had been fighting in Germany, and the signing of a peace treaty in May 1814.

It seems that the British army under Wellington's command entered Auch sometime in the spring of 1814 and paraded in the town square. According to Gabent, Darré, who was present, went up to the duke and presented his compliments. Darré was greeted cordially by the duke, who clearly knew him. Gabent goes so far as to speculate that the town was spared from inhumane treatment by the invading soldiers on account of this meeting of old acquaintances, but on the basis of circulars that were published in advance of the invasion, it appears that Wellington was anxious to behave as graciously as possible if the populace did not resist. Gabent speculates that Darré may have met Wellington during the latter's appointment as chief secretary to Ireland between 1807 and 1809 (Gabent's chronology of 1805-1808 is somewhat wrong here). It is certainly true that Wellington had strong family ties to Ireland, as he was born there, and married his Irish-born wife in 1806.

5. Comments on Darré's *Elements of Geometry*

Our interest in Darré was sparked by the fact that he had written an early textbook on geometry, specifically for his students at Maynooth. The published letters have not given us much insight into his teaching or his writing of the book, although they have opened up aspects of his life in Ireland hitherto little known. Gabent does provide us with one anecdote concerning the book and its role in the science curriculum in Maynooth.

In 1870, two nephews of Darré, both of whom were priests living in the vicinity of Auch, attended the First Vatican Council in Rome. One day they visited a church in the city, where they met a bishop, who entered into conversation with them. He proved to be Irish. They told him they had an uncle, who used to talk to them of Ireland and who had spent 18 years there as a professor at Maynooth. "This must be Darré", the bishop replied. "Your uncle has left us with the reputation of a scholar; I learnt my mathematics from his treatise; we don't know a better one in Ireland and it is still found in the hands of students and teachers in Maynooth". The Reverend Nicholas Callan had brought out a revised and improved version of Darré's text in 1844, and it was to this that the bishop was probably referring.

Let us conclude this essay by giving an appraisal of Darré's text. Barry and O'Farrell make a brief mention of Darré in [1]. They opine that *The organisation of Darré's text leaves something to be desired, even apart from the quality of the English, for which he frankly begs indulgence*. Certainly, some of his terminology does not seem standard. The title *Elements of Geometry* suggests some adherence to Euclid's model, but as Barry and O'Farrell observe, Darré makes little attempt to follow the Euclidean postulates and his proofs are occasionally inadequate. There is also some discussion of Darré's work in [9], provided by Reverend Dr Lennon of Maynooth, but the focus is mainly on Euclid's shortcomings and not on Darré's merits.

One strong impression that strikes us on reading Darré's text is an emphasis on numerical work. This was relatively uncommon in geometry textbooks before the 19th century, especially those based on Euclid, which were largely deductive and eschewed the use of such concepts as angle measurement or solving triangles. On p.120, he mentions *Hutton's tables* (London, 1811) as an aid to calculations. On p.52, Darré has a discussion on π (not so denoted) and methods of approximating it. A substantial part of the text is devoted to plane and spherical trigonometry, subjects which might be considered to be more practical, say for a surveyor or astronomer. There is also a feeling that Darré had his students of natural philosophy in mind, as he mentions topics from hydrostatics to illustrate uses of mathematics.

It would be instructive to know what authors Darré studied in France and how these may have influenced his approach to writing a textbook in geometry, especially one that did not follow the English-language tradition of little or no deviation from the classical text. We described letters of 1804 and 1805, in which Darré sought to recover his library left in Auch, several years before his return. This suggests that he may have had a substantial library, one that conceivably contained mathematics books he had studied or used in his teaching at Auch.

A possible influence on Darré's approach to teaching geometry is the book *Élémens de géométrie* by Alexis-Claude Clairault (1713-1765). This was first published in Paris in 1741 and thus may have been seen, and even studied, by Darré. It was an influential text that advocated an intuitive approach, using problems of land measurement, to motivate concepts and propositions in geometry. Without such motivation, it was felt that many students might struggle with the more traditional, dogmatic approach, generally favoured in Britain and Ireland. (An English language translation of parts of Clairault's book was available to pupils at Irish National Schools by 1833, but not at the time Darré lived in Ireland.)

The author owns a copy of Thomas Elrington's edition of Euclid, published in Dublin in 1802 for the use of students at Trinity College Dublin. It is written in Latin and has no numerical aspect whatsoever, as was the case for texts that served to emphasize logical argument, not mensuration. Darré steers a course that avoids this somewhat arid, albeit traditional, road to geometry. Nowhere does he espouse a philosophy or theory of teaching mathematics, but we feel that he might have approved of Clairault's method, on the basis of his choice of illustrative material.

Darré strikes an interesting contemporary note on p.5 of *Elements of Geometry*. He writes: *Mr Laplace, in his celebrated Mechanique Celeste (sic, Mécanique céleste) has introduced a new division of the circle into 400 degrees, the degree into 100 minutes, . . . , and so on.* Now, in 1795, following scientific recommendations, the French government decreed that the right angle would henceforward measure 100 degrees. There was a similar change to a metric system of measuring time. Laplace enthusiastically adopted this extended metric system and made use of it in his *Traité de mécanique céleste*. Other French scientists were less enchanted by the new idea and it did not last long.

The first volumes of Laplace's treatise only appeared in 1799, several years after Darré left France, and we wonder how he became aware of this work, which was presumably difficult to obtain during the period of European-wide war. Possibly it could have been purchased during the truce of 1802-1803. The Russell Library at Maynooth houses several volumes of the *Mécanique céleste*, dating between 1799 and 1805, and it seemed reasonable to ask if any of these might possibly have been owned or used by Darré. We consulted the appropriate volumes in Maynooth in the course of writing this work. Two volumes had been donated by family of the mathematical physicist Arthur Conway, who worked at Maynooth and subsequently UCD, and these displayed his marginal notes, but there were no signs of earlier ownership or usage on anything we examined. We would like to thank the staff at the Russell Library for their assistance during our investigations.

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The Story of Ireland's Participation in the 29th International Mathematical Olympiad in Australia

FINBARR HOLLAND, THOMAS LAFFEY

Abstract. We present a short sequential account of the major events that led to Ireland's participation in its first International Mathematical Olympiad when this was held in Canberra in 1988 as part of the celebrations to mark the bicentenary of the settlement of Australia by Europeans.

1. Introduction

In 1788, a fleet of British convict ships sailed into Sydney harbour to land the first Europeans to settle in Australia. To mark the bicentenary of this momentous event, the Australian Government decided in 1980 to host a series of cultural events at major Australian cities. The International Mathematical Olympiad (IMO) was one such event that was approved to be held in the capital Canberra, and Mr Peter O' Halloran, of Canberra College of Education, was appointed to organise it. Not only did O' Halloran set about attracting competitors from countries in the vicinity of Australia that had never before taken part in the IMO—which, up to then, had largely attracted contestants from European countries that had a long tradition of participating in it—but, being proud of his Irish ancestry, and conscious that 20% of the Australian population was deemed to be of Irish descent, he was anxious that Ireland be represented at the 29th IMO in Canberra in 1988.

With this in mind, O' Halloran wrote to the Royal Irish Academy (RIA) in 1985, requesting information about mathematical contests being run in Ireland. His request was duly passed to the Irish National Mathematics Committee, a sub-committee of the RIA, of which both of us were then members. As luck would have it, we

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were also co-organisers of the Irish National Mathematics Contest, an annual competition for secondary school pupils in the whole of Ireland, which we had commenced in 1979. This initial contact with O' Halloran, which we followed up, was the first step in Ireland's future involvement in the IMO.

What follows is an account of the sequencing of the major events that led to Ireland's participation in the 29th IMO in Australia. It is a short historical account of the back story leading to Ireland's involvement in its first IMO, the 30th anniversary of which, *inter alia*, was commemorated in Cork at the Award Ceremony organised by the Irish Mathematical Trust in May, 2018; it incorporates elements of separate speeches each of us gave on that occasion.

2. The Formation of the Irish Mathematical Society

The story begins with the formation of the Irish Mathematical Society (IMS). This grew out of biannual meetings of mathematicians of different hues which were held during university Christmas and Easter vacations at the Dublin Institute for Advanced Studies (DIAS), established by Eamon de Valera in 1940. In the early years of its existence, the DIAS was located in 64/65 Merrion Square, Dublin, before moving to its present site at 10, Burlington Road, Dublin. These Mathematical Symposia were organised initially by John Lighton Synge, and later by John Lewis, and attracted theoretical physicists, mathematical physicists and mathematicians from university colleges north and south of the border, as well as from across the water. Those who contributed lectures received honoraria, and all attendees were able to recoup their travel expenses. These incentives to participate and contribute were greatly appreciated by those who attended, and appealed to university staff and graduate students alike, especially Irish graduates studying abroad for higher degrees who came home at Christmas and Easter when the Symposia were held. These meetings were both cultural and social events: as well as exchanging ideas in the lecture room, the latest mathematical theories and academic gossip were exchanged over a pint or two afterwards in a nearby hostelry! These occasions were the breeding ground for the inception of the IMS, which was formed in 1975. It was well established by 1977, and the annual accounts for that year show that 80 people each paid a membership fee of 2 Irish pounds.

Early in the life of the IMS a decision was taken to run a National Mathematical Contest for secondary students, to promote their interest in mathematics and increase their awareness of its importance by posing challenging mathematical problems in a competitive environment. Part of the motivation for initiating such a contest was also to identify and enter a team in the IMO, a passionate desire of the authors, who were given the task of developing this idea. They were faced with some obvious questions: What would be a suitable model for such a contest? Who would administer it? Where would it be held? Who would set the problems? Examine the solutions? Cover the cost? A myriad of similar questions would need to be examined before the idea could be brought to fruition.

3. The Origins of the Irish National Mathematics Contest

Of course, by that time, there were many mathematical contests of various kinds being run in different countries, all inspired by those run for many years in Hungary, their chief aim being to foster interest in Mathematics at second level. From about 1977 the first author (FH) investigated many of the known international contests and his searches led him, for example, to examine, in particular, the format and structure of the Scottish Mathematical Challenge, which was conducted by correspondence, and that being run in England by the Mathematical Association. The structure of the latter was appealing: it was a 90-minute multiple choice test, marked by teachers. And even better still, the problems and their solutions were imported from the USA! In September, 1978, FH contacted Mr Gray, Secretary of the UK Mathematical Association for more details, and raised the possibility of extending the same test to Irish pupils. It turned out that they were using the American High School Mathematics Examination (AHSME) paper which was administered by Dr Walter Mientka, University of Nebraska. On foot of this information, FH wrote directly to Mientka, and obtained his permission to model the INMC on this paper. Sometime in the Autumn of 1978, Mientka forwarded a master copy of the paper, and an accompanying solution booklet for AHSME 1979 to FH. Upon receipt of the master copy of the AHSME and the solutions booklet, the rest was relatively straightforward. The second author (TL) assented to it, the officers of the fledging IMS adopted our proposal to model the INMC on the AHSME, and wholeheartedly backed the idea. Both of us were appointed to implement it, and we agreed to share the administrative load between us, TL taking responsibility for administering the contest in Leinster, and FH in the rest of Ireland. By this stage, too, the Irish Mathematics Teachers Association (IMTA) was also on board, and we wish to acknowledge, in particular, the support we received from Fred Holland, of Holland & Madden fame—Fred was a highly influential member of the IMTA, and he played an important role in persuading its members to endorse the project. The cooperation of school teachers was essential to administer the examination during a suitable slot in the school timetable, and report the results to one of us afterwards. We're very conscious of all the necessary assistance we received from many different teachers over the years, without which the INMC could not have been run, and record our gratitude to all of them, who are too numerous to name.

In late 1978, letters were dispatched by both of us to secondary schools north and south of the border inviting them to enter pupils in the 1979 contest, which was to be given on March 6, 1979, the same day it was to be administered in the USA, and to pay a small fee per entry to cover postage and printing costs. Based on the response from schools, and armed with the number of entries, FH then made small editorial adjustments to the master copy, and placed an order with the UCC printer for an appropriate number of copies of the test paper and duplicates of the solution booklet.

A batch of these were fast-tracked by train to TL some days before the due examination date, and he mailed them out to schools in the Leinster area; FH likewise looked after schools in Munster, Connacht and Ulster. Results were compiled, circulated to participating schools, published in the IMS Newsletter (as the IMS Bulletin was then called), and prizes awarded to the top scorers at a ceremony organised for them and their teachers by TL in UCD early in December 1979.

So, the first INMC took place on March 6, 1979, and continued for many years thereafter with the same format, with the assistance and support of Fergus Gaines in UCD, and Donal Hurley in UCC, who took over the production and distribution of the examination papers whenever FH was absent for any reason. For instance, the 20th INMC occurred on Tuesday, 9th February, 1999, and coincided with the holding of the 50th AHSME. On the occasion of the prize-giving ceremony for the top scorers held that year on December 10, 1999, a perpetual trophy donated by the IMS was presented to the winner of the INMC. Incidentally, from 1983 onwards, a second competition, called the Irish Invitational Mathematics Contest, was offered to the top scorers of the INMC, the materials for which we also received from Mientka.

4. The Role of Peter O' Halloran

Sometime in 1985, the RIA received a copy of the 2nd issue of the Newsletter produced by the World Federation of National Mathematics Competitions, and the cover letter with it sought information about mathematics contests in Ireland. This letter came from Peter O' Halloran, who had been appointed Chairman of the Australian Commission set up to organise the IMO in Australia in 1988. Being proud of his Irish heritage Peter was anxious that Ireland participate in the celebrations, and entering an IMO team that year would be an appropriate way to do this. FH responded to O' Halloran's letter, expressing our interest in the idea, but pointing out reasons why we didn't think it was a feasible prospect at that time. We felt that suitable team members were very likely to be engaged in sitting Matriculation examinations for the NUI and/or TCD during the first fortnight of July, followed by University Entrance Scholarship examinations. They couldn't possibly be in two places at the same time, and, more importantly, couldn't be expected to run the risk of forfeiting their chances of getting a university place and possibly a scholarship as well. In addition, we felt that it would be nigh to impossible to get a sponsor who would be prepared to cover the travel costs. But Peter, waiving aside our perceived difficulties and objections—which he felt could be overcome—persisted, expressing a strong wish for an Irish team to be entered in the competition, given the history of the two countries. The year 1988 marked the Bicentennial of the settlement of Australia by Europeans, and was going to be celebrated hugely in Australia a fifth of whose population was of Irish descent. It was important therefore that Ireland mark the occasion, and sending a team to the 29th IMO would be a fitting way to do so.

We continued to exchange correspondence with O' Halloran about the prospect of sending an Irish MO team. (FH made the mistake of giving him his telephone number, which meant that he started getting early morning phone calls all the way from Australia!) Peter informed us that he was coming to Ireland in the Summer of 1986, ostensibly to trace his ancestors, and we agreed to meet him and explore further the possibility of Ireland sending a team. In the meantime, he was busy working behind the diplomatic scenes, and had made contact with the Irish Ambassador to Australia urging him to progress his ideas. In particular, he arranged to visit the Australian Ambassador to Ireland during his own visit to Ireland.

As promised, Peter and his wife Marjorie duly arrived in Ireland in the middle of July, 1986, and we met them. We accompanied Peter to discuss the matter with Paddy Cooney, TD, then Minister for Education, who had by then received a formal invitation from the Minister assisting the Prime Minister of Australia for Bicentennial Affairs to participate in the IMO, and subsequently Peter Barry, TD, then Minister for Foreign Affairs. These meetings had been set up by the Australian Ambassador to Ireland, whom Peter had met in advance of meeting us. At our meetings with Barry and Cooney, we set out our tentative plans and a strategy for identifying and preparing an Irish team for the 29th IMO, and provided rough costs of the venture. Our meeting with Paddy Cooney was very relaxed, partly aided by the fact that Paddy and the Australian Ambassador to Ireland both had great respect for the education provided by the Marist Brothers in their respective countries—a topic of conversation that was sparked by O' Halloran! By contrast, we were whisked in and out of Minister Barry's presence and our memory of the meeting is a blur. In the ante room we furnished a rough guide to one of his officials of the amount of money we thought we might need, but subsequently mentioned a larger estimate to the Minister, and were somewhat taken aback when the official expressed his annoyance with us about the disparity in amounts when we were leaving! But the essential thing that emerged from the meetings was that both Ministers gave their approval for the venture and indicated that a small committee would be set up to progress the matter; and, more significantly, indicated that finance would be forthcoming. Shortly afterwards, a committee comprising of both of us, Bill Nolan from the Department of Foreign Affairs and an t-Uasal Conchubhair Ó Chaoimh from the Department of Education, was set up, under the latter's chairmanship, to deal with the logistics. (The presence of a representative from Foreign Affairs proved invaluable in dealing with late applications for passports for team members, and visas to the country hosting the IMO. Much later, for instance, when the IMO was held in Argentina, the Irish Ambassador in Buenos Aires arrived at the airport to greet the Irish team led by the Deputy Leader, Gordon Lessells, only to discover that Raja Mukherji, a team member, wasn't being allowed to enter the country because—though born in Ireland, and therefore entitled to an Irish one—he was travelling on an Indian passport! The Ambassador quickly resolved the matter by issuing an Irish passport to Raja which qualified him for a visa to enter Argentina as an Irish MO team member.)

5. Attendance at the 28th IMO in Cuba

Following a series of meetings of the committee of four, strategies were agreed for identifying talented students, for coaching them and raising funds. It was also decided to send an observer to the 28th IMO which was to be held in Cuba in 1987. FH was elected to go there and he wrote to the organiser, Luis Davidson, for particulars about the 1987 IMO. But by the end of May, 1987, FH still hadn't heard anything from Davidson. However, his trip had been sanctioned by that time, and FH made plans to go to Havana on the advice of Peter O' Halloran, with whom FH had shared his concerns about the lack of information from Cuba.

His attendance there was an invaluable learning experience. He saw at first hand how everything worked at an IMO, and as well as renewing acquaintanceship with O' Halloran made contact with Leaders, and Observers from other countries, such as Emanuel Strzelecki from Australia, Ronald Dunkley from Canada and Derek Holton, from New Zealand—who, like FH, was also attending his first IMO—all of whom shared their own experiences of identifying and training their teams. He made lasting friendships with people from other countries as well, and came home armed with advice, problem sets and training materials—and cigars very few of which he smoked!—but minus several items of clothing which went missing at the airport whence he departed!

6. Preparations for 29th IMO in Australia

These began in earnest on FH's return from the 28th IMO in Cuba! A selection of the top scorers of the INMC for 1987 who were still in school, and other talented students identified to us by their teachers, were invited to attend special coaching sessions in UCD and UCC on specific topics not covered by the Leaving Certificate programme. These were given by us with the assistance of some staff members of the UCC and UCD Mathematics Departments, such as Fergus Gaines and Donal Hurley, on a voluntary basis, and by local teachers who helped from time to time in the venture. The first Irish Mathematical Olympiad papers were set, and the top six scorers formed the team to represent Ireland at the 29th IMO. The Minister for Education covered the travel costs for six team members and two leaders, and the rest is history! Every year since then, the Minister has provided just enough funding to cover travel costs of 8 people, and sometimes an observer, to represent Ireland at that year's IMO, the country hosting the event bearing local accommodation and travel costs associated with the contest.

In July of 1988, a group of ten Irish people from different parts of Ireland gathered in Heathrow Airport to board a Qantas Airlines plane which was to take us to Sydney to compete in the 29th IMO which was being hosted in Canberra, the first IMO to be held in the southern hemisphere. The party included two female and four male students, TL as Deputy Leader, FH as Team Leader, and his wife Mae and daughter Allison as accompanying persons.

We were all giddy with excitement when we assembled at the airport, and very nearly missed our flight because we ended up in the wrong terminal! But we recovered our bearings quickly enough and boarded our flight on time, to begin our adventure.

On arrival in Sydney we spent approximately one day there before being transported by bus to Canberra, where our party was split up: along with other deputy leaders and team members, Allison, TL and the six team members were accommodated in a Hall of Residence for students in the College of Education, where the contest was to be held, while, some distance away from the students, FH and Mae were allocated to a separate apartment in a residential complex attached to the Australian Academy of Science, which is where the Team Leaders were billeted. This was adjacent to the Academy's Shine Dome, where for about two days the Jury deliberated over the rules and regulations of the IMO, and composed and translated the examination papers, which consisted of six problems selected from a short list prepared by the local coordinators from those submitted by different countries. One such problem had been submitted by Ireland, and following the examination it fell to TL and FH to decide what marks to award to each Australian team member who attempted it. This problem traces back to an integral identity established by George Boole on the eve of his taking up his appointment as the first Professor of Mathematics at the Queen's College, Cork, in 1849, and the key idea for this also crops up in Lynn Loomis's real variable treatment of the Hilbert transform in 1946, about which FH learnt following Adriano Garsia's seminar about this transform when he was in Caltech during 1964–65. This concerned the size of the set of intervals where the finite Hilbert transform

$$\sum_{j=1}^{n} \frac{c_j}{x - a_j}, \; a_j \in (-\infty, \infty), \; c_j > 0, \; j = 1, 2, \ldots, n,$$

is bigger than a given positive real number y. The 4th problem chosen for the 29th IMO was a special case of this result. Of the attempts made on it by the Australian team members, one stood out—that by Terry Tao, then aged 13, who was competing at an IMO for the third time, having previously won a bronze medal in 1986, and a silver one in 1987. Tao's solution of Problem 4 was a model of clarity, rigour and elegance, and left an indelible mark on both of us; even then he was clearly destined for a bright future, and his subsequent career hasn't disappointed. Since winning an IMO gold medal in Canberra—the youngest ever to win one—Tao has made outstanding contributions to several areas of mathematics, has published more than 300 papers with as many as 60 different authors, maintains a very active blog, which has spawned several of his 15 or so books, and has won many honours, including a Fields medal in 2006. Our examination of Tao's solution of Problem 4 at the 29th IMO in Canberra was the mathematical highlight of our involvement in this event.

In conclusion, that journey to Canberra made thirty years ago was the first of many subsequent trips made abroad by different sets of Irish students, different leaders and deputy leaders every year since 1988 to a country hosting the IMO, to compete in an event which has taken place every year, excluding 1980, since the first was held in Romania in 1959, when only 7 countries competed. Nowadays, that number has swelled to more than 100 from 5 continents, and nearly 600 students participated this year in the 59th IMO held in Cluj-Napoca, Romania.

Finbarr Holland graduated from UCC with a BSc in Mathematics and Mathematical Physics in 1961. He was awarded a Travelling Studentship, and the MSc degree, by the NUI in 1962. The Studentship enabled him to study for his PhD in Harmonic Analysis under the supervision of Lionel B. Cooper at University College, Cardiff; and the National University of Wales conferred this degree on him in 1964. He joined the staff of the Mathematics Department at UCC in 1965, where he currently holds the rank of Professor Emeritus.

Thomas Laffey graduated from UCG with a BSc in Mathematical Science in 1964 and an MSc in Mathematical Science in 1965. After one year teaching in the Department of Mathematics at UCG, he went on a tutorial studentship to Sussex University and, under the supervision of Walter Ledermann, graduated with a D.Phil. in Finite Group Theory in 1968. He joined the staff of the Department of Mathematics at UCD later that year, and he currently holds the rank of Professor Emeritus there.

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Computing the Varchenko Determinant of a Bilinear Form

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Abstract. The Varchenko determinant is the determinant of the bilinear form associated to a real hyperplane arrangement. We show that we can obtain the exact value of this determinant for certain hyperplane arrangements if we know the edges which are relevant.

1. Introduction

Let $x = (x_1, \dots, x_n)$ be a variable of the Euclidean space $\mathbb{R}^n$, and $a_1, \dots, a_n, b$ real coefficients such that $(a_1, \dots, a_n) \neq (0, \dots, 0)$.
A hyperplane H of $\mathbb{R}^n$ is a $(n-1)$–dimensional affine subspace $H := \{x \in \mathbb{R}^n \mid a_1x_1 + \cdots + a_nx_n = b\}$. An arrangement of hyperplanes in $\mathbb{R}^n$ is a finite set of hyperplanes. The most famous hyperplane arrangement is certainly the braid arrangement $\mathcal{B}_n = \big\{\{x \in \mathbb{R}^n \mid x_i - x_j = 0\}\big\}_{1 \leq i < j \leq n}$. Hyperplane arrangement theory is currently a very active area of research, combining ideas from algebraic combinatorics, algebraic topology, and algebraic geometry. In the preface of their book [3], Orlik and Terao wrote *Arrangements are easily defined and may be enjoyed at levels ranging from the recreational to the expert, yet these simple objects lead to deep and beautiful results. Their study combines methods from many areas of mathematics and reveals unexpected connections.* The bilinear form of a hyperplane arrangement defined by Varchenko confirms their affirmation. The Varchenko determinant is the determinant of this bilinear form.

An edge of a hyperplane arrangement $\mathcal{A}$ is a nonempty intersection of some of its hyperplanes. Denote by $L(\mathcal{A})$ the set of all edges of

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$\mathcal{A}$. The arrangement of hyperplanes in $\mathcal{A}$ containing an edge E in $L(\mathcal{A})$ is

$$\mathcal{A}_E := \{H \in \mathcal{A} \mid E \subseteq H\}.$$

The hyperplane arrangement in the edge E cut by $\mathcal{A}$ is

$$\mathcal{A}^E := \{H \cap E \mid H \in \mathcal{A},\, E \nsubseteq H\}.$$

A chamber of a hyperplane arrangement $\mathcal{A}$ is a connected component of the complement $\mathbb{R}^n \setminus \bigcup_{H \in \mathcal{A}} H$. Denote the set of all chambers of $\mathcal{A}$ by $\mathfrak{C}(\mathcal{A})$.

Assign a variable a_H called weight to each hyperplane H of an arrangement $\mathcal{A}$. Define the weight $\mathsf{a}(E)$ of an edge E by

$$\mathsf{a}(E) := \prod_{\substack{H \in \mathcal{A} \\ E \subseteq H}} a_H.$$

The multiplicity of an edge E is

$$l(E) := n(E)p(E),$$

where $n(E) := |\mathfrak{C}(\mathcal{A}^E)|$, and $p(E)$ is defined below.
For every edge E of codimension r, let N be an r-dimensional normal subspace to E. All hyperplanes of the resulting arrangement $(\mathcal{A}_E)^N$ pass through the point $\{v\} = E \cap N$. Consider the hyperplane arrangement which $(\mathcal{A}_E)^N$ induces in the tangent space T_vN. It determines another hyperplane arrangement $P\mathcal{A}_E$ in the projectivization of T_vN.
A chamber of an arrangement is bounded with respect to a hyperplane if the closure of this chamber does not intersect the hyperplane. For any arrangement $\mathcal{A}'$ in a real projective space, the numbers of chambers which are bounded with respect to its hyperplanes are all the same [5, Theorem 1.5], and we denote this number by $e(\mathcal{A}')$.
Finally, for an edge E in $L(\mathcal{A})$, define $p(E) := e(P\mathcal{A}_E)$.

Let $R_{\mathcal{A}} = \mathbb{Z}[a_H \mid H \in \mathcal{A}]$ be the ring of polynomials in variables a_H. The module of $R_{\mathcal{A}}$-linear combinations of chambers of the hyperplane arrangement $\mathcal{A}$ is

$$M_{\mathcal{A}} := \{\sum_{C \in \mathfrak{C}(\mathcal{A})} x_C C \mid x_C \in R_{\mathcal{A}}\}.$$

Let $\mathcal{H}(C, D)$ be the set of hyperplanes separating the chambers C and D in $\mathfrak{C}(\mathcal{A})$. Define the $R_{\mathcal{A}}$-bilinear symmetric form

$\mathsf{B} : M_{\mathcal{A}} \times M_{\mathcal{A}} \to R_{\mathcal{A}}$ by

$$\mathsf{B}(C,C) := 1, \text{ and } \mathsf{B}(C,D) := \prod_{H \in \mathcal{H}(C,D)} a_H \text{ if } C \neq D.$$

The Varchenko determinant of the hyperplane arrangement $\mathcal{A}$ is the determinant

$$\det \mathcal{A} := \det \big(\mathsf{B}(C,D)\big)_{C,D \in \mathfrak{C}(\mathcal{A})}$$

of the matrix associated to the bilinear symmetric form B. The formula of this determinant due to Varchenko is [6, (1.1) Theorem]

$$\det \mathcal{A} = \prod_{E \in L(\mathcal{A})} \big(1 - \mathsf{a}(E)^2\big)^{l(E)}.$$

It is, however, not feasible to directly use this formula to compute a determinant from a certain complexity level. For example, one can not deduce $\det \mathcal{B}_{24}$ directly from it. In this article, we show that we can work around this difficulty for certain hyperplane arrangements if we know the edges which are relevant. In this purpose, we use a clearer definition of the multiplicity $l(E)$ written in an article of Denham and Hanlon [1, 2. The Nullspace of the B Matrices]: First choose a hyperplane H containing E. Then $l(E)$ is half the number of chambers C which have the property that E is the minimal edge containing $\bar{C} \cap H$.

We determine the relevant edges in the next section. Then, we compute the Varchenko determinants of some hyperplane arrangements in the last section.

2. The Relevant Edges

In this section, we remove in the Varchenko determinant the factors $\big(1 - \mathsf{a}(E)^2\big)^{l(E)}$ such that $l(E) = 0$. In the edge set $L(\mathcal{B}_7)$, for example, we do not need to consider the edges $\{x \in \mathbb{R}^7 \mid x_1 = x_2,\, x_4 = x_5\}$ and $\{x \in \mathbb{R}^7 \mid x_2 = x_4 = x_5,\, x_1 = x_7\}$ whose multiplicity is 0. This removal simplifies the computation of the Varchenko determinant.

Take a hyperplane arrangement $\mathcal{A}$ in $\mathbb{R}^n$. We say that an edge E of $\mathcal{A}$ is relevant if $l(E) \neq 0$. Denote the relevant edge set of $\mathcal{A}$ by

$$\mathfrak{R}_{\mathcal{A}} := \{E \in L_{\mathcal{A}} \mid l(E) \neq 0\}.$$

To determine $\mathfrak{R}_{\mathcal{A}}$, we have to consider the faces of the chambers. Recall that the face set of a chamber C in $\mathfrak{C}_{\mathcal{A}}$ resp. of the chambers in $\mathfrak{C}_{\mathcal{A}}$ is

$$\mathcal{F}(C) := \{\bar{C}\cap E \mid E \in L_{\mathcal{A}},\, \bar{C}\cap E \neq \emptyset\} \quad \text{resp.} \quad \mathcal{F}(\mathfrak{C}_{\mathcal{A}}) := \bigcup_{C\in\mathfrak{C}_{\mathcal{A}}} \mathcal{F}(C).$$

Define the following subset of $\mathcal{F}(C)$ resp. $\mathcal{F}(\mathfrak{C}_{\mathcal{A}})$

$$\mathcal{S}(C) := \{\bar{C}\cap H \mid H \in \mathcal{A},\, \bar{C}\cap H \neq \emptyset\} \quad \text{resp.} \quad \mathcal{S}(\mathfrak{C}_{\mathcal{A}}) := \bigcup_{C\in\mathfrak{C}_{\mathcal{A}}} \mathcal{S}(C).$$

Lemma 2.1. *Let $\mathcal{A}$ be a hyperplane arrangement in $\mathbb{R}^n$. Then*

$$\mathfrak{R}_{\mathcal{A}} = \big\{\langle F\rangle \mid F \in \mathcal{S}(\mathfrak{C}_{\mathcal{A}})\big\}.$$

Proof. Let $F \in \mathcal{S}(\mathfrak{C}_{\mathcal{A}})$. We have,

$$\langle F\rangle = \bigcap_{\substack{H\in\mathcal{A}\\ F\subseteq H}} H.$$

By the definition, there exist a chamber C and a hyperplane H such that $\bar{C}\cap H = F$. Since $\langle F\rangle$ is the minimal edge containing F, then $l\big(\langle F\rangle\big) \geq 1$.

Now, take $E \in L_{\mathcal{A}} \setminus \big\{\langle F\rangle \mid F \in \mathcal{S}(\mathfrak{C}_{\mathcal{A}})\big\}$. Suppose that there exist a chamber C and a hyperplane H such that E is the minimal edge containing $F = \bar{C}\cap H$. It means that $E \subsetneq \langle F\rangle$, which impossible since $F \subseteq E$. ☐

Remark that, in the formula $l(E) = n(E)p(E)$, we always have $n(E) \geq 1$. So only the factor $p(E)$ could be considered for deciding which edges are relevant.

Proposition 2.2. *Let $\mathcal{A}$ be a hyperplane arrangement in $\mathbb{R}^n$. For every relevant edge E, we fix a hyperplane H_E of $\mathcal{A}$ containing it. Then,*

$$\det\mathcal{A} = \prod_{E\in\mathfrak{R}_{\mathcal{A}}} \big(1-\mathsf{a}(E)^2\big)^{l(E)}$$

$$\textit{with } l(E) = \frac{1}{2}\,\Big|\big\{C\in\mathfrak{C}_{\mathcal{A}} \mid \langle\bar{C}\cap H_E\rangle = E\big\}\Big|.$$

Proof. It is clear that

$$\prod_{E\in L_{\mathcal{A}}} \big(1-\mathsf{a}(E)^2\big)^{l(E)} = \prod_{E\in\mathfrak{R}_{\mathcal{A}}} \big(1-\mathsf{a}(E)^2\big)^{l(E)}.$$

Recall that $l(E)$ is half the number of chambers C which have the property that E is the minimal edge containing $\bar{C}\cap H_E$. From

Lemma 2.1, we deduce that E is the minimal edge containing $\bar{C}\cap H_E$ if and only if $\langle \bar{C} \cap H_E \rangle = E$. □

3. Some Varchenko Determinants

We compute some Varchenko determinants in this section. Furthermore, the Varchenko determinants of the braid arrangement, and of the hyperplane arrangement associated to the hyperoctahedral group were also computed by Pfeiffer, and Randriamaro [4, 6 Computing the determinants of finite Coxeter groups], but by using tools from group theory.

Central 2–Dimensional Arrangement. This is a hyperplane arrangement $\mathcal{A} = \{H_1, \dots, H_m\}$ in $\mathbb{R}^2$ such that the intersection $\bigcap_{i=1}^m H_i$ is the origin $\{0\}$. An example is the hyperplane arrangement associated to the dihedral group D_m having $2m$ elements with

$$H_i = \Big\{x \in \mathbb{R}^2 \mid x_1 \cos \frac{(i-1)\pi}{m} + x_2 \sin \frac{(i-1)\pi}{m} = 0\Big\}.$$

Assign the weight a_i to the hyperplane H_i. Then,
$\mathfrak{R}_{\mathcal{A}} = \big\{H_1, \dots, H_m, \{0\}\big\}$, and

- $\mathsf{a}(H_i) = a_i$ with $l(H_i) = 2$,
- $\mathsf{a}(\{0\}) = \prod_{i=1}^m a_i$ with $l(\{0\}) = m - 2$.

Then,

$$\det \mathcal{A} = \big(1 - \prod_{i=1}^{m} a_i^2\big)^{m-2} \prod_{j=1}^{m} (1 - a_j^2)^2.$$

General Position and the Hypercubic Arrangement. A hyperplane arrangement $\mathcal{G}_n = \{H_1, \dots, H_{n+1}\}$ in $\mathbb{R}^n$ is in general position if, for every subset P of $[n]$ such that $|P| = p$, we have $\dim \bigcap_{i\in P} H_i = n-p$. This is the case for the hyperplane arrangement such that

- $\forall i \in [n]$, $H_i = \{x \in \mathbb{R}^n \mid x_i = 0\}$,
- $H_{n+1} = \{x \in \mathbb{R}^n \mid x_1 + \dots + x_n = 1\}$.

Assign the weight a_i to the hyperplane H_i.

And for α, β in $\mathbb{R}$ with $\alpha \neq \beta$, the hypercubic arrangement is the hyperplane arrangement $\mathcal{C}_n = \{H_{1,\alpha}, H_{1,\beta}, \dots, H_{n,\alpha}, H_{n,\beta}\}$ in $\mathbb{R}^n$ such that $H_{i,\alpha\,\text{resp.}\,\beta} = \{x \in \mathbb{R}^n \mid x_i = \alpha \text{ resp. } \beta\}$.
Assign the weight $a_{i,\alpha}$ resp. $a_{i,\beta}$ to the hyperplane $H_{i,\alpha}$ resp. $H_{i,\beta}$.

Both hyperplane arrangements have the property

$$\forall H \in \mathcal{G}_n \text{ resp. } \mathcal{C}_n,\ \forall C \in \mathfrak{C}(\mathcal{G}_n) \text{ resp. } \mathfrak{C}(\mathcal{C}_n),\ \langle C\cap H\rangle = H \text{ or } \langle C\cap H\rangle = \emptyset.$$

From Lemma 2.1, we deduce that $\mathfrak{R}(\mathcal{G}_n) = \mathcal{G}_n$, and $\mathfrak{R}(\mathcal{C}_n) = \mathcal{C}_n$.
Moreover, since $|\mathcal{G}_n| = 2^n - 1$, and $|\mathcal{C}_n| = 3^n$, then

$$\Big|\{C \in \mathcal{G}_n \mid H \in \mathcal{G}_n,\ \langle C \cap H\rangle = H\}\Big| = 2^n - 2$$
$$\text{and } \Big|\{C \in \mathcal{C}_n \mid H \in \mathcal{C}_n,\ \langle C \cap H\rangle = H\}\Big| = 2 \times 3^{n-1}.$$

Thus

$$\det \mathcal{G}_n = \prod_{i=1}^{n} (1 - a_i^2)^{2^{n-1}-1} \text{ and } \det \mathcal{C}_n = \prod_{i=1}^{n} (1 - a_{i,\alpha}^2)^{3^{n-1}} (1 - a_{i,\beta}^2)^{3^{n-1}}.$$

Braid Arrangement. This consists of the $\binom{n}{2}$ hyperplanes $H_{i,j} = \{x \in \mathbb{R}^n \mid x_i - x_j = 0\}$, with $1 \leq i < j \leq n$. We assign the weight $a_{i,j}$ to the hyperplane $H_{i,j}$.

Proposition 3.1. *Let $n \geq 2$. We have*

$$\det \mathcal{B}_n = \prod_{\substack{I \in 2^{[n]} \\ |I| \geq 2}} \Big(1 - \prod_{\{i,j\} \in \binom{I}{2}} a_{i,j}^2\Big)^{(|I|-2)!\,(n-|I|+1)!}.$$

This determinant was also calculated by Duchamp et al. [2, 6.4.2 A Decomposition of B_n] using the diagonal solutions of the Yang-Baxter equation.

Each chamber of $\mathcal{B}_n$ is defined by $\{x \in \mathbb{R}^n \mid x_{\sigma(1)} > x_{\sigma(2)} > \cdots > x_{\sigma(n)}\}$, where σ is a permutation of $[n]$. We write $\{x_{\sigma(1)} > x_{\sigma(2)} > \cdots > x_{\sigma(n)}\}$ for simplicity.
Let $I = \{i_1, \ldots, i_r\}$ be a subset of $[n]$, with $|I| \geq 2$. Denote by $E(I)$ the edge

$$E(I) := \bigcap_{\{i,j\} \in \binom{\{i_1,\ldots,i_r\}}{2}} H_{i,j}.$$

Lemma 3.2. *Let $n \geq 2$. We have*

$$\mathfrak{R}(\mathcal{B}_n) = \big\{E(I) \mid I \subseteq [n],\ |I| \geq 2\big\}.$$

Proof. Consider a hyperplane $H_{s,t}$ of $\mathcal{B}_n$, and a permutation σ of $[n]$ such that $\sigma(i) = s$ and $\sigma(j) = t$ with $i < j$. Then,

$$\begin{aligned}
&\big\langle H_{s,t} \cap \overline{\{x_{\sigma(1)} > x_{\sigma(2)} > \cdots > x_{\sigma(n)}\}}\big\rangle \\
&\quad = \big\langle\{x_{\sigma(1)} > \cdots > x_{\sigma(i)} = \cdots = x_{\sigma(j)} > \cdots > x_{\sigma(n)}\}\big\rangle \\
&\quad = \{x \in \mathbb{R}^n \mid x_{\sigma(i)} = x_{\sigma(i+1)} = \cdots = x_{\sigma(j)}\} \\
&\quad = E\big(\{\sigma(i), \sigma(i+1), \ldots, \sigma(j)\}\big).
\end{aligned}$$

Hence, $E\big(\{\sigma(i), \sigma(i+1), \ldots, \sigma(j)\}\big)$ is the minimal edge containing the face $H_{i_s,i_t} \cap \overline{\{x_{\sigma(1)} > x_{\sigma(2)} > \cdots > x_{\sigma(n)}\}}$. □

Lemma 3.3. *Let $I \subseteq [n]$. Then, $l\big(E(I)\big) = (|I|-2)!\,(n-|I|+1)!$.*

Proof. Let H_{i_1,i_r} be a hyperplane containing $E(I)$. We have to count the chambers $\{x_{\sigma(1)} > x_{\sigma(2)} > \cdots > x_{\sigma(n)}\}$ such that

$$\big\langle H_{i_1,i_r} \cap \overline{\{x_{\sigma(1)} > x_{\sigma(2)} > \cdots > x_{\sigma(n)}\}}\big\rangle = E(I).$$

Let ν be a permutation of $\{2,\ldots,r-1\}$. These chambers correspond to the chambers having the forms

$$\begin{aligned}&\{\cdots > x_{i_1} > x_{i_{\nu(2)}} > \cdots > x_{i_{\nu(r-1)}} > x_{i_r} > \ldots\}\\ &\text{and } \{\cdots > x_{i_r} > x_{i_{\nu(2)}} > \cdots > x_{i_{\nu(r-1)}} > x_{i_1} > \ldots\}.\end{aligned}$$

Because of the coefficient $\frac{1}{2}$ in the multiplicity, we just need to consider the chambers

$$\begin{aligned}&\{x_{\sigma(1)} > x_{\sigma(2)} > \cdots > x_{\sigma(n)}\}\\ &= \{\cdots > x_{i_1} > x_{i_{\nu(2)}} > \cdots > x_{i_{\nu(r-1)}} > x_{i_r} > \ldots\}.\end{aligned}$$

Let $i \in [n]$ such that $\sigma(i) = i_1$. We have:

- $(n-r)!$ possibilities for the sequence $\big(\sigma(1), \ldots, \sigma(i-1), \sigma(i+r), \ldots, \sigma(n)\big)$,
- $(r-2)!$ possibilities for the sequence $\big(\sigma(i+1), \ldots, \sigma(i+r-2)\big)$,
- and $n-r+1$ possibilities to choose i since we must have $i \in [n-r+1]$.

Then $l\big(E(I)\big) = (n-r)!\times(r-2)!\times(n-r+1) = (r-2)!\,(n-r+1)!$. □

We obtain Proposition 3.1 by combining Proposition 2.2 and Lemma 3.3.

Hyperplane Arrangement Associated to Hyperoctahedral Group. Let $[\pm n] := \{-n,\ldots,-2,-1,1,2,\ldots,n\}$. Denote $\overline{2^{[\pm n]}}$ the subset of $2^{[\pm n]}$ having the following properties:

- the elements of $\overline{2^{[\pm n]}}$ are the elements $\{i_1,\ldots,i_t\}$ of $2^{[\pm n]}$ such that $|i_r| \neq |i_s|$ if $r \neq s$,
- and if $\{i_1,\ldots,i_t\} \in \overline{2^{[\pm n]}}$, then $\{-i_1,\ldots,-i_t\} \notin \overline{2^{[\pm n]}}$.

For example,

$$\begin{aligned}\overline{2^{[\pm 3]}} = \big\{&\emptyset,\ \{1\},\ \{2\},\ \{3\},\\ &\{1,2\},\ \{-1,2\},\ \{1,3\},\ \{-1,3\},\ \{2,3\},\ \{-2,3\},\\ &\{1,2,3\},\ \{-1,2,3\},\ \{-1,-2,3\},\ \{1,-2,3\}\big\}.\end{aligned}$$

The hyperplane arrangement $\mathcal{O}_n$ associated to the hyperoctahedral group B_n consists of

- the $\binom{n}{2}$ hyperplanes $H_{i,j} = \{x \in \mathbb{R}^n \mid x_i - x_j = 0\}$ with $1 \leq i < j \leq n$,
- the $\binom{n}{2}$ hyperplanes $H_{-i,j} = \{x \in \mathbb{R}^n \mid x_i + x_j = 0\}$ with $1 \leq i < j \leq n$,
- the n hyperplanes $H_i = \{x \in \mathbb{R}^n \mid x_i = 0\}$ with $i \in [n]$.

We assign the weights $a_{i,j}$ to the hyperplanes $H_{i,j}$, the weights $a_{-i,j}$ to the hyperplanes $H_{-i,j}$, and the weights a_i to the hyperplanes H_i.

Proposition 3.4. *Let $n \geq 2$. We have*

$$\det \mathcal{O}_n = \prod_{\substack{J \in \overline{2^{[\pm n]}} \\ |J| \geq 2}} \Big(1 - \prod_{\{i,j\} \in \binom{J}{2}} a_{i,j}^2\Big)^{2^{n-|J|+1}\,(|J|-2)!\,(n-|J|+1)!}$$
$$\prod_{\substack{I \in 2^{[n]} \\ |I| \geq 1}} \Big(1 - \prod_{i \in I} a_i^2 \prod_{\{i,j\} \in \binom{I}{2}} a_{i,j}^2\, a_{-i,j}^2\Big)^{2^{n-1}\,(|I|-1)!\,(n-|I|)!}.$$

Each chamber of $\mathcal{O}_n$ is defined by

$$\{x \in \mathbb{R}^n \mid \epsilon_1 x_{\sigma(1)} > \epsilon_2 x_{\sigma(2)} > \cdots > \epsilon_n x_{\sigma(n)} > 0\},$$

where $\epsilon_i \in [\pm 1]$ and σ is a permutation of $[n]$.
For $J = \{\epsilon_1 i_1,\, \epsilon_2 i_2,\, \ldots,\, \epsilon_r i_r\} \in \overline{2^{[\pm n]}}$, with $|J| \geq 2$, denote $E(J)$ the edge

$$E(J) := \{x \in \mathbb{R}^n \mid \epsilon_1 x_{i_1} = \epsilon_2 x_{i_2} = \cdots = \epsilon_r x_{i_r}\}.$$

And for $I = \{i_1, i_2, \ldots, i_r\} \subseteq 2^{[n]}$, with $r \geq 1$, denote $E(I_0)$ the edge

$$E(I_0) := \{x \in \mathbb{R}^n \mid x_{i_1} = x_{i_2} = \cdots = x_{i_r} = 0\}.$$

Lemma 3.5. *Let $n \geq 2$. We have*

$$\mathfrak{R}(\mathcal{O}_n) = \Big\{E(J) \mid J \in \overline{2^{[\pm n]}},\ |J| \geq 2\Big\} \cup \Big\{E(I_0) \mid I \in 2^{[n]},\ |I| \geq 1\Big\}.$$

Proof. Consider a hyperplane $H_{\epsilon_s s, \epsilon_t t}$ of $\mathcal{O}_n$, and a permutation σ of $[n]$ such that $\sigma(i) = s$ and $\sigma(j) = t$ with $i < j$. If

- $\{\epsilon_s s, \epsilon_t t\} = \{\epsilon_i \sigma(i), \epsilon_j \sigma(j)\}$, then

$$\begin{aligned}&\Big\langle H_{\epsilon_s s,\epsilon_t t} \cap \overline{\{\epsilon_1 x_{\sigma(1)} > \epsilon_2 x_{\sigma(2)} > \cdots > \epsilon_n x_{\sigma(n)} > 0\}}\Big\rangle\\ &= \Big\langle\{\epsilon_1 x_{\sigma(1)} > \cdots > \epsilon_i x_{\sigma(i)} = \cdots = \epsilon_j x_{\sigma(j)} > \cdots > \epsilon_n x_{\sigma(n)} > 0\}\Big\rangle\\ &= \{\epsilon_i x_{\sigma(i)} = \epsilon_{i+1} x_{\sigma(i+1)} = \cdots = \epsilon_j x_{\sigma(j)}\}\\ &= E\big(\{\epsilon_i \sigma(i),\, \epsilon_{i+1}\sigma(i+1),\, \ldots,\, \epsilon_j \sigma(j)\}\big).\end{aligned}$$

- $\{\epsilon_s i_s, \epsilon_t i_t\} \neq \{\epsilon_i \sigma(i), \epsilon_j \sigma(j)\}$, then

$$\begin{aligned}&\Big\langle H_{\epsilon_s s,\epsilon_t t} \cap \overline{\{\epsilon_1 x_{\sigma(1)} > \epsilon_2 x_{\sigma(2)} > \cdots > \epsilon_n x_{\sigma(n)} > 0\}}\Big\rangle\\ &= \Big\langle\{\epsilon_1 x_{\sigma(1)} > \cdots > \epsilon_{i-1} x_{\sigma(i-1)} > x_{\sigma(i)} = \cdots = x_{\sigma(n)} = 0\}\Big\rangle\\ &= \{x_{\sigma(i)} = x_{\sigma(i+1)} = \cdots = x_{\sigma(n)} = 0\}\\ &= E\big(\{0,\, \sigma(i),\, \sigma(i+1),\, \ldots,\, \sigma(n)\}\big).\end{aligned}$$

Consider a hyperplane H_u of $\mathcal{O}_n$, and a permutation σ of $[n]$ such that $\sigma(i) = u$. Then,

$$\begin{aligned}&\Big\langle H_u \cap \overline{\{\epsilon_1 x_{\sigma(1)} > \epsilon_2 x_{\sigma(2)} > \cdots > \epsilon_n x_{\sigma(n)} > 0\}}\Big\rangle\\ &= \Big\langle\{\epsilon_1 x_{\sigma(1)} > \cdots > \epsilon_{i-1} x_{\sigma(i-1)} > x_{\sigma(i)} = \cdots = x_{\sigma(n)} = 0\}\Big\rangle\\ &= \{x_{\sigma(i)} = x_{\sigma(i+1)} = \cdots = x_{\sigma(n)} = 0\}\\ &= E\big(\{0,\, \sigma(i),\, \sigma(i+1),\, \ldots,\, \sigma(n)\}\big).\end{aligned}$$

□

Lemma 3.6. *Let $E(J)$, $E(I_0) \in \mathfrak{R}(O_n)$. Then,*

$$\begin{aligned}\mathsf{a}\big(E(J)\big) &= \prod_{\{s,t\}\in\binom{J}{2}} a_{s,t}\\ &\quad \textit{with} \quad l\big(E(J)\big) = 2^{n-|J|+1}(|J|-2)!\,(n-|J|+1)!,\\ \mathsf{a}\big(E(I_0)\big) &= \prod_{u\in I} a_u \prod_{\{s,t\}\in\binom{I}{2}} a_{s,t}a_{-s,t}\\ &\quad \textit{with} \quad l\big(E(I_0)\big) = 2^{n-1}(|I|-1)!\,(n-|I|)!.\end{aligned}$$

Proof. We have

$$\mathsf{a}\big(E(J)\big) = \prod_{\substack{H\in\mathcal{O}_n\\ E(J)\subseteq H}} \mathsf{a}(H) = \prod_{\{s,t\}\in\binom{J}{2}} a_{s,t},$$

and

$$\mathsf{a}\big(E(I_0)\big) = \prod_{\substack{H \in \mathcal{O}_n \\ E(I_0) \subseteq H}} \mathsf{a}(H) = \prod_{u \in I} a_u \prod_{\{s,t\} \in \binom{I}{2}} a_{s,t} a_{-s,t}.$$

To the edge $E(J)$, assign the hyperplane $H_{\epsilon_1 i_1,\, \epsilon_r i_r}$ containing it. We first have to count the chambers $\{\epsilon_1 x_{\sigma(1)} > \epsilon_2 x_{\sigma(2)} > \cdots > \epsilon_n x_{\sigma(n)} > 0\}$ such that

$$\Big\langle H_{\epsilon_1 i_1,\, \epsilon_r i_r} \cap \overline{\{\epsilon_1 x_{\sigma(1)} > \epsilon_2 x_{\sigma(2)} > \cdots > \epsilon_n x_{\sigma(n)} > 0\}} \Big\rangle = E(J).$$

Let ν be a permutation of $\{2, \dots, r-1\}$. These chambers correspond to the chambers having the forms

$$\{\cdots > \epsilon_1 x_{i_1} > \epsilon_{\nu(2)} x_{i_{\nu(2)}} > \cdots > \epsilon_{\nu(r-1)} x_{i_{\nu(r-1)}} > \epsilon_r x_{i_r} > \dots\},$$
$$\{\cdots > \epsilon_r x_{i_r} > \epsilon_{\nu(2)} x_{i_{\nu(2)}} > \cdots > \epsilon_{\nu(r-1)} x_{i_{\nu(r-1)}} > \epsilon_1 x_{i_1} > \dots\},$$
$$\{\cdots > -\epsilon_1 x_{i_1} > \epsilon_{\nu(2)} x_{i_{\nu(2)}} > \cdots > \epsilon_{\nu(r-1)} x_{i_{\nu(r-1)}} > -\epsilon_r x_{i_r} > \dots\},$$

and

$$\{\cdots > -\epsilon_r x_{i_r} > \epsilon_{\nu(2)} x_{i_{\nu(2)}} > \cdots > \epsilon_{\nu(r-1)} x_{i_{\nu(r-1)}} > -\epsilon_1 x_{i_1} > \dots\}.$$

Because of the coefficient $\frac{1}{2}$ in the multiplicity and the symmetry of the signed permutation, we just need to consider the chambers

$$\begin{aligned}&\{\epsilon_1 x_{\sigma(1)} > \epsilon_2 x_{\sigma(2)} > \cdots > \epsilon_n x_{\sigma(n)} > 0\} \\ &= \{\cdots > \epsilon_1 x_{i_1} > \epsilon_{\nu(2)} x_{i_{\nu(2)}} > \cdots > \epsilon_{\nu(r-1)} x_{i_{\nu(r-1)}} > \epsilon_r x_{i_r} > \dots\}\end{aligned}$$

and multiply the obtained cardinality by 2. Let $i \in [n]$ such that $\sigma(i) = i_1$. We have:

- $2^{n-r}(n-r)!$ possibilities for the sequence $\big(\epsilon_1 \sigma(1),\, \dots,\, \epsilon_{i-1}\sigma(i-1),\, \epsilon_{i+r}\sigma(i+r),\, \dots,\, \epsilon_n \sigma(n)\big)$,
- $(r-2)!$ possibilities for the sequence $\big(\epsilon_{i+1}\sigma(i+1),\, \dots,\, \epsilon_{i+r-2}\sigma(i+r-2)\big)$,
- and $n-r+1$ possibilities to choose i since we must have $i \in [n-r+1]$.

Then

$$\begin{aligned} l\big(E(J)\big) &= 2 \times 2^{n-r}(n-r)! \times (r-2)! \times (n-r+1) \\ &= 2^{n-r+1}(r-2)!(n-r+1)!. \end{aligned}$$

To the edge $E(I_0)$, assign the hyperplane H_{i_r} containing it. Now, we have to count the chambers $\{\epsilon_1 x_{\sigma(1)} > \epsilon_2 x_{\sigma(2)} > \cdots > \epsilon_n x_{\sigma(n)} > 0\}$ such that

$$\Big\langle H_{i_r} \cap \overline{\{\epsilon_1 x_{\sigma(1)} > \epsilon_2 x_{\sigma(2)} > \cdots > \epsilon_n x_{\sigma(n)} > 0\}} \Big\rangle = E(I_0).$$

Let ν be a permutation of $[r-1]$. Those chambers correspond to the chambers having the forms

$$\{\cdots > x_{i_r} > \epsilon_{\nu(r-1)} x_{i_{\nu(r-1)}} > \cdots > \epsilon_{\nu(2)} x_{i_{\nu(2)}} > \epsilon_{\nu(1)} x_{i_{\nu(1)}} > 0\}$$

$$\text{and} \quad \{\cdots > -x_{i_r} > \epsilon_{\nu(r-1)} x_{i_{\nu(r-1)}} > \cdots > \epsilon_{\nu(2)} x_{i_{\nu(2)}} > \epsilon_{\nu(1)} x_{i_{\nu(1)}} > 0\}.$$

Because of the coefficient $\frac{1}{2}$ in the multiplicity, we just need to consider the chambers

$$\begin{aligned} &\{\epsilon_1 x_{\sigma(1)} > \epsilon_2 x_{\sigma(2)} > \cdots > \epsilon_n x_{\sigma(n)} > 0\} \\ &= \{\cdots > x_{i_r} > \epsilon_{\nu(r-1)} x_{i_{\nu(r-1)}} > \cdots > \epsilon_{\nu(2)} x_{i_{\nu(2)}} > \epsilon_{\nu(1)} x_{i_{\nu(1)}} > 0\}. \end{aligned}$$

We have:

- $2^{n-r}(n-r)!$ possibilities for the sequence $\big(\epsilon_1\sigma(1), \ldots, \epsilon_{n-r}\sigma(n-r)\big)$,
- and $2^{r-1}(r-1)!$ possibilities for the sequence $\big((\epsilon_{n-r+2}\sigma(n-r+2), \ldots, \epsilon_n\sigma(n)\big)$.

Then $l\big(E(I_0)\big) = 2^{n-r}(n-r)! \times 2^{r-1}(r-1)! = 2^{n-1}(r-1)!(n-r)!$. $\square$

We obtain Proposition 3.4 by combining Proposition 2.2 and Lemma 3.6.

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Helen Bayly and Catherine Disney as influences in the life of Sir William Rowan Hamilton

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Abstract. In the 1880s Robert Graves published a biography about Sir William Rowan Hamilton (1805-1865), to which in a 1980 biography Thomas Hankins added further information. From these biographies a picture emerged of a man who was unhappily married because he had lost the love of his life, which raised the question how such an unhappy man could produce so much beautiful mathematics. In this article it is stated that a main cause for the unhappy picture is that Graves ignored the influence on one another of Hamilton and his wife Helen Bayly, and Hankins that of Hamilton and his first and lost love Catherine Disney. It is then shown that if these influences are taken into account a very different view on Hamilton's private life arises, in which he was happily married to a wife who enabled him to work as he needed to.

1. Introduction

In this article a largely ignored aspect of the private life of Sir William Rowan Hamilton (1805-1865) will be discussed, namely the influence on one another of Hamilton and the two most important women in his life, his first love Catherine Disney and his wife Helen Bayly. These two women have usually been described very statically; Catherine as Hamilton's only love, and Helen as a woman who simply was not strong enough for the marriage she was in. It led to a likewise static picture of Hamilton as a hopeless romantic for whom only his own feelings were guiding; an actually lonely man in an unhappy marriage because he had lost the love of his life.

It was wondered often how on earth such a man could produce so much beautiful mathematics and write so many kind and cordial letters to his correspondents. It was assumed to be due to his deeply religious feelings and his love for his mathematics, but it should have been a signal that something was wrong. The story of a life cannot

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be told while leaving out the influence of the people closest to the subject. It will be shown hereafter that taking this influence into account the picture of Hamilton becomes much more vivid, making it obvious that he led a life with ups and downs, as most people do.

The ideas in this article are based on the ideas developed in the essay *A Victorian Marriage : Sir William Rowan Hamilton* [3], and summarized in the first part of the article 'A most gossiped about genius: Sir William Rowan Hamilton' [4]. These ideas were in turn based on the enormous, three-volumed biography which was published in the 1880s, about twenty years after Hamilton's death, by Robert Graves [1], who mainly focused on Hamilton's private life. A second biography was published in 1980 by Thomas Hankins [2], who mainly focused on Hamilton's mathematics and metaphysics, but also gave parts of Hamilton's private life not given by Graves, especially concerning Catherine Disney.

Hamilton's private life will be discussed here while including the facts which somehow could be verified, but without the gossipy parts which were added in the course of years, as was shown in [4]. Hamilton lived most of his adult life in a time in which temperance was an important issue, and the original Dublin gossip might have been friendlier if Hamilton had stopped drinking alcohol.[1] Yet he did not want to stop; he liked drinking wine at dinners, and apparently took his reputation in that respect not very seriously.[2] Because his way of drinking alcohol as looked at from our times shows nothing out of the ordinary, anything related to the alcohol will be left out; it was extensively discussed in [3].

[1]Apart from the alcohol, the main view in the gossip seems to have been that Hamilton was a totally unworldly genius. That was a widespread idea, which became part of the later caricatural view on Hamilton. For instance, Hamilton once joked about baking bread from sawdust and so surviving being snowed in. That story has been used as an example of how unworldly he was, see for instance p. 88 of O'Donnell (1983), *William Rowan Hamilton: portrait of a prodigy.* Dublin: Boole Press. But it was something John Herschel had actually written about, see *The Cabinet Cyclopædia*, 1831, p 65. https://archive.org/details/preliminary disco00hers_0. That it was a joke can be seen easily; Hamilton continued about baking pancakes of snow and then imagining that every day would be Shrove Tuesday, or Pancake Day.

[2]As regards his reputation as a mathematician that was almost the opposite: Hamilton took very much trouble to make sure that he was not plagiarizing, and to credit everyone who could be entitled to it.

Deviations from Graves' and Hankins' biographies will be given without further comments, and therefore also referencing will be omitted; all following statements can easily be found in the aforementioned sources. However, reading original letters, many of which are kept in the Library of Trinity College Dublin, or other original sources, may still throw another light on this matter. If it would be discovered that original sources indicate actual errors in the sources used here, then these errors will also be found in this article.

2. An ideal and a bystander

In Graves' biography Hamilton is described as having had a good control over the work-related and public part of his life; according to Graves he was, for instance, a very good public speaker and lecturer, and effective in many presidencies, including that of the Royal Irish Academy. But Graves' opinion was completely different as regards Hamilton's private life.

Hamilton fell in love with Catherine Disney when he was nineteen years old. But she married the reverend Barlow instead, and it took Hamilton years of trying to cope with this "wound of his affections," as Graves called it. At first glance, Graves is very complimentary about Catherine, in the few sentences he writes about her mentioning that she was "of singular beauty, amiable, sensitive, and pious;" the ideal Victorian woman.

But love and marriage could not be discussed openly in Victorian times and when Graves relates that, shortly before Catherine died, Hamilton was allowed to speak with her twice, he is very cautious, Hamilton then was, after all, a married man. Graves takes care not to reveal her identity and not to say anything about her directly, he only vaguely indicates that the "interviews" contained "explanations" by "the departing Christian." Thus by describing Catherine as beautiful and pious yet telling hardly anything about herself, Graves diminished her into a romantic ideal.

This ideal image of Catherine was further enhanced by Graves' negative view on Helen Bayly. Also about her he could not be very open because of the times they lived in, but he did not abstain from relating his very critical feelings for her. Graves was of the opinion that Helen should have forced her husband to live an orderly life, instead of allowing him to work through the night and skip meals when some mathematical investigation was too interesting to stop, or sip beer in order to stay awake when he became tired yet wanted to finish something. This irregular lifestyle and the regularity of sipping beer was, according to Graves, the very cause of everything that went wrong as regards Hamilton's bad Dublin reputation.[3]

In Graves' eyes Helen had been a weak wife; weak of body because she was very often ill, and weak of mind because she had not been able to keep her husband under control. Although Graves sometimes has to remark that she was a good woman, for instance because other people said she was, he immediately drenches these notions in criticisms, as if afraid that the reader would not understand well enough that Hamilton's bad reputation was due to her shortcomings. Yet in this way portraying Hamilton as someone who could not take care of himself and had to be controlled by his wife, while calling him a "simple, zealous great man," Graves diminished Hamilton into a genius without self-insight or full personal relationships.

In the biography Graves introduces Helen with the remark that when in the summer of 1832 she had been very ill, Hamilton's 'anxiousness for her recovery,' "coming at a time when he had felt obliged to suppress his former passion,[4] prepared the way for tenderer and warmer feelings." Nothing is left of Hamilton's love for her, even though he was very open about it in his poems. But what is worse, because Graves hardly gives information about Helen as a person by which the reader would have been able to learn to know her, it can be concluded from the biography that Helen had remained "a shadowy figure in Hamilton's life" as Hankins expressed it,[5] diminishing her into a bystander in her own marriage.

Graves' biography consists of letters by Hamilton and correspondents, commented on by Graves. His goal was to restore Hamilton's bad reputation, but because of this constant focus his descriptions became static. In his biography people do not develop; in the forty-one years between Hamilton's meeting Catherine and his death, and the thirty-two years of the Hamilton marriage, not anything changes as a result of what happens between these people. But Graves does not seem to have recognised this very strange picture; he was trying mightily to show how wonderful Hamilton's character had been, something he repeats throughout the biography.

[3] See for the details the article 'A most gossiped about genius' [4].

[4] Graves alludes here to Ellen de Vere, see p. 96.

[5] [2, 114]. Next to discussing Catherine, Hankins also added information about Helen. However, it was not enough to form a good picture of them.

3. Three loves

Through his letters much is known about Hamilton, but because so little is known about Catherine Disney and Helen Bayly, the influence they had on each other can only be shown by retelling and reinterpreting what happened, while leaving out Graves' comments. It appears that Helen Bayly was not just a weak woman Hamilton happened to marry; Hamilton loved her and trusted her because she was very truthful. And Catherine was not simply Hamilton's only love as Hankins concluded; while being happily married he felt deeply for her when over the years he slowly discovered how extremely unhappy her marriage was.

Next to Catherine and Helen, Hamilton also fell in love with Ellen de Vere, yet that was different; although Ellen did influence Hamilton's life it was indirectly, and he did not evidently influence hers. Moreover, a few years later she married happily, and there thus was no reason for Hamilton to become worried about her and actively bring her back into his life, as he did with Catherine.

Catherine Disney and Ellen de Vere. Hamilton saw Catherine for the first time in August 1824 when he visited Summerhill with his uncle, and he immediately fell in love with her. He had befriended some of her brothers who were, like him, attending Trinity College, and for some months Hamilton and Catherine seem to have seen each other often. Surrounded by her close family they talked, and she played the harp while he listened intensely. Until, on a terrible day in February 1825, her mother told him that Catherine was going to marry in May.

Catherine was forced by her family to marry the reverend Barlow, but from his poems it is clear that Hamilton did not know that. Feeling devastated he did not attend the wedding; had he done so he might have seen her utter unhappiness. He wished her a very happy life in a poem he did not send to her and tried to get on with his life while "maintaining a philosophical calm." That worked partially, in the sense that he was able to work on his mathematics, but it did not help to come to terms with his loss.

Having become Astronomer Royal in 1827, Hamilton moved into Dunsink Observatory. In 1830 he visited Armagh Observatory, and because Catherine lived with her family in the neighbourhood he decided to visit her, obviously expecting a happy family. During the visit Hamilton saw, as he wrote in a poem, a "meek and tender sorrow" in Catherine's eyes, and he realized that the marriage which had "threw a gloom" over his "once bright way" had not made her as happy as he had expected. After the visit Hamilton became 'morbidly despondent,' and he had to be comforted by his friend Lady Pamela Campbell. But because marriages were sacred and revered then, there was nothing he could do for Catherine.

Towards the end of 1831, when visiting the parents of his pupil Lord Adare, Hamilton fell in love with Ellen de Vere. But just before he could ask her she rejected him. Hamilton later wrote to Lady Campbell, "I have had another affliction of the same kind and indeed of the same degree, except that my mind had been a little better disciplined to receive it."

Nevertheless, he became very melancholic for some months, until in the summer of 1832 he made a remarkable psychological discovery. He finally understood that having been for "nearly eight years in a state of mental suffering, with lucid intervals indeed," had yielded him a "passion-wasted life." He wrote to his friend Aubrey de Vere, one of Ellen's brothers, "I determined that I would vigilantly and resolutely exclude all voluntary recollection of your sister, and refuse, so far as in me lay, to indulge myself by dwelling on involuntary remembrance. The determination was well fulfilled: and this vigilant and resolute self-denial, combined with ardent and persevering exertion during some months in abstract science, had its effect in restoring my tone of mind and even my health of body, which had begun to suffer sensibly. The power of hope revived." Only a few weeks thereafter Hamilton discovered conical refraction for which he would be knighted, and fell in love with Helen Bayly.

Helen Bayly. Hamilton learned to know Helen Bayly during visits she made to two of her elder sisters. She came from a very large family, and although she was born in Nenagh, Tipperary, many of her relatives lived in Dublin. The two sisters lived with their families in the townland of Scribblestown, and both their houses were located within a ten-minute walk from Dunsink Observatory. After having moved into the observatory Hamilton soon befriended them, apparently especially the family at Scripplestown House. He became impressed by Helen's truthful character because, for instance, she said to him that she preferred his sister's poetry over his, which then was not a very common thing to do.

They seem to have met each other regularly, and Helen will have seen how Hamilton tried to cope with the loss of his first love, Catherine Disney. It is known from Hamilton's poems that in 1831 she saw him fall in love with Ellen de Vere, and early in 1832 how difficult it was for him to come to terms with her rejection. Hamilton indeed never kept silent about these feelings, sending the poems about how he felt to many people.

When early in November Hamilton asked Helen to marry him, she was very hesitant. It had nothing to do with whether or not she thought him loveable enough, and she will doubtlessly have noticed how Hamilton changed after his psychological discovery; she seems to have been insecure about her weak health. Helen was very often ill and sometimes her illnesses could not be diagnosed, which means that she may have suffered from some chronic illness, or perhaps from allergies, of which nothing was known then. As mentioned, in the summer of 1832 she had fallen dangerously ill. She then was staying at Scripplestown and Hamilton had been very worried about her; he was completely aware of her weak health.

Trying to convince her to marry him Hamilton assured her that he did not think her weak health would be a reason for them not to become happy together, but at that time he was becoming more famous by the day. Helen obviously realized what a household for such a man would mean for her, and she knew that having to receive and organize visits to very many important people would be far too strenuous for her.[6] She apparently decided only to accept if Hamilton would promise her that they would live a retired life at the observatory. Hamilton made this promise, and that was the marriage they agreed upon.

The retiredness concerned official dinners, balls, and Victorian high-society life; it had nothing to do with visiting and being visited by friends and family, connections they had in abundance. It is known that Helen frequently received guests or made visits to family members and friends, such visits often taking multiple days. Hamilton's uncle wrote about Helen that she had "“won golden opinions” from her extensive circle of acquaintance," and even Graves wrote that she won the "good opinion of [Hamilton's] friends." But as they had agreed upon, his eminent friends hardly knew her.

[6]Many of these people brought staff and servants with them; organizing such visits must have been enormous undertakings.

4. Marriage

They married in April 1833, and the first years of the marriage were very happy indeed. One of the things which apparently made the marriage a success was that Helen understood what Hamilton needed to be able to work so hard. Already before the marriage Hamilton had told her about how he worked, writing that he "was up almost the whole of Monday night, in the pains or pleasures of thought-birth, mathematical views springing up in almost oppressive variety: so that even when I went to bed I could scarcely sleep, and was greatly exhausted the next morning." When he was in one of his "mathematical trances" she did not force him to come to dinner or to bed, she instead took care that some food was brought into his study if he did not react to the dinner bell.

But with Hamilton's growing fame he befriended more and more members of the higher classes, and on a beautiful day in September 1838 he asked her to accompany him on a three-week visit to Lord Northampton at Castle Ashby in England, to which she agreed. Already a few days later, on the same day a party of scientific men visited the observatory, she fell ill again, and she must have felt terrible being reminded of her weak health so soon after having agreed to such a high-class visit.

The visit took place, and at the castle Hamilton enjoyed himself greatly. Thereafter he started to fantasize about many more of such enjoyable visits with her at his side; he was clearly forgetting his ante-nuptial promise. Towards the end of 1839, being pregnant with their third child, Helen fell ill again, and was diagnosed with her first nervous illness. Pregnancies having been outright dangerous then she apparently felt very vulnerable, and because of the unrest in the country she became afraid of staying at such a remote place as the observatory. They took lodgings for her in Dublin.

She came back in August 1840 to give birth to their youngest child, then stayed at Scripplestown for some months, presumably to breastfeed her baby, and then went to England to stay with a sister. Hamilton missed her terribly, and in May 1841 he wrote to a friend, "As to scientific work [...] I have done very little, for a whole year past, that is, since Lady Hamilton's health obliged her to leave the Observatory." When in the summer Helen also became physically very ill Hamilton hurried to England to be with her. Towards the end of the year she finally regained her health, and in January 1842 Hamilton went to England to bring her home again.[7] According to Graves, "with renewed cheerfulness" Hamilton "resumed his mathematical studies."

Although Helen was used to speaking her mind, she apparently could not speak easily about her own needs in the marriage. That may have had to do with the then large inequality between men and women, in which a woman's reputation, socially and as a Christian, solely depended on being a good and obedient wife and mother. Having been very pious and wanting to be a good wife she may have tried to grant Hamilton the pleasure of her company on his visits to high-classed friends, but such visits often lasting a week or more, and the journeys taking days and having been very exhausting, it is easy to imagine that she had become afraid of falling ill during a journey or a visit; she could fall ill within a few hours. It seems very well possible that only in England, or on the long way home, she could finally remind her husband of his promise about their retired life. After they came home he never asked that of her again.[8]

5. Renewed contact with Catherine

When in 1847 Catherine's eldest son received his bachelor's degree, he decided to compete for a fellowship. Hamilton tutored him in mathematics, and in July 1848 Catherine wrote to him to thank him for it. It led to a correspondence which lasted for six weeks, according to Graves the only time Hamilton really interrupted his work. The correspondence was distressing for both of them, and apparently having assumed that her marriage had started well but had become unhappy, Hamilton then learned that Catherine's marriage had been unhappy from the start.

After six weeks Catherine confessed to Barlow that she had been corresponding with Hamilton, something he had forbidden in 1830, when after Hamilton's visit in Armagh she had asked him whether she could keep contact with Hamilton. Having vowed obedience at

[7]To see these events in context, it must not be forgotten that hardly anything was known about psychology; not about nervous breakdowns and not about the importance of bonds between parents and children. In their circles the children were usually taken care of by personnel, a very common thing to do; also during the three-week visit to Castle Ashby the children seem to have stayed in Dublin.

[8]In 1853 Hamilton successfully persuaded her to come with him to meet the Queen. But the invitation had come earlier on the same day; she may simply have felt well that day.

the altar she seems to have considered her secret correspondence as unforgivable; her heartbreaking last line to Hamilton was, "To the mercy of God in Christ I look alone, for pardon for all my sins."

Catherine decided to commit suicide, and wanted Hamilton to play a part in it. Early in October 1848 she sent him a letter, containing a letter she had written to her husband, a stamped envelope, and instructions to send the letter in the stamped envelope to her husband, who would then receive it after her death. But Hamilton did not send her suicide letter to Barlow. He believed that she had mentally broken down, and called the letter, which was written in "open defiance of her husband," her "last rational letter." Catherine survived, but thereafter lived mainly with family.

Through her brother Thomas and his wife Dora every now and then Catherine and Hamilton sent each other gifts, and in October 1853 she sent him a gift through which he knew she was dying. He went to visit her, and was allowed to speak with her twice. Only during these interviews she could finally tell him that her family had forced her to marry Barlow, that she had loved Hamilton and had wanted to marry him. It is easy to understand what impact that had on Hamilton; he had not just lost her, something he had to learn to cope with for years, but he had lost a marriage they both had wanted.

It took him quite some time to come to terms with such devastating knowledge, yet it had nothing to do with not having been married happily. Catherine had been forced into a marriage by the family she trusted, and Barlow had insisted on marrying Catherine although she did not want him; these realizations must have made it even more difficult for Hamilton, who suffered when people close to him were unwell, and was known for respecting people more than was usual for a man in his position.[9]

6. Difficult memories and quiet last years

After Catherine's death in November 1853, Hamilton wrote very many letters trying to cope with his feelings. Early in 1854 he calmed down again, and life finally seemed to become a bit easier,

[9]As an example of his respect for every human being, Hamilton's friend Augustus De Morgan wrote: "When some housebreakers were caught on the premises, and detained until they could be carried before a magistrate, [Hamilton directed] that the felons should be asked whether they preferred tea or milk for breakfast."

making Helen openly happy and satisfied with her husband. She proudly declared that her husband had "grown quite a good boy of late – so sociable and neighbourly," although she feared that "it was too good a thing to last." But then, in 1855, the reading of old letters his deceased sister Eliza had left him triggered Hamilton to dwell on the past again and open up to his friend Aubrey de Vere about all that had happened with Catherine, and that whole summer they corresponded intensely.

Most likely not having been able to talk about it with his "local friends" because marriages were an almost forbidden subject, Hamilton started more correspondences about Catherine, for instance with Catherine's sister-in-law Dora Disney. Helen had always accepted that Hamilton had also loved Catherine and Ellen; he had never withheld that from her. Neither had it been a problem that he had correspondences and close friendships with women. But somehow this was different, and when in June 1855 Helen found one of Dora's letters in her husband's pocket she became jealous and made a row.

It still took Hamilton some months before he took her seriously and was able to tear himself out of the past again, but by that time it was too late. Apparently frustrated by not being able to reach or comfort him and starting to lose all hope, Helen had slipped into her second 'nervous illness.' Hamilton became very worried about her, and he nursed her for months.

To fall so ill in such a situation is again not as strange as it may seem. Next to fearing that she would lose a loving husband, who took her weak health with good spirits, nursed her when she needed him to and did not ask impossible things from her, if women in their times were abandoned or granted a divorce, they lost everything. With Helen's weak health that would have been even more problematic than it was for other women because she would hardly be able to earn herself an income; she must have felt terrible.

Hamilton in the meantime will have felt very guilty of what he had done; he revered marriage and saw their bond as sacred, yet he had not honoured and kept her, as he had promised at the altar. In the summer of 1856 Helen recovered, and thereafter there were no more letters about Catherine. Except in the summer of 1861, when one of the younger Disney sisters heard about the love story and wanted Hamilton to tell her about it. Hamilton found it difficult again, but now it took him much less time to come back into his own life.

The last decade of the marriage therefore seems to have been generally quiet and trustful, as is often the case when partners have managed to survive heavy crises. The socially very strict time they lived in gave both Hamiltons very little room to move, which in Helen's case meant that she could not easily show that she was becoming unhappy, and in Hamilton's case that he could not openly express his grief about Catherine's unhappiness. But it did not mean at all that they did not love each other; according to Graves, the Hamiltons remained attached to each other until the end.

7. Conclusion

It has been argued here that one of the reasons for the widely accepted view on Hamilton, as having been unhappily married, is that in his two main biographies the influence which Hamilton and the two most important women in his life had on each other was largely ignored, making their descriptions static. For instance, the fact that Helen became ill did not seem to have had anything to do with what happened between her and Hamilton, as if she just was weak and he had unfortunately chosen the wrong wife. And Hamilton's feelings for Catherine did not seem to have had anything to do with her gruesome fate, as if from the day he met her he just loved her no matter what, and the wonderfully loving romantic became unhappy for the rest of his life.

It then was shown, by retelling their story while taking the influences these people had on each other into account this static picture changes, and Hamilton can be seen as a man who was married happily to a loving and understanding wife, and who had a conceivable deep grief about his first love whose life had been made miserable by the people she had trusted.

Various conclusions can be drawn here. In the Victorian era women were subordinate to men, and therefore the readers of Graves' biography were not alarmed at all; it was perfectly accepted that a woman's influence on a man's ideas could be ignored, as if he would have had those ideas regardless of which women surrounded him. But after the Victorian era was over and even intensively studied, the story of Hamilton's life was not adjusted to contemporary knowledge. One conclusion therefore is that many more widely accepted views on people or events may contain flaws because the role played by women was ignored.

Another conclusion is that if a description of a person or an event seems illogical although the facts seem to have been well represented, there will be aspects or nuances which have been misinterpreted or misjudged. In Hamilton's case it was often remarked that who Hamilton was as a mathematician seemed irreconcilable with how he was described as a person. Uneasy feelings were hushed by the notion that Graves had been a friend of Hamilton, and that therefore his description must have been the truth. But what was ignored or overlooked was Graves' motive for the biography; he focused very hard and very constantly on restoring Hamilton's reputation. It caused his truth to become coloured, with Hamilton's unhappy description as a result. Illogicality should be something to react upon, even in biographies.

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Anne van Weerden Works in the Utrecht University Library and also studied physics. After a 2014 seminar about the history of vector analysis she became intrigued with the very unhappy reputation of Sir William Rowan Hamilton. Having become convinced that many claims about his private life could not be true she decided to try to find out where they came from, and show how the information given in his two main biographies can be read very differently.

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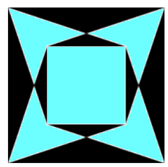

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PROBLEMS

IAN SHORT

Problems

We begin with two problems by Finbarr Holland of University College Cork. The first of these uses the usual notation $x_1, x_2, \ldots, x_n$ for the components of a vector x in $\mathbb{R}^n$.

Problem 82.1. Suppose that u and v are linearly independent vectors in $\mathbb{R}^n$ with

$$0 < u_1 \leqslant u_2 \leqslant \cdots \leqslant u_n \quad \text{and} \quad v_1 > v_2 > \cdots > v_n.$$

Given $x \in \mathbb{R}^n$, let y be the orthogonal projection of x onto the subspace spanned by u and v; thus $y = \lambda u + \mu v$, for uniquely determined real numbers λ and μ. Prove that if

$$x_1 > x_2 > \cdots > x_n,$$

then μ is positive.

Problem 82.2. Prove that

$$\int_0^\infty \frac{\sinh x - x}{x^2 \sinh x}\, dx = \log 2.$$

Readers may also like to attempt to prove the integral formula

$$\int_0^\infty \frac{\sinh x - x}{x^2 \sinh x}\, e^{-x}\, dx = \log \pi - 1.$$

The third problem appeared in the magazine of the M500 Society a few years ago. The M500 society is a mathematical society for those associated to the Open University.

Problem 82.3. Prove that

$$\sum_{n=1}^{\infty} \frac{1}{(5n-3)(5n-2)} = \frac{\pi}{5}\sqrt{1 - \frac{2}{\sqrt{5}}}.$$

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Solutions

Here are solutions to the problems from *Bulletin* Number 80. The first problem was solved by the North Kildare Mathematics Problem Club and the proposer, Finbarr Holland. We present the solution of the problem club.

*Problem 80.1***.** Let $x_0, x_1, x_2, \dots$ be a null sequence generated by the recurrence relation

$$(n+1)(x_{n+1}+x_n) = 1, \quad n = 0, 1, 2, \dots .$$

Prove that the series

$$\sum_{n=0}^{\infty} (-1)^n x_n$$

converges, and determine its sum.

Solution 80.1. For $n = 0, 1, 2, \dots$, we have

$$x_{n+1} = -x_n + \frac{1}{n+1},$$

so

$$x_{n+2} = x_n - \frac{1}{n+1} + \frac{1}{n+2}.$$

Therefore, by induction,

$$x_{2n} = x_0 - S_{2n}, \quad \text{for } n = 0, 1, 2, \dots,$$

where S_n is the sum to n terms of the alternating harmonic series:

$$S_n = 1 - \frac{1}{2} + \frac{1}{3} - \cdots + (-1)^{n+1}\frac{1}{n}.$$

Since $x_0, x_1, x_2, \dots$ is a null sequence, we have $x_0 = \lim_{n\to\infty} S_n = \ln 2$, so now we just have to show that

$$\sum_{n=0}^{\infty} (x_{2n} - x_{2n+1})$$

converges, and to determine its sum.

We have

$$x_{2n} - x_{2n+1} = 2(\ln 2 - S_{2n}) - \frac{1}{2n+1}.$$

For the next part we use an argument of N. Kazarinoff (*Analytic Inequalities*, Holt, Reinhart and Winston, New York, 1961, 45–46) that evaluates $\ln 2 - S_n$ in terms of the integral

$$I(n) = \int_0^{\frac{\pi}{4}} \tan^n \theta \, d\theta.$$

In fact, a simple recursive argument gives

$$I(2n+1) = \frac{1}{2}\ln 2 - \frac{1}{2}S_n,$$

so

$$x_{2n} - x_{2n+1} = 4I(4n+1) - \frac{1}{2n+1}.$$

Observing that

$$\frac{1}{2n+1} = 2\int_0^{\pi/4} \tan^{4n+1}\theta \sec^2\theta = 2I(4n+1) + 2I(4n+3),$$

we obtain

$$x_{2n} - x_{2n+1} = 2(I(4n+1) - I(4n+3)).$$

Hence

$$\sum_{n=0}^{\infty}(x_{2n} - x_{2n+1}) = 2\int_0^{\pi/4} \sum_{m=0}^{\infty}(-1)^m \tan^{2m+1}\theta \, d\theta,$$

which equals

$$2\int_0^{\pi/4} \frac{\tan\theta}{1+\tan^2\theta}\, d\theta = \int_0^{\pi/4} \sin 2\theta \, d\theta = \frac{1}{2}. \qquad \square$$

No correct solutions have been received for the second problem, besides the solution of the proposer, J.P. McCarthy of the Cork Institute of Technology. Here is the problem again in case you want a second crack at it.

Problem 80.2. Let

$$S(\sigma) = \sum_{i=1}^{n} \frac{1}{\sqrt{n^{i+\sigma(i)}}},$$

where σ is a nonidentity permutation of $\{1, 2, \ldots, n\}$. Find the maximum of S over all such permutations.

The third problem was quoted verbatim from *Lectures and Problems: A Gift to Young Mathematicians*, by V.I. Arnold. It was solved by Henry Ricardo of the Westchester Area Math Circle, New York, USA and the North Kildare Mathematics Problem Club. Both solutions were essentially the same.

Problem 80.3. Two volumes of Pushkin, the first and the second, are side-by-side on a bookshelf. The pages of each volume are 2cm thick, and the front and back covers are each 2mm thick. A bookworm has gnawed through (perpendicular to the pages) from the first page of volume 1 to the last page of volume 2. How long is the bookworm's track?

Solution 80.3. It was pointed out that the problem is ambiguous in various ways; however, with the reasonable assumption that volume 1 is to the left of volume 2 on the shelf the answer is 4mm, since the bookworm gnaws through two covers. □

Arnold comments in his book that he posed this problem in a published paper, along with the solution 4mm. However, apparently the journal editors did not trust this answer, so they edited the second-last sentence of the question to say 'from the *last* page of volume 1 to the *first* page of volume 2', thereby creating an error which appeared in the published manuscript!

We invite readers to submit problems and solutions. Please email submissions to `imsproblems@gmail.com` in any format (we prefer Latex). Submissions for the summer Bulletin should arrive before the end of April, and submissions for the winter Bulletin should arrive by October. The solution to a problem is published two issues after the issue in which the problem first appeared. Please include solutions to any problems you submit, if you have them.

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