

Leaving Cert Points: Do the Math?

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ABSTRACT. In this paper we look at the Leaving Cert results from 2002–2016. We consider them in the light that many students will be choosing subjects in a way that will maximise their points. We observe that in a market for subjects, similar to the job market, a “pragmatic” student who needs points will choose subjects that offer them the highest points. As English and maths are examined across, more-or-less, the full Leaving Cert student cohort, they are uniquely comparable. Irish, which is studied by many students, also provides a useful reference point. We compare the results of these subjects over the period. In line with other authors, we identify a number of quirks of the grading system. We conclude that the statistics indicate that the mean/mode/median pragmatic student would choose to invest their effort in English over Irish and Irish over mathematics. We also highlight that normalisation of grades can cause feedback, resulting in easy subjects becoming easier and harder subjects becoming harder. We conclude that if the uptake of higher-level mathematics is to be increased, then the subject should be made more attractive in the subject marketplace. We discuss some possible ways that this could be achieved, including the awarding of bonus points.

1. INTRODUCTION

The Leaving Certificate is the final exam of the Irish second-level education system. Places in third level education are generally allocated to students based on a *points system*, where students are awarded points based on the level at which they take a subject and the grade they receive. In this system, apart from some rare exceptions, points are awarded equally for all subjects and a student’s final points are the sum of the points from their best six subjects. These points are important to students. As typically observed [10],

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students often respond to the question “How did you do in the Leaving Cert?” by saying how many points they got, and so make efforts to maximise their points.

Students facing into the Leaving Cert have a number of choices to make. They have options regarding the subjects that they take, and options regarding the level at which they take these subjects. It seems likely that most students will aim to maximise their points, subject to the options available (e.g., subject choices and levels available at the school), the resources available (e.g., time, effort, available educational support) and information regarding how their efforts will be rewarded. This has been observed in interviews with students [20]. It is in the best interests of parents, students and teachers to be well informed about the ‘points race’, so while there will be rumour and hearsay, it seems likely that their information will be based on fact. As the majority of students only take the Leaving Cert examinations once, an important input to their decision will be the performance of previous cohorts of students.

Thus, we find ourselves in a situation where there may be a competitive market in Leaving Cert subjects. Students have a certain amount of effort to spend in order to get as many points as possible. All things being equal, we expect that subjects that give a greater reward for fixed effort will be chosen over more challenging subjects. Of course, directly comparing most subjects is difficult, for example, the availability of French and Ancient Greek teachers in schools will typically be quite different. Previous work [13, 9, 20] has shown that factors including choices in the junior cycle, school size and educational advantage can have an impact on course availability and students’ achievement.

However, three subjects are available to all students: English, Irish and mathematics. Indeed, most students have no choice but to take all three of these subjects. Figure 1 shows how many students were taking these subjects for the years 2002–2016, based on the statistics from the www.examinations.ie website. We can see that the numbers taking English and maths are similar, while Irish lags by about 10% (presumably due to some students being exempt from taking Irish).

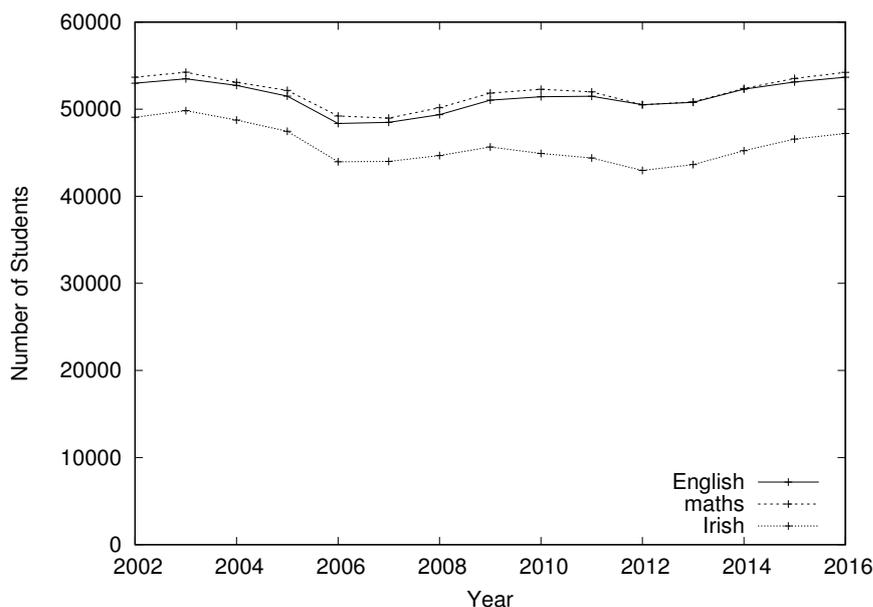


FIGURE 1. The number of students taking English, Irish and maths from 2002–2016.

These students represent the vast majority of the Leaving Cert cohort¹. The students taking English and maths will essentially be the same people. This group will, more-or-less, overlap with all the Irish students. It seems that, by comparing performance in these subjects, we should be able to draw some conclusions about the relative rewards offered by these subjects.

The www.examinations.ie website publishes an annual report containing a summary of the Leaving Cert results. For each subject, it gives the number of students taking the subject at each level and the percentage breakdown of how many students achieve each grade at each level for the three years prior to the report. Historical reports are available, and results presented here are based on the results from 2002–2016. The points awarded for each grade and level are shown in Table 1. Note that from 2012 onwards, 25 “bonus points” were offered to students passing higher-level mathematics. From 2017 onwards there were some changes to the Leaving Cert grading system and points system. While the principles remain similar, the details have changed, and so we do not consider data from 2017 here.

¹Note, a significant fraction of students, between 10–20%, do not reach the Leaving Cert [1].

Grade	A1	A2	B1	B2	B3	C1	C2	C3	D1	D2	D3	E	F	NG
Higher	100	90	85	80	75	70	65	60	55	50	45	0	0	0
Ordinary	60	50	45	40	35	30	25	20	15	10	5	0	0	0
Foundation	20	15	10	5	0	0	0	0	0	0	0	0	0	0

TABLE 1. Points offered for each grade at each level. From 2012 onwards a bonus of 25 points was given for higher-level mathematics at D grades and above. Not all institutions offer points for subjects at foundation level.

We are not the first to analyse and compare results in the Leaving Certificate. The issue of variable grading practices between subjects and levels has been previously observed [12]. In addition to quirks of the marking system, which we also identify here, this work also observes that the “difficulty” of subjects is not simply related to which grades are given out, but how easy these grades are to achieve. Earlier trends in the uptake and points awarded for the twenty most commonly taken Leaving Certificate subjects have also been studied [14]. The same work also identifies substantial variation between subjects, and particularly the significant impact of the proportion of students who choose to take subjects at ordinary and higher level.

2. A LOOK AT THE DATA

Let us consider a number of different ways to compare the performance of students in English, Irish and mathematics. Using the points system, we will first consider the average performance, which indicates there is a significant difference in the points awarded. We will show that this difference arises because of the level at which the students have taken the different subjects, and suggest that one explanation for this is pragmatic students voting with their feet.

However, average performance may not tell the full story [12]. We will next look at the the cumulative performance, showing the number (or fraction) of students achieving at least some number of points. This shows that the quantiles of English, Irish and maths are strictly ordered², with English awarding higher points to more candidates.

Finally, we consider how many students achieve a particular number of points. This lets us study the mode number of points awarded,

²This can be termed stochastic dominance.

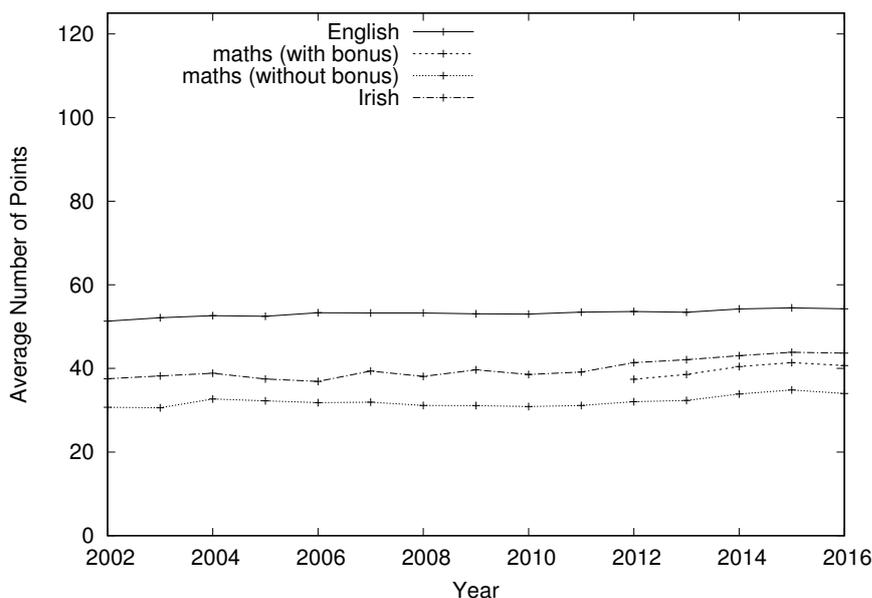


FIGURE 2. The average number of points given to students taking English, Irish and maths from 2002–2016.

and makes clearer the impact of fail grades, which may again impact the choices of pragmatic students. It also reveals a number of quirks of the Leaving Certificate marking system.

2.1. The Average Student. Figure 2 shows the average number of points given to students of English, Irish and maths (i.e., the total number of points given divided by the total number of students taking the subject at any level), from 2002–2016. We see that English gives about 53 points per student, Irish gives about 40 points per student and maths about 32 points per student (or 38 in 2012–2016 when the bonus is factored in). Clearly there is something different about these three subjects.

While, technically, students are not required to take English, Irish and maths, for a substantial majority of students these subjects are effectively compulsory. Consequently, the only choice available to most students regarding English, Irish and maths is the level at which they take these subjects. On the whole, students will be making choices about the level they take these subjects based on the effort to reward tradeoff. If we look at the average number of points at each level in these subjects (see Figure 3), we see that the higher-level subjects all give about 70 points per student and the ordinary-level subjects all give about 25 points per student. Foundation level

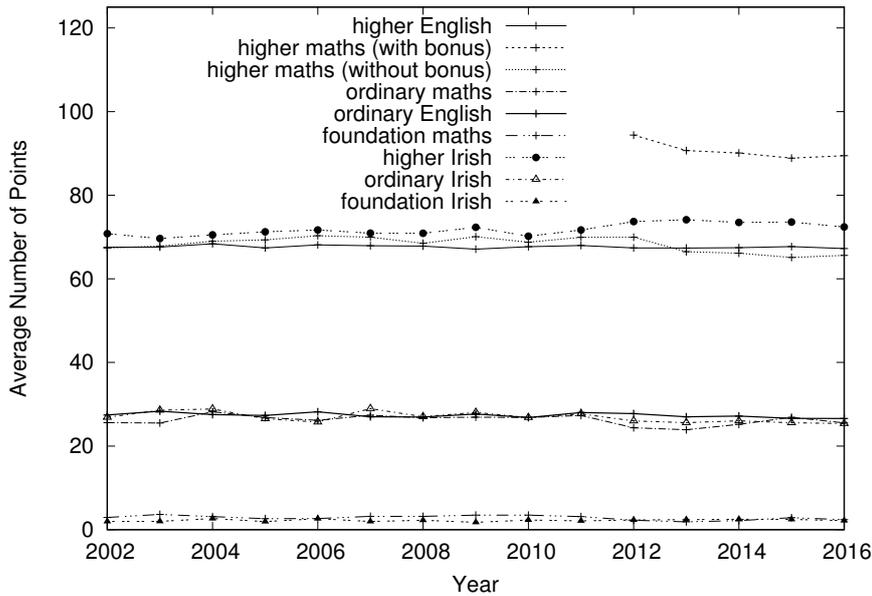


FIGURE 3. The average number of points given to students taking English, Irish and maths at a particular level from 2002–2016.

Irish and maths give about 2 points per student (English is not offered at foundation level³). The results at each level are quite consistent across subjects.

So, where does the difference in average reward come from? It arises from students voting with their feet. Figure 4 shows a consistent picture from year-to-year, where more than half of Leaving Cert candidates take English at higher level. From 2002–2013 around one quarter take higher-level Irish and one fifth take higher-level maths, but there has been an increase in the fraction taking higher Irish and maths since 2012, corresponding to changes in the courses and points. The smaller numbers of students at higher level for Irish and maths then result in lower overall average points. The importance of the level at which students take a subject to the average performance has also been highlighted before [14].

Why are students shying away from Irish and maths at higher level? While both Irish and maths might be considered to have an

³We believe it is important to include foundation level in this study, because to omit these students would make the maths and Irish cohort an artificially smaller group of stronger students. We also note that some institutions do not offer points for foundation level Irish/Maths. As we will see, considering foundation level to offer zero points would not make a significant difference to our results.

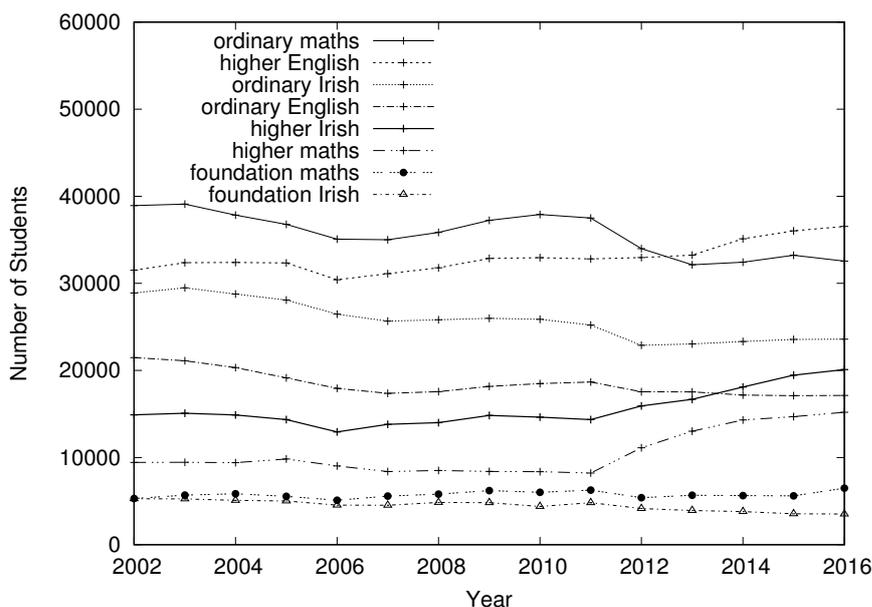


FIGURE 4. The number of students taking English, Irish and maths at a particular level from 2002–2016.

image problem, the competition for points means that students are unlikely to abandon these subjects because of image alone. Certainly, there are some differences in the availability of higher-level subjects in various schools⁴, but it seems unlikely that image or availability can explain such a large difference in numbers.

One possible explanation is that students have determined that the effort required to get a high maths grade will be better rewarded by if it is spent on other subjects. This could be English (or Irish), but might also be other optional subjects. There is anecdotal evidence of students dropping from higher maths to ordinary maths and taking up an optional subject at higher level to boost their points. This behaviour has been reported in a survey of UCD Economics and Finance Students [16] and is probably more widespread [6, 15].

2.2. On the Curve. So far, we have looked at average points, but other statistics tell a similar story. As students aim for *at least* a particular number of points, rather than an exact target, the cumulative statistics are particularly useful. Figure 5 shows the cumulative statistics for 2003, i.e. the number of students achieving at least some

⁴Information on how many schools provide each subject is available in Section 4 of the Department of Education statistics [4] and also reported annually at <http://www.education.ie/en/Publications/Statistics/Statistical-Reports/>.

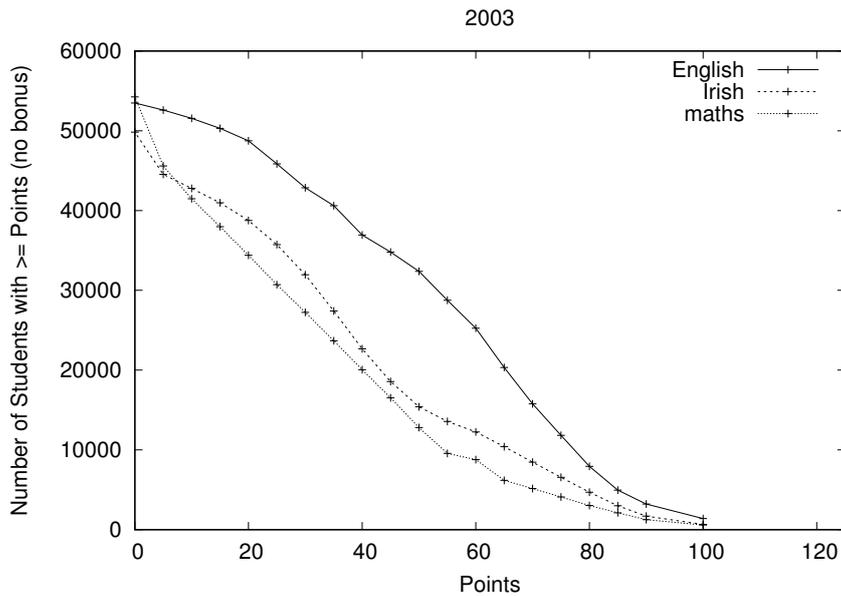


FIGURE 5. Number of points versus how many students have at least this many points by subject for 2003.

number of points at any level. For example, if we draw a vertical line at 80 points, we see that there are 7900 students on more than 80 points in English, but just 4700 in Irish and 3000 in maths.

Alternatively, if we draw a horizontal line at 10000 students, we see that the top 10000 students in English are on 75 points or more, in Irish the top 10000 students are on 65 points or more and in maths they are on 50 points or more. The mathematics curve is always under the other curves until 5 points, where mathematics rises above English/Irish, mainly because it has a larger overall number of students than other subjects.

These trends are broadly repeated from year-to-year, as can be seen from Figure 6, which shows the equivalent graph for each year between 2002–2016. From 2012 onwards, we show two curves for mathematics, corresponding to the points value with and without the bonus.

For all years, the number of students attaining at least some number of points in Irish is consistently below the number in English. Without the bonus, maths is generally lower again. The only exception is at the very lowest grades where the maths curve crosses above the others. This is simply due to the number of students sitting maths; there are more students overall studying maths than Irish or English and they all must achieve zero or more points. If the

bonus points are included for mathematics, we find that the maths curve crosses the Irish curve at about 70 points and the English curve at about 80 points.

If we plot these curves in terms of the fraction of students taking the subject, rather than the total number (see Figure 7 and Figure 6 respectively), then the curves without the bonus do not cross. These graphs tell us that the median points, or indeed, any percentile for points for English is greater than that for Irish, which is greater than that for maths. For 2012-2016, where the bonus for maths applies, we see that the top 20–25% of students would get more points for maths than Irish/English.

2.3. Making the Grade. While harder to interpret in terms of a student's aims, we can also study the number of students with a particular number of points, as shown in Figure 9. Of course, a particular number of points may correspond to different grades at different subject levels, and we will look at these subcategories shortly.

One motivation for considering the number of students with a particular number of points is to find the mode, i.e. the most-commonly-awarded number of points. We identify the number of points where the maximum is achieved to find the mode of the points distribution. For Irish and mathematics, this is 0 points, while for English this is 60 or 65 points. Again, Figure 10 shows the curves for 2002–2016, and the trend is similar from year to year. Note that we do not show the bonus points for mathematics in these figures for two reasons. First, the students being spread over different numbers of categories make the graphs hard to compare in a meaningful way. Second, it will make no difference to the mode, as students who get 0 points do not get a bonus.

While there are rewards associated with achieving particular grades at higher level, there are also risks associated with taking an exam at higher level. Failing English, Irish or maths will preclude a student from many third level courses, careers and apprenticeships. By plotting the number of students achieving particular grades, we can get some idea of what the risks are.

To plot a curve for higher-level subjects, we simply mimic the CAO points system, taking the top percentage within that grade, i.e. A1=100, A2=90, ... D3=45. Although no CAO points are awarded below the D3 grade, in order to see the tail of the curve,

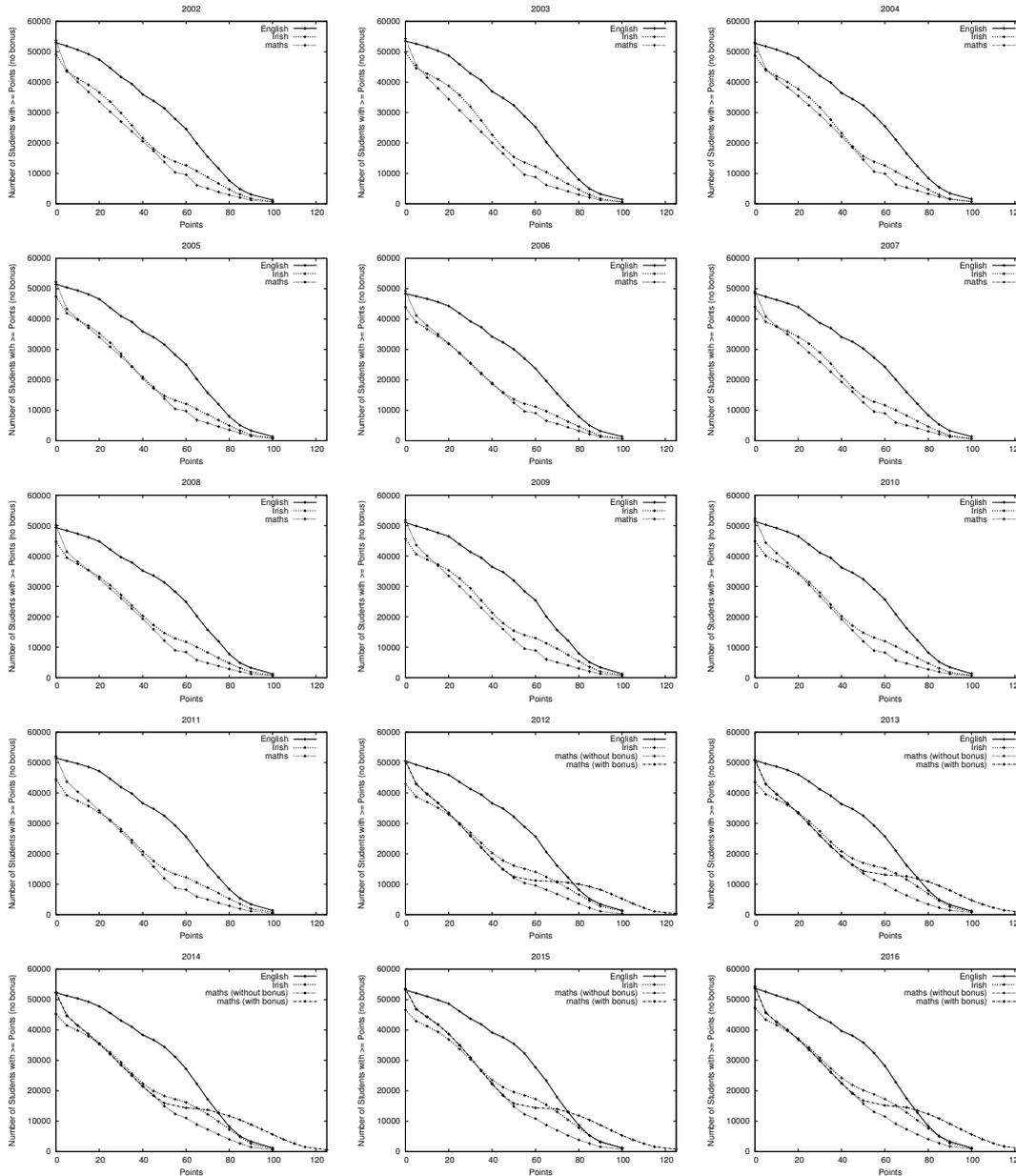


FIGURE 6. Number of points versus how many students have at least this many points by subject for 2002–2016.

we continue on assigning the lower grades their upper percentage, $E=40$, $F=25$, $NG=10$.

The CAO system awards 60 points for an ordinary-level A1, effectively equating that grade with a C3 at higher level. This offset of 40 points is applied to every other grade, resulting in $A2=50$, ... $D3=5$. In order to compare the curves of the different levels we do the same here. Again, we want to see the tail, so although points are not awarded, we continue on and assign $E=0$, $F=-15$, $NG=-30$.

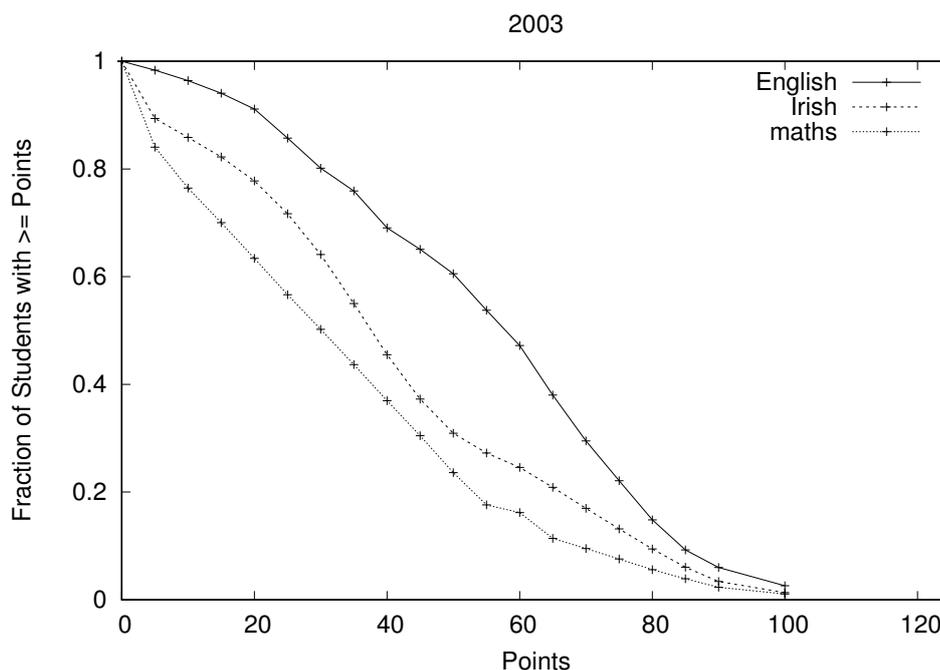


FIGURE 7. Number of points versus the fraction of students that have at least this many points by subject for 2003.

Foundation is treated similarly, with an offset of 75, as per the CAO points scheme. This formula agrees with the number of points given for all non-zero-points grades except for an A1 at foundation level⁵. Put a little more formally, we give a value for each grade at each level:

$$\text{value}(\text{grade}, \text{level}) = \text{cutoff}(\text{grade}) - \text{correction}(\text{level})$$

where the cutoff for a grade is the upper percentage that you can get for the grade and the correction is 0 for higher, 40 for ordinary and 75 for foundation.

Figure 11 shows the number of students achieving a particular value for English and maths (Irish has been omitted for clarity). Moving from the right-to-left along any curve, the first point is the number of A1s, the next A2s, then B1, B2, B3, C1, C2, C3, D1, D2, D3, E, F and NG. The E, F and NG grades are fail grades.

Looking at the right-hand side of this graph (40–100 points), we can see that the curve for higher English is far above the curve for

⁵An A1 at foundation level is actually worth only 20 points, but we give it a value of 25, so the foundation level curves are spaced in the same way as the other levels.

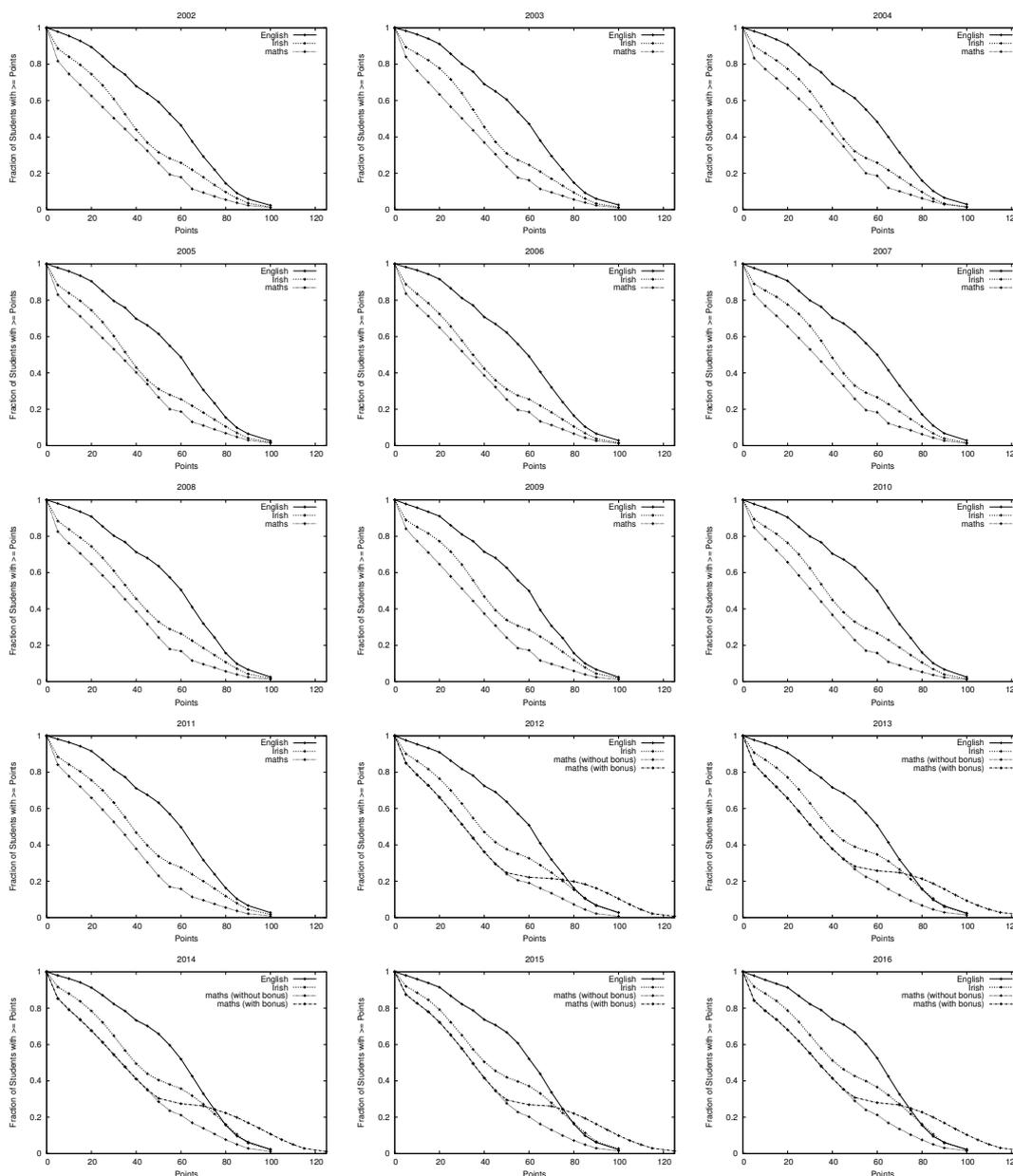


FIGURE 8. Number of points versus the fraction of students that have at least this many points by subject for 2002–2016.

higher maths, indicating the large difference in the numbers achieving particular higher-level grades in these two subjects. We have seen this in the previous figures. However, the part of the curves for higher-level English and maths between 10–40 almost over-lie one another in most years. Indeed, in some years the higher maths curve even rises above that for higher English. This means that

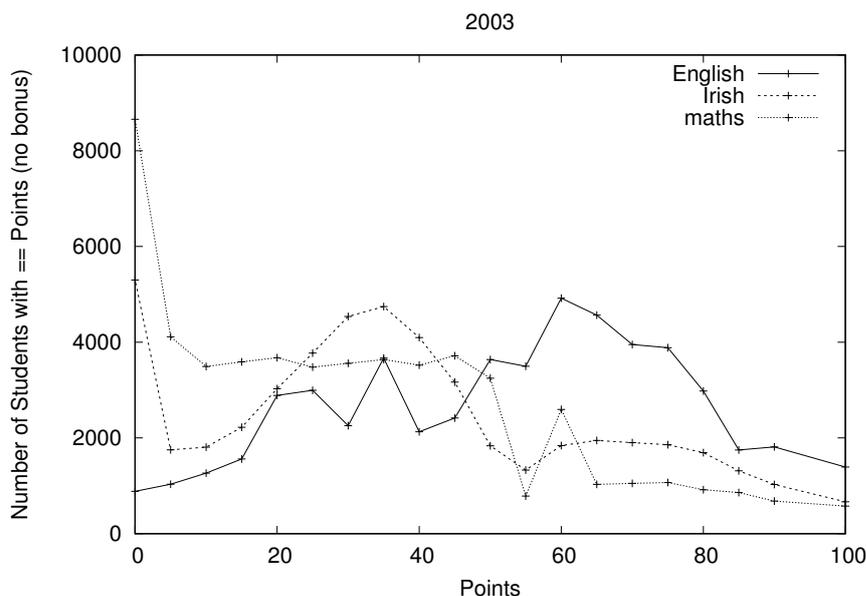


FIGURE 9. The number of students achieving a particular number of points in English, Irish and maths in 2003 (excluding bonus).

number of people failing higher maths is comparable to the number failing higher English, even though about three times as many people take higher English. One way to view this is that the risk of failing higher maths is about three times that of failing higher English. As expected, this is not peculiar to 2003, and we can see broadly similar trends repeated in Figure 12 for years 2002–2016.

The curves for ordinary English and maths are more in line with what one might expect given the greater number taking ordinary maths: the number of students achieving a particular grade in ordinary maths is usually well above the number achieving that grade in ordinary English.

There are some interesting peaks in the curves, which persist from year-to-year. Consider the English curves: there seems to be a dip where the point for A2 seems to be high relative to the point for B1; similarly, the point for B3 seems to be high relative to the point for C1. This can be observed in both the higher and ordinary curves. A likely explanation for this is that either the marking process or the appeals process is ‘marking up’ students across the boundary between A and B grades, and C and B grades. If this explanation is correct, it would provide evidence that the desirability of particular

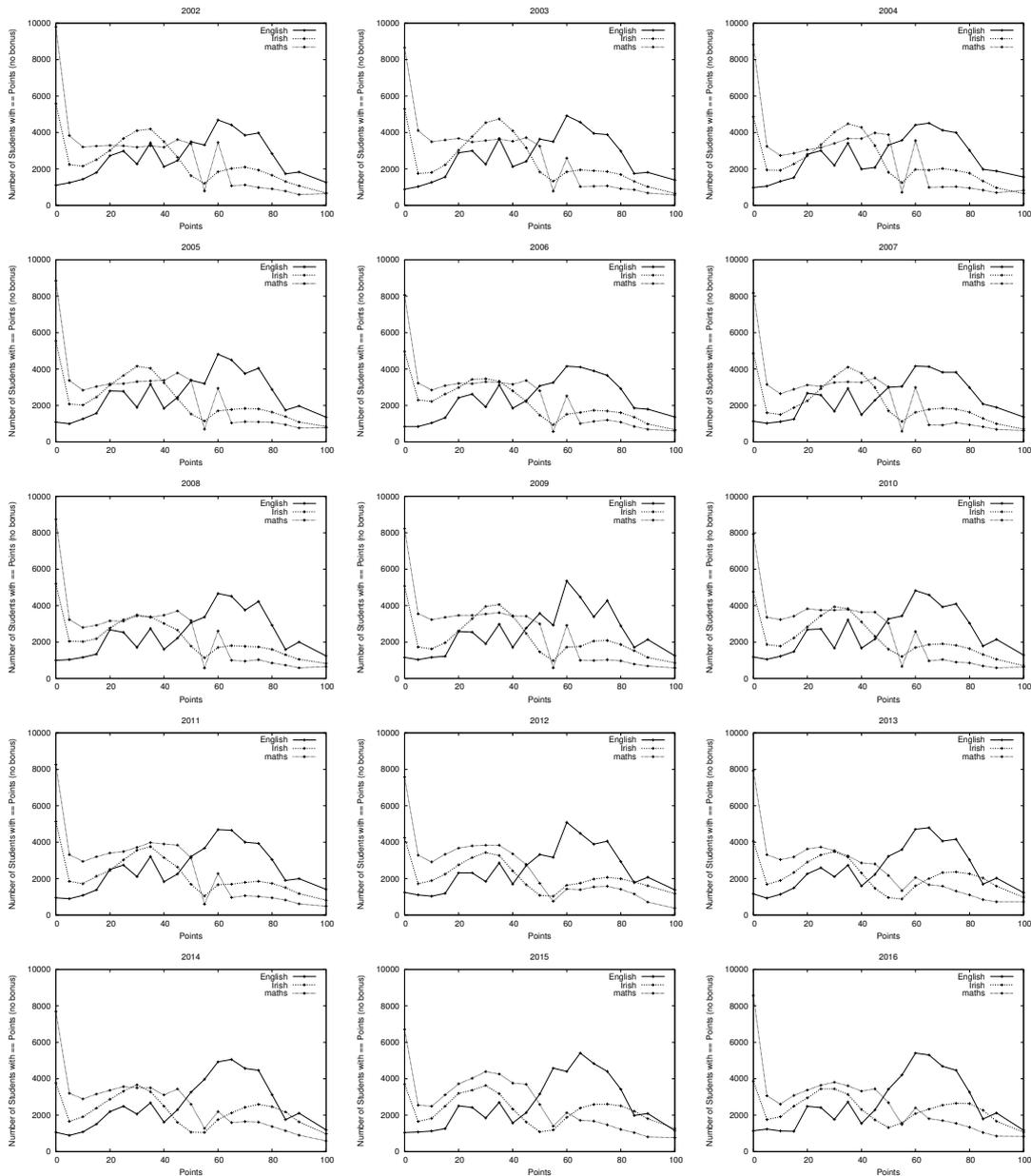


FIGURE 10. The number of students achieving a particular number of points in English, Irish and maths in 2002–2016 (excluding bonus points).

grades is having an impact on the rewards offered. Similar effects have been observed previously [14].

While the higher maths curve does not obviously show such features, possibly because of a more rigid marking scheme in mathematics, the ordinary-level maths curve does show one interesting persistent feature: the numbers achieving a D2 grade always seem to dip relative to the numbers achieving a D3 or E grade. While

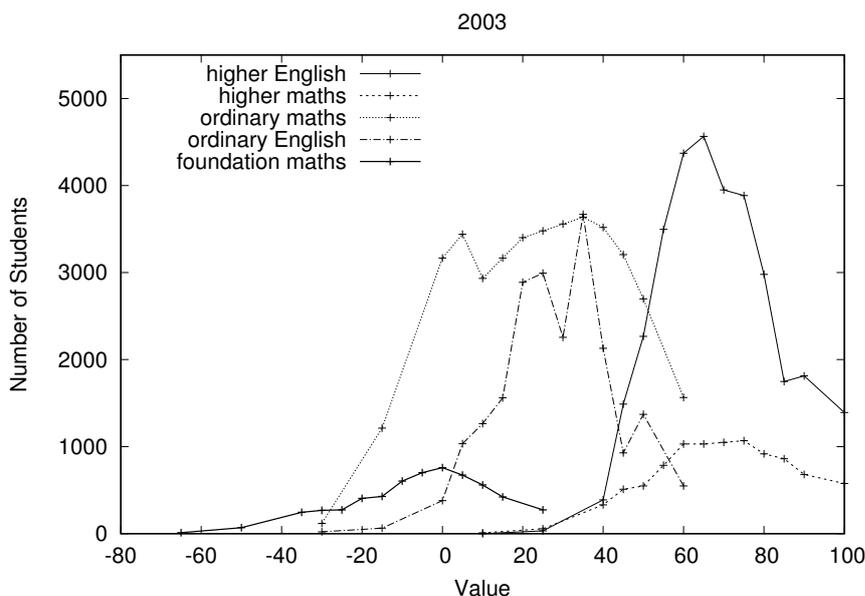


FIGURE 11. The number of students achieving different grades in each subject in English and maths in 2003.

the reason for this is not clear, it may be related to the “attempt mark” system often used for grading maths. A minimum attempt mark is assigned to all students who make a certain basic level of progress with each question, which awards about one third of the marks for the question⁶. The surge in these grades may represent students who are getting the majority of their marks from attempt marks.

3. DISCUSSION

It is fairly clear that the three (practically) compulsory subjects are different in terms of the rewards achieved by students. Consider the following situations, in light of what we have seen.

The Price of Success: A student estimates she needs a B3 average (75 points) across 6 subjects to get her chosen college course. She believes that she will get a B3 in each of 5 optional subjects. Her Irish isn’t good enough, but she can choose maths or English as her sixth points subject and take the other two at ordinary level. If she chooses English, she needs to be in the top 24% of people. If she chooses maths she needs to be in the top 8% of people in the country to get

⁶The exact operation of this system has varied slightly over the period.

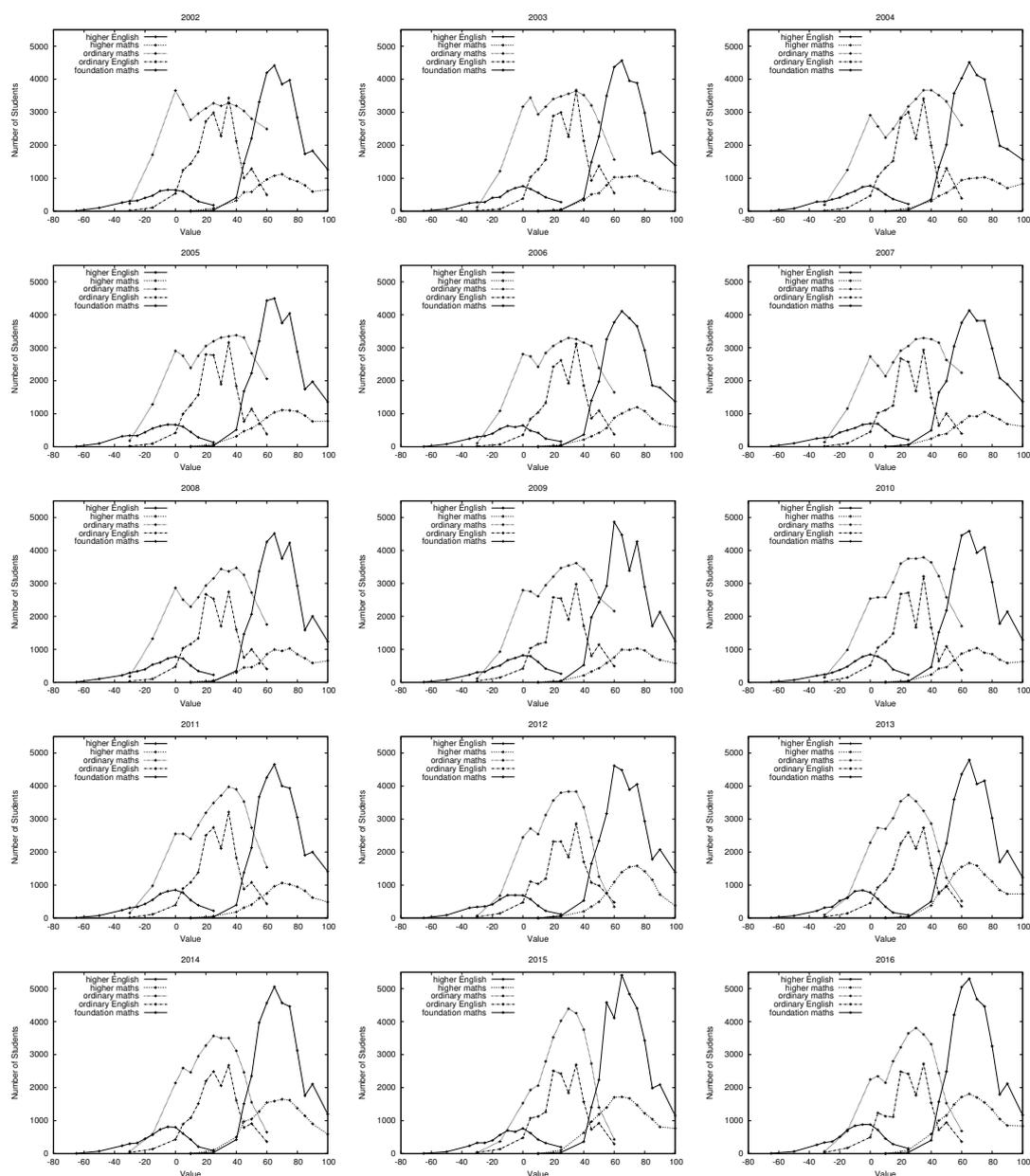


FIGURE 12. The number of students achieving different grades in each subject in English and maths from 2002–2016.

a B3, or top 22% to get 75 points with the higher level bonus. Which subject does she choose to take at higher level?

Fear of Failure: A student is borderline between higher maths and English. He *must* not fail either. He asks both teachers' advice. The maths teacher knows from experience that about 3.8% of people fail higher maths and that about 5–7% get As in ordinary maths, which are equivalent in points to a D1 or

D2 in higher level. The English teacher, from experience, has observed about a 1.4% failure rate at higher level and 7–9% of people getting an A at ordinary level.

Of course, where students can make these decisions, they may not have direct knowledge of such statistics. However the significant differences we have shown may provide anecdotal evidence that inform their decisions.

If there really is a marketplace for subjects, then this would indicate that students are not choosing higher maths because their effort can be better applied elsewhere. Increasing participation in higher maths is desirable for a range of reasons [5]. If we want to improve the take-up of maths at higher level, what can we do? Some possibilities present themselves.

- (1) Reduce the effort required to study higher maths by making it easier,
- (2) Reduce the effort required to study higher maths by improving educational support and accessibility,
- (3) Increase the rewards associated with higher maths by increasing points offered,
- (4) Decrease the risks associated with failing higher maths,
- (5) Make other subjects less desirable.

We will not comment in detail on the last option, as it is unlikely to be popular with anyone.

Another proposed option [7] is to adjust the market, so that mathematics *must* be used as a points subject. This would mean that ordinary level maths becomes less attractive for students who are targeting high points.

We note that one could argue that the results we see are simply because our students are better at English and weaker at mathematics. However, this assumes that there is some absolute way to measure the difficulty of a Leaving Cert subject, other than having students study the subject and take the exam. We see no other obvious objective way of measuring the difficulty of, or level of reward associated with, a particular subject.

3.1. Making Things Easy. Making the mathematics course easier is unpalatable to many of the consumers of Leaving Certificate students. Universities, high-tech companies and professional bodies want students who have a certain level of mathematics, and

some already find that not all students have a sufficiently high level of mathematics. There have been concerns raised about the new *Project Maths* course examined from 2012 onwards [21] and there is ongoing study on its impact on competence at third level [22].

There is also an obvious race to the bottom associated with making courses easier in order to make them more popular: if every subject repeatedly does this, all courses will gradually become easier and easier. This suggests that all Leaving Cert subjects should have some group of stakeholders who have a direct interest in the balance between the numbers taking the subject and the difficulty of the exam.

3.2. Making Maths Better. Improving the educational support available to students of mathematics has been on the agenda for many years. Studies have shown that there are a surprisingly large number of teachers of mathematics at second level who are teaching *out of field* [3, 19]. In particular, some commentators have suggested that the level of mathematics training required by a teacher before they can teach mathematics should be higher. Indeed, a number of courses at third level have been set up to facilitate the up-skilling of interested teachers (e.g. in UL and Maynooth). Perhaps measures that provide incentives for highly-numerate graduates to go into teaching could also be used, as they are already in high demand in the jobs marketplace. Schemes like this are in use in the UK. Certainly, it would be hard to argue against better teachers in any area, but any improvements will continue to be a long-term project.

The Project Maths course has largely been rolled out, and has been the primary examination in mathematics since 2012. It aims for students to achieve a better understanding of the material and how it is applied. The hope is that by making the applications of what is learned more obvious, that it will become more attractive to students. A more complete description of Project Maths is available [18].

It seems that changes to the emphasis and style of examination can have an impact on uptake. In 2012 the examination of Irish was changed to place more value on the Irish Oral examination. Figure 4 shows that this appears to have increased the uptake in higher Irish. While higher mathematics also shows an increase, corresponding to the introduction of Project Maths, the impact is confounded by the introduction of bonus points.

3.3. Adjusting Points. Increasing the points for maths has, of course, been often proposed, and bonus points were available for mathematics before⁷. When the mathematics syllabus was changed in the 1990s, bonus points were dropped because it was assumed that the new course would be more in line with other courses in terms of its difficulty. We can see that this does not seem to have been true. Indeed, looking at Leaving Cert exams from the 1990s up to 2011, there are many similarities. The latter part of each question ('part c') was still challenging and required students to combine understanding of various parts of the course. The style of questions in Project Maths is quite different, however there are still questions combining various parts of the syllabus.

Would bonus points actually encourage students to take higher maths? The evidence is not completely clear [11]. Before 2012, the University of Limerick (UL) offered bonus points for several years, but this did not seem to have significantly increased uptake among students who subsequently attend UL. However, it is not clear whether UL alone was big enough to have an impact on students' subject choices and whether those students affected ultimately went to UL. Since 2012, a bonus of 25 points for pass grades of higher maths has been in place, which we may be able to shed some light on. Indeed, it has been suggested that the points table might have to be extended to 120 or 150 points in order to account for differences between subjects [14], so it is worth considering the impact of any bonus and what aim it might have.

With the data available, we can calculate the effect of offering bonus points for maths, under the assumption demand for higher maths would remain unchanged. Say we aim to equalise the average points given by maths and English, we know that we need to raise the pre-2012 average by about 20 points. Since only about one in five students were taking higher maths, that suggests we need to give about 100 points, on average, to these higher-level students. In fact, if we give 100 extra points to all students passing higher maths, the average points for maths moves to about 50 points, which is only slightly below English. An alternative scheme, to multiply the points for higher maths by 2.5, also moves the average to around 50. This

⁷It was shown some time ago that Leaving Certificate maths was a good indicator of first year university performance [17], which provided a reason for weighting maths more highly for points up to the early 1990s.

also has some advantages over simply adding a fixed bonus, such as providing a slightly smaller step in points between a D3 and an E grade.

We expect that offering a substantial bonus would increase the numbers of students taking maths at higher level. However, this should also increase the average number of points given for maths, so equalising points using a pre-2012 year's statistics overestimates what is really required. Offering 60 bonus points at higher, or doubling points, would move the average for maths to about 40 points if the demand didn't change. This might be more acceptable and allow for change in demand. However, without understanding the feedback between the extra points and students' choices, it is impossible to say exactly what would be required to level the rewards offered by maths and English.

Our calculation suggests that the 25 bonus points offered from 2012 would fall significantly short of what would be required to equalise the average points, which we see confirmed in Figure 2. However, we do see an increase in the numbers taking higher maths in the first years after 25 bonus points were offered (see Figure 4), indicating that increased reward can indeed increase the numbers choosing to take a subject. It is worth observing that 2012 was also the first year in which Project Maths was examined on a wide scale, so the impact of this can not easily be separated from the offer of bonus points.

It is also worth noting that there might be other collateral effects of introducing bonus points for higher mathematics, from desirable effects (e.g. increasing the availability of higher maths) to the undesirable (e.g. over-influence of the third-level system on second-level studies, mathematics being used as a "booster" subject for points). Previous studies have discussed some of these issues [2, 17, 12]. Of course, there is also the option of decoupling the Leaving Cert and points, and so avoiding any impact that points could have on subject preferences. Previously, independent matriculation exams were available through various universities and admissions were based on these. However, it is unlikely there is an appetite to reintroduce separate matriculation examinations for our school leavers, and it seems likely that any anomalies created by the current points system would be inherited by the matriculation system.

3.4. Bell Curving? Note that there is some evidence that points are currently effectively being adjusted. Compare the consistency in points for higher, ordinary and foundation (Figure 3), even over significant course changes, to the lack of consistency seen between English, Irish and maths (Figure 2). This suggests that the grades are already being adjusted to ensure consistency at a particular level across subjects.

If this sort of adjustment is being applied, it raises a potentially serious issue when combined with a subject marketplace. If there is an easier alternative to higher level maths, some weaker students will move to ordinary level. The adjustment will then be applied, thus offering the same points to the remaining stronger students, which will effectively make higher-level maths even harder. This will cause more students to abandon higher maths, and so on. Meanwhile, the students moving from higher to ordinary are relatively strong for ordinary level, and so increase the standard there, pushing students who previously did ordinary into the lower grades there. If this positive feedback exists in practice, any difference between subjects will tend to be amplified by a combination of grade adjustment and student choices. This may even offer an explanation of the failure to achieve a 20–25% takeup rate in higher mathematics, which was the expected takeup for the syllabus introduced in 1992 [6].

We also expect that a better-rewarded subject will see the opposite happening. If a subject is seen to be an easier choice at higher level, then more students will take it at higher level. The average mark will remain the same, despite larger numbers taking it, so the marking will be adjusted to be easier, making the subject more attractive.

Normalising marking between subjects is a challenge. In optional subjects where the cohort taking the subjects is highly self-selecting, it seems that nothing short of introducing some sort of “jury service” would allow you to compare how the average student would cope with taking the course. However, given that basically everyone has to take English, Irish and maths, it should be possible to achieve better normalisation than is currently observed. One way to do this would be to adjust the points awarded for each subject so that the average across all students taking the subject is 50 points.

3.5. Fewer Fails. Some have suggested that an E grade at higher-level maths should be considered as a pass in certain circumstances, in order to reduce the risk associated with higher maths [8]. This has

been adopted in the new Leaving Cert grading system introduced in 2017, which awards some points for a higher H7 grade (30–40% at higher level), which was previously regarded as a fail grade. The Irish University Association’s guide to the grading system says this is to encourage uptake at higher level and also to reduce the risks associated with failing.

An alternative possibility that has been suggested is to allow students a second-chance exam if they fail mathematics at higher level, or to have a base-level maths exam that all students take followed by an advanced exam for those who wish to earn a higher grade.

4. CONCLUSION

We believe that looking at Leaving Cert subject/level choice as a market may offer interesting insights into the choices made by students. To understand subject uptake, it seems that regarding points as the reward offered by subjects, and considering the relative demands made by each subject should be a useful strategy. When we compare English, Irish and mathematics, we see that there are significant differences in the rewards offered to students, after the demands have been accounted for by student choice of subject level. In addition to quirks at grade boundaries, we also see evidence of grade normalisation, which could have a dangerous effect when combined with this subject market.

We have also tried to identify possible courses of action to improve the uptake of higher mathematics. It seems unlikely that a realistic application of any one of these would be sufficient to put mathematics and English on a par in this points market. It is also likely that any reform that improves mathematics in the Leaving Cert through syllabus reform, improved teaching, bonus points, etc., must take account of the subject market in which maths exists to be fully effective.

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APPENDIX A. NOTES ON METHOD

The statistics presented here are derived from the following files from the <http://www.examinations.ie/> website:

2002_LC_GradesAwardedbySubject.pdf
 2004_LC_Breakdown_Candidates_by_grade_Higher_Ordinary_and_Foundation_Level.pdf
 2005_LC_Breakdown_Candidates_by_grade_Higher_Ordinary_and_Foundation_Level.pdf
 2006_Breakdown_Candidates_by_grade_Higher_Ordinary_and_Foundation.pdf
 2007_LC_2007_breakdownResults_10_or_More.pdf
 2008_LCGrade_over_3_years.pdf
 2009_lc_nat_stats_2009_211009_excluding_10.pdf
 Provisional_Results_2010_excluding_subjects_with_less_than_10_candidates.pdf
 Provisional_Results_LC_2011_National_Stats.doc
 Provisional_Results_2012_less10.doc
 2011_2013_LC_Statistics_no_less_than-10619.doc
 Leaving_Certificate_2015_Provisional_Results_Less_Than_10_Candidates_Word.docx
 BI-ST-9486403.csv
 EN-ST-20970104.csv

Each file presents results from multiple years. A number of discrepancies in the reported results appear, presumably due to appeals of results being resolved. Where these arise, the results from the most recent file have been used. The breakdown for each year is given as a percentage, to two decimal places. However the least significant digit is always zero, so it seems likely they have been rounded to one decimal place. The sums of the percentages are between 99.7% and 100.3%, so some small discrepancies arise between the listed total number of students and cumulative figures over grades.

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