Reports of Sponsored Meetings

Minimal Surfaces: Integrable Systems and Visualization. Spring 2017 Workshop
27–29 March 2017, University College Cork

m:iv is an international network funded by The Leverhulme Trust. The network partners consist of Tim Hoffman (Munich), Martin Kilian (Cork), Katrin Leschke (Leicester), Francisco Martín (Granada) and Katsuhiro Moriya (Tsukuba). The network will run a series of workshops to bring together experts from the fields of minimal surfaces, integrable systems and computer visualization. More details can be found at `http://www2.le.ac.uk/projects/miv`. The first workshop, held in Cork, ran over three days and consisted of twelve talks by speakers from around the globe. The speakers, titles and abstract were as follows:

Benoît Daniel (Université de Lorraine)
Minimal isometric immersions into $\mathbb{S}^2 \times \mathbb{R}$ *and* $\mathbb{H}^2 \times \mathbb{R}$.
For a given simply connected Riemannian surface Σ, we relate the problem of finding minimal isometric immersions of Σ into $\mathbb{S}^2 \times \mathbb{R}$ or $\mathbb{H}^2 \times \mathbb{R}$ to a system of two partial differential equations on Σ. We prove that a constant intrinsic curvature minimal surface in $\mathbb{S}^2 \times \mathbb{R}$ or $\mathbb{H}^2 \times \mathbb{R}$ is either totally geodesic or part of an associate surface of a certain limit of catenoids in $\mathbb{H}^2 \times \mathbb{R}$. We also prove that if a non constant curvature Riemannian surface admits a continuous one-parameter family of minimal isometric immersions into $\mathbb{S}^2 \times \mathbb{R}$ or $\mathbb{H}^2 \times \mathbb{R}$, then all these immersions are associate.

Isabel Fernández (University of Seville)
Surfaces of critical constant mean curvature and harmonic maps.
Minimal surfaces (H=0) in euclidean 3-space and Bryant surfaces (H=1) in hyperbolic 3-space are a special family among all the CMC surfaces in spaces forms. Similarly, surfaces of critical CMC in homogenous 3-spaces present a special behavior among all the CMC surfaces. For example, Fernández and Mira found a hyperbolic Gauss map for surfaces in $\mathbb{H}^2 \times \mathbb{R}$ that turns out to be harmonic for surfaces of critical CMC. Later on, Daniel discovered a Gauss map for surfaces

in Heisenberg space that is also harmonic when the mean curvature is critical. However, both definitions are quite different and it was unclear how to extend them to the general setting. In this talk we will review some properties of critical CMC surfaces in homogeneous 3-spaces and present a unified definition of a Gauss map for surfaces in these ambient spaces that is harmonic when the mean curvature of the surface is critical.

Leonor Ferrer (University of Granada)

Properly embedded minimal annuli in $\mathbb{H}^2 \times \mathbb{R}$.

In this talk we ask for properly embedded minimal annuli in $\mathbb{H}^2 \times \mathbb{R}$ which bound a pair of vertical graphs over $\partial_\infty \mathbb{H}^2 \equiv \mathbb{S}^1$. We present some compactness results for these surfaces. We also give some existence results for proper, Alexandrov-embedded, minimal annuli. Contrary to what might be expected, we show that, in general, one can not prescribe the two components of the boundary at infinity. However, we can prescribe one of the boundary data, the position of the neck and the vertical flux of the annulus. This is a joint work with F. Martín, R. Mazzeo and M. Rodríguez.

Sebastian Klein (University College Cork)

Asymptotic methods for finite gap curves in the 3-dimensional space forms.

I will discuss how asymptotic methods can be used to study closed curves, and in particular finite gap curves, in the 3-dimensional space forms.

Laurent Hauswirth (Université Paris-Est)

Asymptoticity of Minimal surfaces in Heisenberg space.

I will describe the asymptotic behavior of minimal ends using the Dirac operator. Joint work with Taimanov.

Felix Knöppel (Technical University Berlin)

Complex line bundles over simplicial complexes.

Over the last years we could apply bundle theory to several problems in Computer Graphics. This led to a quite good understanding of the underlying discrete theory: We present a discrete analogue of a classification theorem due to Kobayashi and then focus on hermitian line bundles with curvature. For these a discrete analogue of Weil's

theorem and a discrete Poincaré-Hopf theorem hold. Furthermore, we generalize the well-known cotan-Laplace operator.

Ben McKay (University College Cork)
Isometric immersions and integrability
Some examples of surfaces for which isometric immersion to a space form is an integrable system.

Barbara Nelli (University of L'Aquila)
Minimal Surfaces in the Heisenberg Space.
We discuss the behaviour of some minimal surfaces in the Heisenberg space. In particular, we deal with existence and growth of non compact graphs and stability properties.

Pablo Mira (Technical University of Cartagena)
Constant mean curvature spheres in homogeneous three-manifolds, I
The aim of these two talks (which are based on joint work with Bill Meeks and Antonio Ros) is to prove the following theorem: any two spheres of the same constant mean curvature immersed in a homogeneous three-manifold only differ by an ambient isometry. Our study will also determine the exact values of the mean curvature for which such CMC spheres exist, together with some of their most important geometric properties. For instance, we will show that CMC spheres in simply connected metric Lie groups have index one, are Alexandrov embedded and maximally symmetric, their left invariant Gauss maps are diffeos, and the corresponding moduli space of CMC spheres is a connected one-dimensional manifold.

Franz Pedit (University of Massachusetts)
Energy quantization for harmonic 2-spheres in non-compact symmetric spaces.
It is well known from results by Uhlenbeck, Chern-Wolfson and Burstall-Rawnsley that harmonic 2-spheres in compact symmetric spaces have quantized energies. Using the reformulation of the harmonic map equation as a family of flat connections, we construct an energy preserving duality between harmonic maps into a non-compact symmetric space (typically pseudo-Riemannian) and harmonic maps into compact real forms of its complexification. Applying this construction to the conformal Gauss maps of Willmore 2-spheres in the n-sphere provides a generalization and unifying approach to existing

quantization results in special cases: Bryant for n=3; Montiel for n=4; and Ejiri for Willmore 2-spheres admitting a dual Willmore surface.

Joaquín Pérez (University of Granada)
Constant mean curvature spheres in homogeneous three-manifolds, II
See Pablo Mira.

Ulrich Pinkall (Technical University Berlin)
From Smoke Ring Flow to Real Fluids.
The so-called "smoke ring flow" for space curves was introduced in 1910 by Da Rios (who was a PhD student of Levi-Civita) for describing the time evolution of vortex filaments in an ideal fluid. Starting from the 1970's it became clear that the smoke ring flow actually is the most basic integrable system that originates in Differential Geometry. It is closely related to the one-dimensional Landau-Lifshitz equation and to the one-dimensional nonlinear Schrödinger equation. As an asymptotic limit it is also crucial for understanding the geometry of CMC surfaces in space forms. Vortex filaments are the solitons of fluid dynamics, so in this sense fluid flow can be viewed as a perturbed integrable system. In this talk we will show a method for fluid simulation that reflects this fact. This method is closely related to the three-dimensional Landau-Lifshitz equation and to the three-dimensional nonlinear Schrödinger equation.

Martin Schmidt (University of Mannheim)
Willmore energy of conformal maps $f : \mathbb{C}/\Gamma \to \mathbb{R}^4$

Report by Martin Kilian, UCC
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Fourteenth Irish Geometry Conference
25–26 May 2017, Maynooth University

The Irish Geometry Conference was held at Maynooth University on 25-26 May 2017, and was organized by David Wraith. This annual meeting in now in its fourteenth year, and this was the fourth time it had been held in Maynooth. As well as participants based in Ireland, this year's event included two speakers from Germany and

one from Argentina, with talks covering a diverse range of geometric and topological research topics. The speakers, titles and abstracts were as follows:

Stefan Bechtluft-Sachs, N.U.I.M.

Green's function of Dirac Operators on rank-1-symmetric spaces.

In harmonic spaces the introduction of radial functions reduces the scalar Laplace equation to an ordinary differential equation. The Laplace operator has a right inverse whose integral kernel, Green's function, is radial in the sense that it is a function of the distance only. Our main aim is to extend this radial calculus to Dirac operators and operators that can be derived from these, like the Cartan differential and the Laplacian on differential forms.

On compact manifolds a solution of the Dirac equation on differential forms yields a solution of the Cartan equation via Hodge Theory. This in turn provides a density for Gauss-type formulas for the linking number (in 3 dimensions this is essentially the Biot-Savart law for the magnetic flux induced by a stationary current). On non-compact rank one symmetric spaces, in the absence of Hodge Theory, we get the same results from our radial equation. (joint with E.Samiou)

Nikos Georgiou, W.I.T.

Minimal surfaces in the product of 2-manifolds

In this talk we first introduce a Kaehler structure of neutral signature on the product of two pseudo-Riemannian manifolds. Then we discuss surface theory in such a product, and in particular we describe a classification result about Lagrangian surfaces with parallel mean curvature. Finally, we present some recent results on minimal surfaces in the product of a two-dimensional real space form with itself.

Thomas Huettemann, Q.U.B.

From topology to strongly graded algebra

Finiteness properties of topological spaces are relevant in many areas of topology (e.g. manifold theory) and algebra (e.g. geometric group theory). Starting from a specific homological result on "finite domination" going back to Ranicki and others, I will explain how seemingly simple generalisations lead to unexpected complications, and how strongly graded algebra provides a natural setting to formulate statements and proofs.

Martin Kerin, University of Bonn and University of Muenster, Germany

Non-negative curvature on exotic spheres
Since their discovery, there has been much interest in the question of precisely which exotic spheres admit a metric with non-negative sectional curvature. In dimension 7, Gromoll and Meyer found the first such example. It was subsequently shown by Grove and Ziller that all of the Milnor spheres admit non-negative curvature. In this talk, it will be demonstrated that the remaining exotic 7-spheres also admit non-negative curvature. This is joint work with K. Shankar and S. Goette.

Martin Kilian, U.C.C.
On finite gap curves
A curve in a 3-dimensional space form is a finite gap curve if it is stationary under some evolution in the self-focusing non-linear Schroedinger hierarchy. I will survey some recent results about such curves and will explain techniques from the theory of integrable systems for this setting.

Adib Makrooni, N.U.I.G.
Compact homogeneous spaces with positive Euler characteristic and their 'strange formulae'.
I will describe a generalisation of the 'strange formula' of Freudenthal and de Vries for compact homogeneous spaces with positive Euler characteristic. I will apply the results to computing a topological invariant used to study hyper-Kaehler structures. In my talk I will make use of a sharpened version of the Borel and de Siebenthal Theorem, describing the isotropy representation of K on the tangent space to G/K, where K denotes a maximal connected subgroup of maximal rank in a compact simple Lie group G. This is joint work with J. Burns.

Carlos Olmos, National University of Cordoba, Argentina
The nullity of homogeneous Riemannian manifolds
the nullity distribution of the curvature tensor of a Riemannian space was defined by Chern and Kuiper in 1952. This distribution turns out to be autoparallel, around the points where the dimension is locally constant. Nevertheless, nothing was known about the nullity distribution in homogeneous spaces. In this talk we will mainly refer to some recent results obtained jointly with Antonio J. Di Scala and Francisco Vittone, that will motivate the presentation of some interesting points of view in homogeneous geometry. Let M be a locally irreducible homogeneous Riemannian manifold. We prove

that if M is either compact, or Kahler, or more generally nearly Kaehler, then the distribution of nullity is trivial. We will present also a general structure theory for homogenous manifolds with non-trivial nullity that predicts the existence of a transvection (i.e. a Killing field which is parallel at some point) with null Jacobi operator and not in the nullity. With the aid of this result we are able to find a one parameter family irreducible homogeneous spaces of dimension 4 with non-trivial distribution of nullity (as far as we know these are the first known examples). By making use of the above mentioned structure theorem, we also show that any homogeneous space with a transitive semisimple subgroup of isometries, has trivial nullity distribution.

Thomas Schick, University of Goettingen, Germany

'Poison' submanifolds prevent positive scalar curvature

The Gauss-Bonnet theorem implies that the sphere is the only compact oriented surface admitting a metric with positive scalar curvature. In higher dimensions, the role of the Gauss-Bonnet theorem is taken by various index-invariants of the Dirac operator.

A particularly intricate way to employ this goes back to Gromov and Lawson: they identify certain types of submanifolds whose existence prevents the existence of a metric of positive scalar curvature. We present this and a generalization (joint work with Hanke and Pape). Then we discuss recent constructions which put these obstructions into the wider context of higher index theory (relating this also to conjectures like the Baum-Connes isomorphism conjecture or the strong Novikov conjecture).

Finally (if time permits), on non-compact manifolds we will comment on the difference between the non-existence of complete metrics with positive scalar curvature versus "scalar curvature bounded below by a positive constant" (joint with Cecchini).

Report by Prof David Wraith, Maynooth University
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Third Irish Linear Algebra and Matrix Theory Meeting September 8th, 2017, Maynooth University Department of Mathematics & Statistics

The 3rd Irish Linear Algebra and Matrix Theory Meeting took place at the Maynooth Univeristy Department of Mathematics &

Statistics on September 8th, 2017. In the spirit of the 2 previous and successful meetings in this series, hosted by NUI Galway (2012) and UCD (2014), the event's principal aim was to provide a relaxed environment in which researchers interested in Linear Algebra and related applications could meet and discuss a range of topics and results from the field. The venue for the meeting was MS2 in Logic House on Maynooth's historic South Campus, and there were numerous opportunities for participants to hold informal discussions over coffee and lunch during the day. The total number of participants at the event was 16, of whom 5 were research students.

Linear Algebra and Matrix Theory Meeting

The meeting consisted of 7 talks covering a wide range of topics across the spectrum of pure and applied linear algebra. The speakers at the meeting and the talk titles are listed below.

- Thomas J. Laffey (UCD), *Generating sets for full matrix algebras.*
- Niall Madden (NUI Galway), *Parameter robust linear solvers for singularly perturbed differential equations.*
- Ollie Mason (Maynooth University), *The joint spectral radius and extremal norms in the max algebra.*
- Cian O'Brien (NUI Galway), *Alternating signed bipartite graphs.*
- Rachel Quinlan (NUI Galway), *Rank problems for entry pattern matrices.*

- Sergey Sergeev (University of Birmingham), *Reachability, circulants and interval analysis in max algebra.*
- Helena Šmigoc (UCD), *Companion type realisations in the nonnegative inverse eigenvalue problem.*

We would like to thank all speakers and participants for contributing to what was a most enjoyable day.

The organizers are very grateful for the support received from the Department of Mathematics & Statistics at Maynooth University and the Irish Mathematical Society.

Report by Oliver Mason, Maynooth University
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