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IRELAND'S PARTICIPATION IN THE 57TH INTERNATIONAL MATHEMATICAL OLYMPIAD

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From 6th until 16th July 2016, the 57th International Mathematical Olympiad took place in Hong Kong. This was the second time that an IMO was hosted in Hong Kong. A total of 602 students (71 of whom were girls) participated from 109 countries. In four categories (number of countries, number of contestants, number of participating girls, percentage of girls) previous IMO records were broken. The following countries sent a team for the first time to the IMO: Iraq, Jamaica, Kenya, Laos, Madagascar, Myanmar and Egypt.

The Irish delegation consisted of six students (see Table 1), the Team Leader, Bernd Kreussler (MIC Limerick) and the Deputy Leader, Anca Mustaţă (UCC).

Name	School	Year
Antonia Huang	Mount Anville Secondary School, Dublin 14	4th
Robert Sparkes	Wesley College, Ballinteer, Dublin 16	6th
Cillian Doherty	Coláiste Eoin, Booterstown, Co. Dublin	5th
Ioana Grigoras	Mount Mercy College, Model Farm Road, Cork	6th
Liam Toebes	Carrigaline Community School, Co. Cork	6th
Anna Mustaţă	Bishopstown Community School, Cork	4th

Table 1. The Irish contestants at the 57th IMO

1. Team selection and preparation

Each year in November, the Irish Mathematical Olympiad starts with Round 1, a contest that is held in schools during a regular class period. In 2015 almost 14,000 students, mostly in their senior cycle, from about 290 second level schools participated in Round 1. Teachers were encouraged to hand out invitations to mathematics enrichment classes to their best performing students.

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At five different locations all over Ireland (UCC, UCD, NUIG, UL and MU), mathematical enrichment programmes are offered to mathematically talented students, usually in their senior cycle of secondary school. These classes run each year from December/January until April and are offered by volunteer academic mathematicians from these universities or nearby third-level institutions.

Rarely, students who participate for the first time in the mathematics enrichment programme qualify for the Irish IMO team – Cillian Doherty is one of these rare exceptions. Usually, those who make it to the team come back after their first enrichment year to get more advanced training.

In order to activate the full potential of these returning students – like in previous years – an Irish Maths Olympiad Squad was formed. It consisted of the 13 best performing students at IrMO 2015, who were eligible to participate in IMO 2016. Between IrMO and the restart of the enrichment classes, for this group of students the following extra training activities were offered: two training camps (one in June and one at the end of August), participation in the Iranian Geometry Olympiad (September), a remote training which runs from September to December and participation in Round 1 of the British Mathematical Olympiad (November).

The centrally organised remote training, which was offered for the first time in 2013, is now an established pillar of the preparation of the Irish IMO team. At the beginning of each of the four months from September to December, two sets of three problems were emailed to the participating students. They emailed back their solutions before the end of the month to the sender of the problems, who gave feedback on their attempts as soon as possible. The eight trainers involved in 2015 were: Mark Flanagan, Eugene Gath, Bernd Kreussler, Gordon Lessells, John Murray, Anca Mustaţă, Andrei Mustaţă and Rachel Quinlan.

An important component of the training for maths olympiads is to expose the students to olympiad-type exams. It is now an established tradition in all five enrichment centres to hold a local contest in February or March. In addition, this year a number of students from Ireland was invited to participate in the British Mathematical Olympiad Round 1 (27 November 2015) and Round 2 (28 January 2016). I would like to thank UKMT, in particular Geoff Smith, for giving our students this opportunity.

For the first time in 2015, some members of the Irish Maths Olympiad Squad participated in the Iranian Geometry Olympiad, which took place on 3 September 2015. Participants of about 20 countries sat the olympiad exam in their home counties. Each participating country received solutions with marking schemes from the organisers and was responsible to grade the exam papers of their own students. The results together with scans of the solutions of the four best students could then be sent by email to the organisers. The exam problems were fairly tough; there was no especially 'easy' problem on the paper. The maximally possible score was 40 points, the Bronze medal cut-off at IGO was 14 points and the best score of an Irish participant was 8 points. More information can be found on `www.igo-official.ir`.

The selection contest for the Irish IMO team is the Irish Mathematical Olympiad (IrMO), which was held for the 29$^{\text{th}}$ time on Saturday, 23$^{\text{rd}}$ April 2016. The IrMO contest consists of two 3-hour papers on one day with five problems on each paper. The participants of the IrMO, who normally also attend the enrichment classes, sat the exam simultaneously in one of the five centres. This year, a total of 88 students took part in the final Round of IrMO. The top performer is awarded the Fergus Gaines cup; this year this was Antonia Huang. The best six students (listed in order in Table 1) were invited to represent Ireland at the IMO in Hong Kong.

In addition to the training camps mentioned above and an IMO-team camp at UCC before departure, immediately before the IMO a four-day joint training camp with the team from Trinidad and Tobago was held in Hong Kong. The sessions were conducted by the two Deputy Leaders Anca Mustaţă and Jagdesh Ramnanan.

2. The days in Hong Kong

The team (including Leader and Deputy Leader) arrived around 10pm on Monday, the 4$^{\text{th}}$ of July in Hong Kong. A bus ride of more than 90 minutes took us to the Holiday Inn Express Hotel in Kowloon East, where the team would carry out their intensive pre-IMO training camp in collaboration with the team from Trinidad and Tobago.

During the camp the students enjoyed the excellent free facilities of some of the local public libraries. On three of the days they took separate mock exams that were similar to IMO exams in duration and difficulty. On each day, different members of the team had particular success in solving the mock exam problems, which indicated that each team member was capable of a good performance at the IMO on their best day.

On Tuesday at noon I was transferred to the Harbour Grand Hotel Kowloon where the Jury resided until the day of the second exam.

The Jury of the IMO, which is composed of the Team Leaders of the participating countries and a Chairperson who is appointed by the organisers, is the prime decision making body for all IMO matters. Its most important task is choosing the six contest problems out of a shortlist of 32 problems provided by a problem selection committee, also appointed by the host country. This year's official Chairperson of the Jury was Prof. Kar-Ping Shum. He was already Chair of the Jury at the IMO in Hong Kong in 1994. He received the Paul Erdös Award 2016 of the World Federation of National Mathematics Competitions for devoting himself for more than 30 years to the promotion of mathematics, mathematics education and mathematics competitions in Hong Kong and all over Southeast Asia.

The Jury sessions this year were conducted by Andy Loo on behalf of the Chairman. With his excellent communication skills, the Silver Medallist at the IMO in Argentina 2012 made the Jury sessions a pleasant experience. A novelty introduced by the organisers was the use of electronic voting devices during the Jury meetings. This sped up the usually very lengthy process of selecting the six contest problems, but also made all votes secret. In situations where a clear majority was expected to vote in favour of a certain motion, Andy Loo used voice votes ("Those in favour of the motion say 'Aye', . . . , those against say 'No', ... I think the Ayes have it, the Ayes have it."). This procedure was even faster than the use of the electronic voting devices. If the voice vote didn't end with an obvious majority for one option, an electronic vote would be conducted.

Before the process of problem selection was begun, the Jury decided if they want to continue the practice of recent years to have one problem from each of the four areas (algebra, combinatorics, geometry and number theory) included in problems 1, 2, 4 and 5. A majority of almost two thirds voted in favour of continuing this practice. Two problems, one from algebra and one from combinatorics, had to be removed from the short list because similar problems with similar solutions were used in recent competitions in Bulgaria and

Russia. From the remaining 30 shortlist problems, the six contest problems were selected in an efficient way during a number of Jury meetings on Friday, 8 July. During the remaining Jury meetings, translations and marking schemes were approved. The creativity of the leaders when translating the exam problems into their native languages becomes evident in Problem 6: about 36 different names were used for the person who claps his or her hands.

During a couple of joint meetings of the Jury with the IMO Advisory Board, important changes to the general regulations were discussed and approved. The most important change concerns eligibility rules for contestants. So far, it was required that Contestants are not formally enrolled at a university. The newly adopted regulations require instead that Contestants must have been normally enrolled in full-time primary or secondary education. Also, the reference date for the age limit is no longer the day of the second Contest paper, but now is the first of July. The new regulations will be phased in within the next two years.

On Saturday, the 9$^{\text{th}}$ of July, the Irish team moved from the Holiday Inn Express to student accommodation on the campus of HKUST and the IMO got under way. The opening ceremony took place at Queen Elizabeth Stadium on Sunday afternoon. During the traditional parade, the teams appeared in order of the first participation of their countries at the IMO. In addition to the usual speeches, there were performances of several pieces of music written especially for this event, such as the IMO 2016 Theme Song "In Love We Are One".

The two exams took place on the 11$^{\text{th}}$ and 12$^{\text{th}}$ of July, starting at 9 o'clock each morning. During the first 30 minutes, the students were allowed to ask questions if they had difficulties in understanding the formulation of a contest problem. The Q&A session on the first day of the contest, where 85 questions were asked by students from 44 countries, was completed at 10.51 am. On the second day, 95 questions from students of 50 countries were answered by 10.45 am. Initial discussions took place about possibilities to streamline the Q&A sessions at future IMOs, for example by dealing with routine questions in a standardised way and by having designated people, involving coordinators, for each of the three questions who follow the process closely.

The student's scripts were available on the evening of the first exam day. Skimming through their work it seemed that Ioana could probably get full marks for Problem 1. Because all our team members were aware of spiral similarity, they could secure at least one mark for the only geometry problem on this year's paper. The work of our students for Problems 2 and 3 didn't look that promising. After joining the contestants at HKUST, Anca and I went into the detailed study of our student's scripts. Anca's excellent knowledge in geometry proved to be crucial for securing all the 23 marks our students deserved for Problem 1.

During the coordination days, the students were entertained with excursions to a variety of interesting places in Hong Kong. On the first day they took advantage of Antonia's familiarity with the place to visit the second tallest building in Hong Kong, the Two International Finance Centre, from where they could view the city from the top. On the second day they went on a bus excursion to The Peak, a unique high point on Hong Kong Island offering spectacular views of the city. The shape of the Peak Tower blended with the letter π forms a major component of the logo of this year's IMO. The students also visited a traditional market and a school.

The final Jury meeting, at which the medal cut-offs were decided, took place on the evening of Thursday, 14$^{\text{th}}$ July. The closing ceremony followed by the IMO Dinner was held on Friday evening at the Hong Kong Convention and Exhibition Centre. The journey back home started for our team on Saturday very early in the morning.

3. The problems

The two exams took place on the 11$^{\text{th}}$ and 12$^{\text{th}}$ of July, starting at 9 o'clock each morning. On each day, $4\frac{1}{2}$ hours were available to solve three problems.

Problem 1. Triangle BCF has a right angle at B. Let A be the point on line CF such that $FA = FB$ and F lies between A and C. Point D is chosen such that $DA = DC$ and AC is the bisector of $\angle DAB$. Point E is chosen such that $EA = ED$ and AD is the bisector of $\angle EAC$. Let M be the midpoint of CF. Let X be the point such that $AMXE$ is a parallelogram (where $AM \parallel EX$ and $AE \parallel MX$). Prove that lines BD, FX, and ME are concurrent. *(Belgium)*

Problem 2. Find all positive integers n for which each cell of an $n \times n$ table can be filled with one of the letters I, M and O in such a way that:

- in each row and each column, one third of the entries are I, one third are M and one third are O; and
- in any diagonal, if the number of entries on the diagonal is a multiple of three, then one third of the entries are I, one third are M and one third are O.

Note: The rows and columns of an $n \times n$ table are each labelled 1 to n in a natural order. Thus each cell corresponds to a pair of positive integers (i, j) with $1 \leqslant i, j \leqslant n$. For $n > 1$, the table has $4n - 2$ *diagonals* of two types. A diagonal of the first type consists of all cells (i, j) for which $i + j$ is a constant, and a diagonal of the second type consists of all cells (i, j) for which $i - j$ is a constant. *(Australia)*

Problem 3. Let $P = A_1A_2 \ldots A_k$ be a convex polygon in the plane. The vertices $A_1, A_2, \ldots, A_k$ have integral coordinates and lie on a circle. Let S be the area of P. An odd positive integer n is given such that the squares of the side lengths of P are integers divisible by n. Prove that $2S$ is an integer divisible by n. *(Russia)*

Problem 4. A set of positive integers is called *fragrant* if it contains at least two elements and each of its elements has a prime factor in common with at least one of the other elements. Let $P(n) = n^2 + n + 1$. What is the least possible value of the positive integer b such that there exists a non-negative integer a for which the set

$$\{P(a+1), P(a+2), \ldots, P(a+b)\}$$

is fragrant? *(Luxembourg)*

Problem 5. The equation

$$(x-1)(x-2)\cdots(x-2016) = (x-1)(x-2)\cdots(x-2016)$$

is written on the board, with 2016 linear factors on each side. What is the least possible value of k for which it is possible to erase exactly k of these 4032 linear factors so that at least one factor remains on each side and the resulting equation has no real solutions? *(Russia)*

Problem 6. There are $n \geqslant 2$ line segments in the plane such that every two segments cross, and no three segments meet at a point. Geoff has to choose an endpoint of each segment and place a frog on it, facing the other endpoint. Then he will clap his hands $n-1$ times. Every time he claps, each frog will immediately jump forward to the next intersection point on its segment. Frogs never change the direction of their jumps. Geoff wishes to place the frogs in such a way that no two of them will ever occupy the same intersection point at the same time.

(a) Prove that Geoff can always fulfil his wish if n is odd.
(b) Prove that Geoff can never fulfil his wish if n is even.

(Czech Republic)

4. The results

The Jury tries to choose the problems in such a way that Problems 1 and 4 are easier than Problems 2 and 5. Problems 3 and 6 are usually designed to be the hardest problems. That this goal was met this year is reflected in the scores achieved by the contestants on the problems (see Table 2).

The medal cut-offs were as follows: 29 points needed for a Gold medal (44 students), 22 for Silver (101 students) and 16 for Bronze (135 students). A further 162 students received Honourable Mentions – a record number. Overall, 35.2 % of the possible points were scored by the contestants, which is in line with the IMOs 2008–2012,

	P1	P2	P3	P4	P5	P6
0	52	277	548	132	353	474
1	63	65	25	22	36	31
2	32	99	14	26	55	9
3	9	30	0	10	21	39
4	6	7	0	26	50	4
5	35	8	2	15	2	4
6	14	9	3	24	4	4
7	391	107	10	347	81	27
average	5.272	2.033	0.251	4.744	1.678	0.806

Table 2. The number of contestants achieving each possible number of points on Problems 1–6

Name	P1	P2	P3	P4	P5	P6	total	ranking
Robert Sparkes	5	0	0	7	3	0	15	281
Anna Mustata	4	0	0	7	3	0	14	312
Ioana Grigoras	7	0	0	2	0	0	9	409
Cillian Doherty	3	2	0	1	0	0	6	469
Liam Toebes	2	0	0	3	0	0	5	481
Antonia Huang	2	0	0	0	0	0	2	515

Table 3. The results of the Irish contestants

but lower than the average scores in 2013 and 2014, and 4.3 points higher than the historically low average of last year.

Table 3 shows the results of the Irish contestants. Writing a complete solution to a problem during the exam is a difficult task at a competition of this level, and is rewarded by the award of an Honourable Mention. The three Honourable Mentions awarded to the Irish contestants this year consolidate a recent trend: 2016 represents the fifth year in a row with at least two Honourable Mentions for the Irish team.

The figures in Table 4 have the following meaning. The first figure after the problem number indicates the percentage of all points scored out of the maximum possible. The second number is the same for the Irish team and the last column indicates the Irish average score as a percentage of the overall average. The last column of this table shows that the Irish Team is approaching a competitive level at Problems 1 (geometry) and 4 (number theory). Improvements in this direction have been seen in recent years; now this seems to be a sustained trend.

Problem	topic	all countries	Ireland	relative
1	geometry	75.3	54.8	72.7
2	combinatorics	29.0	4.8	16.4
3	number theory	3.6	0.0	0.0
4	number theory	67.8	47.6	70.3
5	algebra	24.0	14.3	59.6
6	combinatorics	11.5	0.0	0.0
all		35.2	20.2	57.5

Table 4. Relative results of the Irish team for each problem

Although the IMO is a competition for individuals only, it is interesting to compare the total scores of the participating countries. This year's top teams were from USA (214 points), Republic of Korea (207 points) and China (204 points). Ireland, with 51 points in total achieved the 75$^{\text{th}}$ place. This is the fifth highest team score and the fifth best relative ranking of an Irish team at the IMO.

This year, six student achieved the perfect score of 42 points. The detailed results can be found on the official IMO website `http://www.imo-official.org`.

5. Outlook

The next countries to host the IMO will be

2017	Brazil	12–24 July
2018	Romania	
2019	United Kingdom	
2020	Russia	
2021	USA	

6. Conclusions

This year's results of the Irish IMO team are in line with performance in recent years. When comparing Ireland with other countries, it is more meaningful to consider relative ranks than looking at absolute ranks, because the number of participating countries has increased over the years. This year, 31.48% of the participating teams scored less than the Irish team. After 2005 (the year in which Fiachra Knox achieved a Silver Medal) the Irish team achieved a higher relative rank only twice: in 2007, when Stephen Dolan got a Bronze Medal and in 2014, when all six students received Honourable Mentions.

Since Ireland's first participation in 1988, the Irish teams won eight medals and 37 Honourable Mentions, 18 of these since 2012. This underscores the increased ability level of recent students which is supported by increased training activities. This year, Robert Sparkes only narrowly missed a Bronze Medal.

It should be mentioned that Ireland's involvement in the European Girls' Mathematical Olympiad (EGMO) certainly had a positive impact on the training and performance of the IMO team members. This year, for the first time ever, there were three girls among the top six performers at the Irish Mathematical Olympiad. They came with a lot of international experience: Anna has four EGMO participations under her belt and returned home with a Silver Medal from EGMO 2015 and 2016; Ioana, who participated three times at the EGMO, achieved a Bronze Medal at EGMO 2016; and Antonia, who currently holds the Fergus Gaines cup, has participated twice at EGMO. Two of these three, Anna and Antonia, will be eligible to participate at the IMO for two more years.

To sustain the positive development in the performance of the Irish Team at the IMO we have seen in recent years, more needs to be done to increase the ability and confidence of our students to solve an easy IMO problem in each of the four subject areas (algebra, combinatorics, geometry and number theory).

From successful past Irish IMO contestants we know that a crucial prerequisite for achieving an award at an IMO is the ability to work independently through new training materials and the desire to work intensely on difficult problems in their own time. One of the aims of the remote training, which runs from September until December, is to support students in developing the ability to work on their own. The score boards of the remote training in recent years indicate that only those who qualified for the Irish IMO team had responded regularly to the monthly problems. A challenge for the near future will be to increase the number of those students who engage fully with the remote training.

Experience from a large number of international and national mathematical problem-solving competitions suggest that students who get involved in such activities at an earlier age have a much higher probability to succeed at a high level. The earlier students start to engage independently in mathematical problem-solving activities, the more profoundly their problem solving skills can be developed.

With this in mind, it becomes clear how valuable any initiative is that aims at getting students in their Junior Cycle or students in Primary School involved in mathematical problem solving activities. A notable example is the Maths Circles initiative which was set up for Junior Cycle students in second level schools in the Cork area in 2013. As a follow-up, the maths enrichment centre at UCC now runs Junior Maths Enrichment Classes for students in second and third year.

Currently there is a bid for an SFI grant which aims at extending the Maths Circles initiative nationwide. One could hope that this initiative helps to motivate teachers to support problem-solving activities at a local level so that early-stage mathematical problem solving activities would become more widespread. Such activities for younger students would greatly enhance the general mathematical education of school-level students. Only with a broad base of young students with mathematical problem-solving skills, it will be possible in the long term to lead the best students to an internationally competitive level.

Also worth mentioning in this context is the PRISM (Problem Solving for Post-Primary Schools) competition, which is a multiple-choice contest designed to involve the majority of pupils in mathematical problem solving; it has a paper for Junior Cycle students and one for Senior Cycle students. This contest is organised since 2006 by mathematicians from NUI Galway.

While our students are well equipped to solve problems at the level of the Irish Maths Olympiad, they have less experience in attempting problems at IMO level. This can be disheartening for students who, at the IMO contest, find themselves unable to comfortably deal with the difficulty level as well as aspects of time management within the exam. Students from other countries have more experience in sitting exams of the difficulty and format of the IMO. We have started to build such experience into our training programmes, mainly at some of the training camps for the Irish Maths Olympiad Squad and at the joint camp with the team from Trinidad and Tobago. Ways should be explored in which IMO style exams could be made part of the team selection process.

A number of IMO teams regularly organise joint training camps that take place immediately before the start of the IMO. Joint sessions with other teams strengthen international relationships among mathematically gifted students and enrich the training of all participating teams. The joint training in Hong Kong with the team from Trinidad and Tobago was very successful and everybody agreed that similar camps should be held in future years, provided that sufficient funding is available. Prior to the IMO in Brazil 2017, such a camp could help the Irish contestants to adjust to the different time zone and the tropical climate.

To be able to fund such camps, to continue with all the other training activities mentioned in this report, to send a full team of six students and to restart the practice to send an Official Observer to any of the next IMOs, efforts have to be increased to secure funding.

7. Acknowledgements

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The foundation for the success of the contestants is the work with the students done in the Enrichment Programmes at the five universities. This work is carried out for free by volunteers in their spare time. Thanks go to this year's trainers at the five Irish centres:

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