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# INCREMENTAL ELASTIC SURFACE WAVES AND STATIC WRINKLES

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This is an abstract of the PhD thesis *Incremental elastic surface waves and static wrinkles* written by Artur L. Gower under the supervision of Prof. Michel Destrades at the School of Mathematics, Statistics, and Applied Mathematics, National University of Ireland, Galway and submitted in September 2015.

This *article-based thesis* comprises a collection of four articles, each of which constitutes a chapter written and formatted in manuscript form. The general aim underlying these articles is to understand and predict how incremental elastic surface waves propagate or static wrinkles form on a deformed elastic substrate. The formation of these small-amplitude disturbances can be the end goal, such as in sending signals or creating functional coatings, or they can be used to measure and characterise the underlying elastic substrate. This thesis focuses on using surface waves or static wrinkles to characterise soft solids, such as biological tissues.

For the complete thesis see [1].

Here we summarize the *main conclusion of the thesis.* Chapter 1 predicts a new phenomenon: oblique wrinkles, which should appear in a large range of materials. Yet oblique wrinkles have not been seen experimentally so far on soft solids. Another issue raised was why are the predicted critical strains greater than the experimentally observed critical strains? We showed that this is likely due to a skin effect caused by dehydration.

In Chapter 2 the effects of a stiffer skin on an elastic substrate on surface wrinkles was initially studied, and therein we also studied the

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possibility of using surface wrinkles to characterise fibre reinforced materials.

The results from Chapter 3 show that measuring the propagation speed of surface waves only along the principal directions of deformation leads to many challenges in non-destructive evaluation of strain and stress, because these directions are not necessarily aligned with the directions of fastest and slowest propagation. However, the methods for calculating surface waves along any direction presented in that chapter are now sufficiently mature and robust to be able to use the full Rayleigh wave-field in order to characterise solids. There are now experiments in place that measure surface waves on tissue, and a wide range of techniques to infer the surface elastic properties from these measurements for a range of materials. Yet, to date, surface wave measurements have not been adequately linked to the elastic properties of soft tissue, such as the residual stress or the reinforcing fibre properties.

Chapter 4 shows a surprisingly simple relationship between the angle of the surface wrinkle wave-front and the fibre orientation, a trend which becomes stronger the stiffer the fibres. Yet predicting how these wrinkles appear on soft fibre reinforced solids required a highly technical and involved numerical method. A promising alternative model is that of a soft tissue reinforced by fibres idealized to be infinitely stronger than the surrounding soft matrix.

## References

[1] A. L. Gower (2015) Incremental Elastic Surface Waves and Static Wrinkles. Available at:`http://hdl.handle.net/10379/5303`

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