B. Beauzamy: Archimedes' Modern Works

REVIEWED BY RICHARD M. ARON

This is the fifth in the "Real Life Mathematics" series of the author's company, Société de Calcul Mathématique, the first four being in French and mostly on applied probability. With a title like "Archimedes' Modern Works" it is clear that this volume is intended for a much larger, more general audience. In fact, the book's title provides us with a strong hint about the author's thesis, namely that Archimedes' contributions provide techniques to attack modern problems that are new and sometimes more powerful than current approaches.

The text consists of two main parts, "Archimedes maps" and the longer "Archimedes method," each divided into a number of chapters. These parts have as bookends some introductory remarks about the topic itself and the sources used, followed at the end by Part 3. This last part consists of some 30 pages of material that includes "reasonably unquestionable" material about Archimedes by Plutarch, et al, together with an "objective" description of Archimedes' abilities and some additional material such as an unanswered letter from Beauzamy to the Lord Mayor of Syracuse, requesting that this city give more prominence to its hometown hero.

A major part of Archimedes' work was lost for around 2,000 years. It was rediscovered in 1906 by John Ludwig Heiberg, an expert on Archimedes who travelled to Constantinople to examine it. In fact, the "it" that Heiberg examined was ostensibly not Archimedes' work, but rather a prayer book which had been completed on April 13, 1229. This prayer book had been "palimpsested" from seven treatises by Archimedes. A *palimpsest* is simply an early example of recycling. Centuries ago, people wrote on parchment that was made from animal skin. Parchment was time-consuming and expensive to produce, but it was also quite durable. Instead of throwing it aside when it was no longer needed, it could be scraped again, or "palimpsested," thereby clearing its surface for-in this case-some

thirteenth century writers to write a prayer book. (In some sense, a modern analogy occurs when we delete a file from our PC in order to make room for a new file; in fact, traces of the deleted file still exist in the hard drive from which experts can sometimes reassemble parts of the deleted information.) In 2002, the Archimedes Palimpsest was sold at auction and is now under study in Baltimore, Maryland. Among other things, it contains hitherto undiscovered work of Archimedes on a game called *Stomachion* and on the aforementioned *Archimedes Method*.

The first part of the book, Archimedes maps, begins with a 20 page examination of Archimedes' argument that the area of a sphere is four times the area of a diametrical circle. To modern mathematicians, this is evident, since all that one is saying is that $(4\pi r^2) = (4)(\pi r^2)$. On the other hand, as Beauzamy points out, Archimedes' approach yields a robust method to compare two objects which allows extensions to more complicated situations, something that a mere formula does not do. The approach that Archimedes used can be applied to yield the modern textbook formula

$$2\pi \int_a^b f(t)\sqrt{1 + f'(t)^2}dt,$$

for the area of the surface of revolution obtained by rotating $y = f(x) > 0$, $a \le x \le b$, about the $x-$axis. The author notes (here, and not infrequently throughout the book) that modern mathematicians "simply forgot to indicate that it comes from Archimedes, following exactly his proof." Another instance of such forgetfulness occurs with the so-called Lambert projection (1772), of mapping the northern hemisphere onto a disc such that any two countries having the same area on the earth are mapped into two sets with the same area in the disc.

The second part deals with what is known as Archimedes method, of "comparing the weight" of geometric figures in order to be able to compare their volumes, areas, etc. Here one finds the comparison of the volume of a sphere and that of a circumscribed cylinder. Again, the modern reader may not be impressed at first, since one can calculate each volume and simply divide. But how many of us realize that the ratio is 2 : 3? (Despite misgivings about the rigor of his arguments, Archimedes apparently decided that this was his crowning achievement; a century after his death, Cicero was able to locate his tomb because it contained the engraving of a sphere and

circumscribed cylinder that Archimedes had requested.) Seemingly, Archimedes was dissatisfied with the rigor of this method, preferring to use the method of exhaustion (e.g. computing the area of a circle by instead computing the areas of inscribed and circumscribed polygons). Although the book contains attempts to give descriptions of the method, all are somewhat murky in their formulation (with, e.g., the first such description occupying a half-page of text). On the other hand, there are several applications of the method, both in the historical and the modern contexts. For instance, there is a nice description of the case of Hiero's crown, whereby Archimedes was able to determine that gold had been replaced by a cheaper metal in the production of this object. (Linked to Hiero's crown is the famous "Eureka" story of the naked Archimedes running out of his bath.)

Throughout, the reader is often rewarded with "Beauzamy-isms," such as his promise to the reader that "Archimedes' approaches [to comparison methods] provide interesting and relevant research subjects, the kind of research subject which will be appreciated by all governments and funded by all agencies." I wonder.

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