

ON PROPERTIES OF THE MIRIMANOFF POLYNOMIALS

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This is an abstract of the PhD thesis *On Properties of the Mirimanoff Polynomials* written by D. Lynch under the supervision of Prof Rod Gow at the School of Mathematical Sciences, University College Dublin and submitted in June 2012.

Historically the Mirimanoff polynomials were first considered in connection with solving Fermat's Last Theorem [1], and recently have been generalised further [2]. We show a surprising connection between these polynomials and the established theory of Costas arrays. Over finite fields, we investigate a conjectured bound on the number of roots. This is proven for some special cases. We also consider the polynomials over the complex domain, where it is shown that for odd n (a parameter of the polynomial), the roots all occur in three distinct regions of the complex plane. We then build on these results to show the irreducibility of a specific factor of the Mirimanoff polynomials over the integer polynomial ring. Irreducibility is proven for $n = 2p$ and $n = 3p$ where p is a prime. In addition, partial results are given for $n = 5p$ and $n = 7p$.

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