# Julian Havil: The Irrationals, Princeton University Press, 2012, ISBN:978-0-691-14342-2

### REVIEWED BY ROBIN HARTE

We practicing mathematicians have tended to become a little blasé about real numbers: we remember early encounters with "Dedekind cuts" and "Cauchy sequences", and that "the square root of two" is not a rational number, but are generally happy to have put that sort of angst behind us. This splendid book brings us back to when these tedious details represented unsolved problems, and occupied the serious attention of giants. Arithmetic and geometry were intimately intertwined, and "commensurability" was a vital issue. From our casual present day viewpoint about fundamentals, there are some interesting surprises: how many of us for example were aware what heavy weather was to be made of the irrationality of $\sqrt{17}$ ?

Highlights of this book include the seemingly frivolous "look and say" irrationals built by prescribing their decimal expansions, the status of the golden mean, values of the Riemann zeta function, and a "repellant" property of rationals: if $a/b \neq p/q$ then $|aq - bq| \geq 1$ giving $|a/b - p/q| \geq 1/(bq)$. This eventually leads to a quantitative measure of transcendentalism.

Julian Havil writes delightfully, with a keen sense of irony; this is a book to read through and then go back, again and again, for another look at this Newtonian pebble and that.

For this reviewer there is only one significant omission: Brouwer intuitionism and the extraordinary efforts of Erret Bishop to [*Foundations of constructive analysis*] breathe fresh life into it. In particular the "constructive" version of the reals is built on a "constructive" version of the Cauchy sequences $x = (x_n)$ for which, always, $|x_n - x_m| \leq \frac{1}{n} + \frac{1}{m}$. For some period of time after reading Erret Bishop, we became seriously nervous about least upper bounds - a feeling apparently shared many years earlier by Arthur Allen of TCD. With the passage of time, of course, our non-constructive

habits reasserted themselves, and we came to see a curious similarity between the protocols for writing constructive analysis and "non standard" analysis.

**Robin Harte** . After teaching for some twenty five years at Swansea and Cork, Robin retired and returned, circuitously, via Fairbanks Alaska and UNAM, Mexico to a vaguely emeritus position in Trinity College. He occasionally tries to take himself seriously.

SCHOOL OF MATHEMATICS, TRINITY COLLEGE DUBLIN
*E-mail address*: rharte@maths.tcd.ie