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Alexei V. Pokrovskii

02.06.1948 – 01.09.2010

Alexei Vadimovich Pokrovskii, an outstanding mathematician, a scientist with broad mathematical interests and a pioneer in the mathematical theory of systems with hysteresis, died unexpectedly on September 1, 2010, aged 62. For the last nine years he was Professor and Head of Department of Applied Mathematics at University College Cork.

The main body of Alexei's work belongs to the areas of nonlinear dynamical systems (including systems with hysteresis, discontinuous and nonsmooth systems), control theory, nonlinear functional analysis and applied mathematical modelling. However, the remarkable diversity of his research was broader and included, at different stages of his work, contributions to game theory, stochastic systems, complexity and general functional analysis.

Alexei was born and reared in Voronezh, a city in Central Russia about 500 kilometres south of Moscow. His family came from a medical background. His paternal grandfather, Alexei Ivanovich Pokrovskii (1880-1958), was a professor and Chair of Ophthalmology at Voronezh Medical Academy, the author of more than 90 research publications. Alexei's father, Vadim Alexeevich Pokrovskii, was a professor and Chair of Hygiene in Voronezh Medical Academy; his uncle, Alexei Alexeevich Pokrovskii, was an academician, a vice-president of the Academy of Medical Science of the USSR and the director of the Institute of Nutrition in Moscow. Alexei's mother, Angelina, was a teacher of English. His daughter Olya continuing the family tradition graduated from University College Cork (UCC) with a primary medical degree in 2009; the same year, his son Alexei Jr. received MSc degree in mathematics from University of Cambridge.

Alexei, his wife Natasha, daughter Olya and son Alexei Jr. at the conferring ceremony on the day of Olya's graduation from UCC.

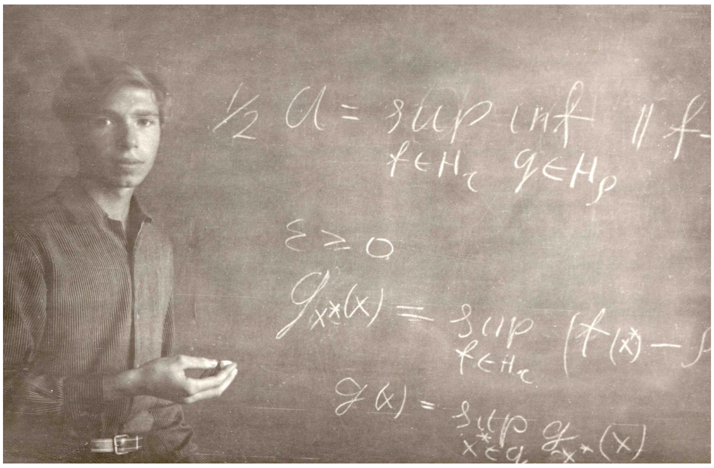
Alexei attended Voronezh State University in 1966-1971, where he received his BSc and MSc degrees in mathematics. He was a student of Mark Alexandrovich Krasnosel'skii, one of Russia's foremost mathematicians of the last century and the founder and the leader of the famous mathematical school of Nonlinear Functional Analysis. Mark Krasnosel'skii received his PhD under the direction of Mark Grigorievich Krein in 1948, and his Dr. Sci. (Dr. habil.) in 1950 at Kiev State University. He was invited to Voronezh by Vladimir Ivanovich Sobolev, a renowned expert in functional analysis, and was offered and accepted the Chair of Functional Analysis at Voronezh State University in 1952 at the age of 32. Later, they, together with Selim Krein, the younger brother of Mark Krein, who moved to Voronezh simultaneously with Mark Krasnosel'skii, organised the Mathematical Institute at Voronezh State University.

Mark Krasnosel'skii (left) and Vladimir Sobolev (right) in Voronezh.

By the time Alexei started his degree, Voronezh had become an important centre of mathematical and applied mathematical research. Alexei's early academic career was notable for the fact that he started publishing original mathematics while still a teenager. Professor Petr Petrovich Zabreiko recalled that a paper of A. F.

Timan on approximation theory was discussed at the Nonlinear Functional Analysis Seminar series (the renowned Krasnosel'skii's seminar). It was a big paper of more than 100 hundred pages. At some point, Alexei, a second year undergraduate student at the time, stepped forward to the blackboard and presented a simple but non-trivial equation. After a brief discussion it became clear that most of the results of Timan's paper follow from this formula. Alexei published this result in his first paper [1], which was frequently cited afterwards.

Alexei's outstanding talent and keen interest in mathematics became widely recognised in the University as he engaged in research with senior colleagues while still an undergraduate. Remarkably, the range of work he published as an undergraduate student included approximation theory, bifurcation theory, positive almost periodic functions, game theory and hysteresis operators. The latter topic is of special importance.



Alexei at Nonlinear Functional Analysis Seminar in Voronezh.

From 1969, when Alexei was yet only a 4$^{\text{th}}$ year student (but already an author of 4 publications), Krasnosel'skii's group began to discuss the phenomenon of hysteresis, first presented at the Nonlinear Functional Analysis Seminar series by a physicist, Boris Darinskii. The term 'hysteresis' was coined by the physicist James Alfred Ewing in his paper on electromagnetism published in 1881. See [2] for the history of the question. Many phenomenological models of hysteretic relationships between physical variables have been known since the beginning of the last century, including the so-called models with local memory, such as Prandtl's ideal plastic element (also known as *stop*) and the *non-ideal relay*, and complex models with non-local memory such as the Preisach model of magnetic hysteresis, similar models of capillary hysteresis based on domain theory, the Prandtl-Ishlinskii model of plasticity and others. Hysteresis effects were vaguely associated with memory and multivalued functions. This memory was, however, different from the memory modelled by convolution operators or delayed systems. The main characterisation of the memory manifested through hysteresis was a permanent effect of certain events in the past on the future; permanent magnetisation of a ferromagnetic material resulting from a single fluctuation of an external magnetic field would be a typical example.

A group of participants of Krasnosel'skii's seminar, including Alexei, made the first step towards the mathematical treatment of this phenomenon by introducing a new class of operators related to Prandtl's model. These operators, now known as *hysterons*, have a simple definition on the class of piecewise monotone continuous functions of time (inputs), which they map to outputs from the same class. A continuity argument was used to extend these operators to the whole space of continuous functions [3]. In a subsequent paper, Krasnosel'skii and Pokrovskii proposed a new class of differential control equations, which include hysterons as a particular case, and found the conditions that ensure the continuity of the input-output relationships defined by these equations with respect to the uniform norm [4].

In the early 1970s, Alexei moved with Mark Krasnosel'skii and a part of his group to Moscow to the Institute for Control Problems of the Russian Academy of Sciences. Here, Alexei completed his PhD under the direction of Mark Krasnosel'skii in 1974. By the time of completion, his published work numbered 16 articles.

The Institute for Control Problems was founded in 1939 with the active participation of Alexander Alexandrovich Andronov, the author of classical results in Nonlinear Oscillations theory and the

A part of Krasnosel'skii's group (Research Laboratory of Mathematical Methods for Analysis of Complex Systems) at Moscow Institute for Control Problems. From left to right, upper row: A. V. Pokrovskii, V. I. Opoitsev, A. Sobolev, M. A. Krasnosel'skii; below: N. A. Bobylev.

founder of the famous Nonlinear Dynamics school in Lobachevsky State University of Nizhny Novgorod where he moved later. The new research environment, which included applied mathematicians, physicists and engineers, stimulated the interest of Krasnosel'skii and Pokrovskii in problems of control as well as reinforcing their research in hysteretic systems and providing a new perspective on them. They formulated a research programme aimed at developing a rigorous mathematical theory, which should deliver efficient mathematical tools for modelling systems with hysteresis, simultaneously making them amenable to the study by methods of differential equations, operator theory and nonlinear functional analysis. In particular, it should resolve the ambiguity about the nature of the permanent memory associated with hysteresis phenomena and the means of modelling it. Moreover, the theory should have the means of describing systems where some relationships between the variables were formulated in terms of differential equations, while other relationships were hysteretic and could be described by the hysteron operator or the like. A typical motivating example is Maxwell's equations coupled to a hysteretic constitutive relationship between the magnetic induction B and the magnetic field H such as the relationship assumed in the Preisach model of ferromagnetic media.

The realisation of this programme took a decade. Publications of Krasnosel'skii and Pokrovskii tell the story of the evolution of their views on the subject over this time. The language and paradigm of systems theory fused with the ideology of nonlinear analysis to create the fundamentals of the theory of systems with hysteresis. A hysteretic relationship with nonlocal memory was represented by a composition of the input-state operator and the state-output function (functional) with an infinite-dimensional (or multi-dimensional) state encoding the memory of the system. In many phenomenological models the output is obtained as the superposition of outputs of infinitely many hysterons such as non-ideal relays with varying parameters in the Preisach model, stops in the Prandtl-Ishlinskii model etc. Krasnosel'skii and Pokrovskii proposed a mathematical formalism of parallel connections and cascades of hysterons which allows one to extend the input-state-output operators from the class of piecewise monotone inputs to all continuous inputs. They studied the regularity properties of these operators using an alternative geometrical description of the evolution of memory states of the Preisach model [5]. A further important step was to extend these ideas to models/operators with vector-valued inputs and outputs [6]. At the same time, they began to study dynamical systems with hysteretic components starting from the example of an oscillator described by a second order differential equation coupled with the Prandtl-Ishlinskii and Preisach operators [7, 8].

The range of Alexei's interests was continuously growing. Together with Mark Krasnosel'skii and other colleagues, he addressed a number of control problems such as, for example, the mathematical formalism of the method of block diagrams [9]; the effect of small (in uniform norm) perturbations, which have finite quadratic variation (energy), on multicomponent systems and sliding modes [10,11]; and the problem of absolute stability [12]. For example, the absolute stability of the zero solution was shown to be equivalent to the absence of nonzero uniformly bounded solutions for a wide class of evolutionary systems. Further results included conditions ensuring the positivity of the impulse-frequency response of linear systems, that is the property allowing one to apply the method of monotone operators to the analysis of such systems and their nonlinear extensions.

Together with Mark Krasnosel'skii, Alexei investigated discontinuous systems. Some typical problems associated with discontinuity are illustrated by their example of the "monster" binary function $f(t,x)$, which they showed to exist if the continuum hypothesis is true [13]. For every t, the "monster" function has the value 1 for all x except at at most a countable number of points; at the same time, for every x, this function is zero for almost every t (moreover, one can require the equality $f(t,x(t)) = 0$ to be true almost everywhere in t for every measurable function $x(t)$). In particular, the "monster" is not a measurable function of the two variables. However, the superposition operator defined by this function maps every measurable function $x(t)$ (input) to a measurable function $f(t,x(t))$ of t (output).

Another group of discontinuous problems is related to monotonic systems and discontinuous monotone operators. The Birkhoff-Tarski theorem guarantees the existence of a fixed point within an invariant interval of a monotone operator A, which acts in a Banach space semiordered by a cone. However, such a fixed point can be a discontinuity point of A. Krasnosel'skii and Pokrovskii showed the existence of a fixed point which is a continuity point of A (a regular fixed point) for general classes of monotone operators [14]. In applications to boundary-value problems, such a fixed point is a solution which passes through continuity points of discontinuous terms almost everywhere. In the context of dynamical systems, such points define stable solutions. Later, Alexei proposed a simple iterative algorithm, the so-called shuttle algorithm, for finding regular fixed points [15, 16].

Many results of Krasnosel'skii and Pokrovskii from this period, including some of the above mentioned, had links to the study of hysteresis phenomena. For example, the differential control equations they studied in relation to the hysteron and Duhem's magnetization model proved to be intimately connected to stochastic differential equations and allowed them to obtain a description of individual trajectories of Itō and Stratonovich stochastic equations [17].

In 1983 Alexei was awarded the prestigious Andronov prize by the USSR Academy of Sciences.

The seminal monograph of M. A. Krasnosel'skii and A. V. Pokrovskii "Systems with Hysteresis" appeared the same year [18]. It laid the foundations of the mathematical theory of hysteresis operators on the basis of the analysis of many phenomenological models of hysteresis, thereby forming the modern concepts of mathematical modelling of hysteretic systems, opening the door to the systematic application of mathematical tools to the analysis of dynamical systems with hysteretic components, and paving the way for the research that followed. The theory of Krasnosel'skii and Pokrovskii is essentially the theory of rate-independent operators[1] (the term and concept introduced later by Augusto Visintin). That is, these operators, acting in spaces of functions of time, are invariant with respect to the action of the group of monotone transformations of the time scale. This general definition entails a set of non-trivial properties of hysteresis operators which are sufficient for developing formal concepts with various applications. In particular, such operators are never differentiable, i.e., either nonsmooth or discontinuous. Rate-independent input-state-output operators have been constructed for models of hysteresis proposed in diverse disciplines.

The density of new ideas and concepts in Krasnosel'skii and Pokrovskii's book is remarkable. Literally every chapter of the book and sometimes even a section or remark pioneered a branch of the mathematical theory of systems with hysteresis. Topics in the book, to mention just some, include the regularity of hysteresis operators; identification theorems; composition and inversion of hysteresis operators and construction of compensators (a topic of importance for engineering and control applications); representation theorems; vibrostable differential equations; links with the theory of sweeping processes and Skorokhod problems; discontinuous transducers; and, the geometrical interpretation of dynamics of states for complex hysteresis models (such as Preisach and Prandtl-Ishlinskii operators).

The publication of the book in 1983 in Russian and of its extended English translation in 1989 [19] was followed by an explosion of interest in the mathematical tools it offered to the applied mathematics community, resulting in a raft of publications in the 1990s

[1]The theory of rate-independent hysteresis operators has also been extended to some classes of rate-dependent models of hysteresis.

continuing into the first decade of this century. Several groups of researchers in Europe, the USA and Japan contributed to the development of the mathematical theory. Important monographs were written by Isaak Mayergoyz, Augusto Visintin, Pavel Krejčı, Martin Brokate and Jürgen Sprekels [20–24]. The convergence of mathematicians with researchers in hysteretic systems from different fields (electromagnetism, phase transitions, mechanics, engineering, economics and others) on the basis of common mathematical language and common understanding of hysteresis phenomena enriched the theory by new problems and methods, leading to the versatile science of hysteresis [25], which has many faces depending on the application. Links with thermodynamics, statistical physics (including the Ising model), multi-rate systems, stochastic systems, optimal control as well as to the mathematical disciplines such as variational inequalities and queueing theory have been discovered and are being explored. The interaction of researchers in hysteresis with different backgrounds benefited from regular interdisciplinary meetings such as the series of conferences in Trento (continued in Berlin), the US-European Hysteresis Modelling and Micromagnetics symposium series and, later, the MURPHYS conference series which Alexei organised in Cork.

Until 1992 Alexei remained in Moscow. He obtained his Dr. Sci (Dr. habil) at the Institute for Control Problems in 1989 and next year became head of the centre responsible for developing mathematical methods in Control at the Institute for Information Transmission Problems of the Russian Academy of Sciences, an institution renowned for many excellent mathematicians working there including three Fields medalists. His mathematical universality became apparent and his collaboration with others was wide-ranging and prolific.

He published a few works on game theory, Kolmogorov's complexity and predictability [26–28]. In particular, he introduced measures of unpredictability of binary sequences using hierarchies of sets of predictors. The measure of unpredictability of a sequence is dependent on a particular choice of a set of predictors (finite automata, Turing machines etc.); however, as Alexei showed, it satisfies certain universal relationships. This work was highly valued by Andrei Nikolaevech Kolmogorov.

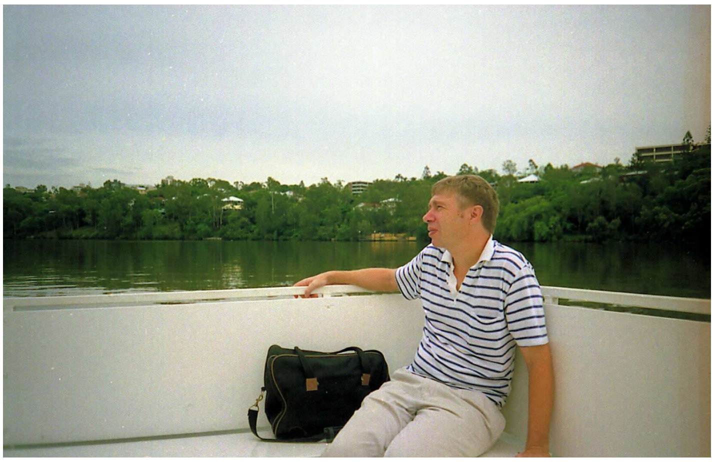
Another body of work addressed linear systems and included methods of identification of a linear system on the basis of a few tests [29];

limit norm of linear operators and its application to control [30]; and, stability of asynchronous systems [31]. The latter problem is related to infinite products of matrices selected from a finite set and the theory of switching systems.

His research of hysteretic systems continued apace with a focus on dynamics of closed systems with hysteretic components, including problems of stability, dissipative properties, oscillations, method of averaging and dynamics of distributed systems of parabolic type [32–35].

From the beginning of 1990s, Russian scientists started to travel abroad. In 1991 Alexei attended the "Models of Hysteresis" meeting organised by Professor Augusto Visintin in Trento. Here, for the first time, he met colleagues from the western hysteresis community and initiated collaboration and friendship, which continued for many years afterwards.

In 1992 Alexei accepted a research position in Australia, where he worked until 1997 dividing his time between being an Adjunct Professor at Deakin University, Geelong, and Director for European Operations at the Centre for Applied Dynamical Systems, Mathematical Analysis and Probability, at the University of Queensland, Brisbane. The Australian period was very productive for him with his research output numbering around 50 papers. The focus of his research during this time shifted towards nonlinear dynamics, specifically, dynamics of discretisations of chaotic systems. The starting point of this research can be illustrated by the observation that the dynamics of the logistic map $x_{n+1} = \mu x_n(1 - x_n)$ are qualitatively different from those of its discretised version (computer realisation) no matter how accurate the discretisation may be. The understanding of this effect was achieved by Alexei and his colleagues Phil Diamond, Peter E. Kloeden and Victor Sergeevich Kozyakin when it became clear that the discretisation acts as a randomising factor. As a result, they developed phenomenological models based on the theory of a class of random maps which were capable of an accurate description of the effect of discretisation on the original system dynamics. In order to ensure that computer models accurately mimicked the dynamics of a map (i.e., robustness to discretisation), they developed new mathematical tools such as split-hyperbolicity and bi-shadowing on the basis of topological degree theory. These ideas have been summarised and further developed in the monograph [36], most part of which was written when the authors lived already far from each other in Ireland, Australia, Germany[2], and Russia. It is now in press and should be published soon.



Alexei on a ferry to the University in Brisbane-apparently, absorbed in another mathematical problem.

Alexei cultivated and enjoyed a collective way of doing research, which is apparent from his publications. He had a talent to identify and consolidate interests of his colleagues and involve them in joint research projects; at the same time, he was invariably interested in the research done by others and truly enthusiastic about their achievements and success. Given his method of work, the huge number of collaborators he had comes as no surprise. Professor Phil Diamond, Professor Peter E. Kloeden and Alexei organised a large scale collaboration with many mathematicians worldwide who came to visit the University of Queensland and Deakin University for varying periods of time. In particular, many Russian colleagues of Alexei visited the research center in Australia and enjoyed the welcoming warm hospitality of Pokrovskii's family during their stay-among

[2]Professor Kloeden moved to the Johann Wolfgang Goethe University in Frankfurt am Main in Germany where he was appointed to the Chair of Applied and Instrumental Mathematics in 1997; simultaneously, Alexei moved to Cork.

them, N. A. Bobylev, V. A. Bondarenko, M. L. Kleptsyna, V. S. Kozyakin, A. M. Krasnosel'skii, N. A. Kuznetsov, B. N. Sadovskii, A. A. Vladimirov and I. G. Vladimirov.

Even though Alexei didn't have security of tenure in Australia, nevertheless, the time spent there was a happy time for his family, who retain fond memories of their stay.[3] However, Alexei probably wanted to be closer to European centres of mathematica research and especially centres of active 'hysteretic life'. Still in Australia, he tested the applicability of the split-hyperbolicity technique to analysis of complex dynamical systems with hysteresis [37].

Alexei first came to Cork in the Spring of 1997 to be interviewed for the Chair of Applied Mathematics at UCC. By that time his research output ran to over 100 papers. As part of the interview process he delivered a lecture about his research interests, during the course of which he paid tribute to the work of his mentor and teacher, Mark Krasnosel'skii, who had just passed away. While his bid for the Chair wasn't successful on that occasion, his exceptional ability as a research scientist of the first rank was recognised.[4] Alexei was offered and accepted a research position, which was created in the Institute of Non-linear Science in UCC with the support of Professor Michael P. Mortell, the UCC President, and Professor John McInerney, Head of the Department of Physics. This post, though not a permanent one, provided a modicum of security for Alexei and his family for the next three or four years, which they used to good effect to settle in Cork. During that period, his expertise in several branches of mathematics, and his capacity to interact productively with a range of experts working in fields outside pure and applied mathematics, such as computer science, physics, engineering and economics, became known within the College. Accordingly, when the Chair of Applied Mathematics became vacant again, and Alexei was an applicant, it came as no surprise that his star-quality was acknowledged by the College, and in 2001 he was appointed to this Chair.

Following his appointment, while continuing to work with former colleagues elsewhere in different parts of the world, he developed

[3]Indeed, Alexei's daughter Olya returned to Australia for a while after graduating from UCC to commence her medical career.

[4]Professor Michael P. Mortell recollects that the Extern of the Selection Committee strongly advised to use every opportunity to retain Alexei.

fruitful collaborations with people in his own department, and in the UCC Departments of Computer Science, Civil, Electrical and Food Engineering, Mathematics, Microbiology, Statistics, Physics and Zoology, which led to a raft of joint publications. In doing so, he displayed a commanding knowledge of several branches of science which enabled him to appreciate the relevance of mathematical advances to the world around us, and the ability to apply them in a host of different areas.

The applied aspect of mathematical research clearly drove Alexei's interests during the Irish period of his career. He enjoyed modelling as much as analysis and immensely enjoyed and valued collaboration with colleagues from other disciplines, finding it most interesting and stimulating. The research themes of the last cohort of his PhD students included modelling hysteresis in macroeconomics (Hugh McNamara), soil-water hysteresis in hydrology (Denis Flynn, Andew Zhezherun), epidemics and seasonal dynamics of wild bird populations (Suzanne O'Reagan), canard solutions and chaos in nonsmooth singularly perturbed systems (Andrew Zhezherun), bifurcations and chaos in systems with Preisach hysteresis operator (Oleg Rasskazov).

He adapted the split-hyperbolicity concept and other techniques developed in Australia to analyse complex dynamics and chaos in laser systems, wave patterns, models of epidemiology and other applied problems using rigorous computer-based proofs [38–42].

Hysteresis, the subject which was always close to Alexei's heart, became again central to his interests. His research was now inspired by challenges of modelling hysteresis in economics, hydrology, epidemiology, biology (population dynamics) and multi-rate systems. He aspired to make the theory available to, and useful for, problems in these new areas of application, in the same way, as it has already proved to be successful in more traditional fields such as magnetism, plasticity, material science, mechanical engineering and control design. He had a very productive collaboration focusing on modelling hysteresis in hydrology [43–46] with J. Philip O'Kane, Professor and Head of Department of Civil & Environmental Engineering in UCC with whom he co-supervised two PhD students; and with Rod Cross, Professor of Economics at University of Strathclyde with whom he developed models of hysteresis in macroeconomics [47–50]. Their memoirs [2, 51] reflecting on Alexei's impact on these subjects are available in open access.

One means Alexei used to establish the UCC Department of Applied Mathematics as a centre of research excellence in applied mathematics and dynamics of hysteretic systems on the international scene was the Multi-rate Processes and Hysteresis conference series. The conference originated from his idea to explore the links between the methods of the theory of multi-rate systems and the theory of systems with hysteresis [52,53]; Van der Pol relaxation oscillations is one classical fundamental example of such links. After initial discussions with Michael P. Mortell and with Robert E. O'Malley and Vladimir Andreevich Sobolev, two world renowned experts in the analytic and geometric theory of singularly perturbed systems, with whom Alexei met at the Industrial Mathematics Congress in Edinburgh in 1999, a pilot workshop was organised in Cork in 2001. From 2002 it grew to a series of regular bi-annual meetings and gradually acquired the acronym MURPHYS, 'coincidentally' the name of an Irish stout brewed in Cork. These successful and truly multi-disciplinary meetings unifying the efforts of the singular perturbation and hysteresis communities for solving new interesting mathematical and applied problems attracted specialists from many places across the world and provided a stimulating forum for researchers in mathematics, applied mathematics, engineering, control, physics, hydrology, combustion processes, economics, financial mathematics, biology, epidemiology and, even, history. Proceedings of the conference, which was enjoyed by all the participants, were edited by the four co-chairmen.

During the last few years of his headship, Alexei put a lot of effort into cultivating biologically oriented mathematical research in his Department. With his usual contagious enthusiasm and energy he set up a regularly meeting working group between the UCC Departments of Applied Mathematics and Zoology, involving Professor Michael J. A. O'Callaghan, Dr. Tom C. Kelly, Dr. Sarah Culloty, Dr. Ruth Ramsay and others. He led several productive research projects [54–57], built collaborations with mathematical biology researchers in Ireland and the USA, co-supervised two PhD theses in environmental science and epidemiology, increased the presence of biological modelling theme in BSc and MSc applied mathematics degrees and launched a mathematical module for Systems Biology students. In one of the latest works he formulated principles of modelling hysteretic response of human population to epidemics [58].

Naturally, wherever Alexei worked, he was a centre and an attractor of excellent research, a leading mind, most esteemed by colleagues

Michael O'Callaghan, Alexei and Jim Grannel, organisers of MURPHYS conference in UCC.

and beloved by students. His charisma was irresistible. Incredibly imaginative and infinitely rich in ideas, he was absolutely generous in sharing them with others. Quoting Professor Finbarr Holland, "He had a child-like curiosity and wonderment for the scientific world, a deep knowledge of several disparate areas which, combined with a penetrating mind, enabled him to make significant progress in whatever problem that took his interest. But he also took a keen interest in other people's work, and whenever somebody shared a surprising new fact with him, his countenance would alter, his eyes would sparkle with delight, and one would get the 'thumbs up', signifying his pleasure. Such a response was very encouraging to the person sharing the information, especially to a young researcher, still unsure of his or her own ability. He was immensely generous with his time and talents, and warm-hearted in attributing to others ideas that were very often his alone, qualities which endeared him to his students. In truth, he was a polymath of the first rank." He was a truly kind and wise man, caring for people, always willing to help, responsive and discreet; and, the best colleague and mentor you could wish to have, invariably supportive and incredibly encouraging. All his countenance and manner radiated comforting friendliness. "Alexei had a most distinctive manner of speaking. Instead of using full stops, he would punctuate his sentences with a variety of smiles, ranging from the rueful to the exuberant. When we talked on the telephone I could always picture the type of smile on his face. I will miss those conversations." (Rod Cross).

Several papers, which commemorate Alexei and pay tribute to his mathematical work, and his full list of publications can be found on the web page

http://euclid.ucc.ie/pages/staff/pokrovskii/alexeipokrovskii.htm

The article by Finbarr Holland in *The Irish Times* and a few memoirs of colleagues and friends published in the open access *Journal of Physics: Conference Series* in the volume of Proceedings of conference MURPHYS'2010 are also available online [59, 60]. A tribute to Alexei has been paid at the International Symposium on Hysteresis Modelling and Micromagnetics in Levico, Italy, in May and at the Nonlinear Dynamics Conference organised in his memory in UCC in September [61]. A special volume dedicated to him will be published in *Discrete and Continuous Dynamical Systems B* next year.

There is a lot of unfinished business; the work Alexei initiated and research he developed is being continued by students whom he taught and mentored and his colleagues whom he continues to inspire.

Alexei was an outstanding mathematician, with a special way with people. He was loved by everyone who knew him; he is sadly missed.

His wife Natasha works in Tyndall National Institute. His daughter Olya is working in Cork University Hospital and studying ophthalmology. Alexei's son Alexei Jr. is doing a PhD in graph theory in the London School of Economics.

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