The IMS September Meeting 2010 at DIT
Abstracts of Invited Lectures

Automorphic Lie algebras and corresponding integrable systems

Alexander Mikhailov (University of Leeds)

We study a new class of infinite dimensional Lie algebras, which has important applications to the theory of integrable equations. The construction of these algebras is very similar to the one for automorphic functions and this motivates the name automorphic Lie algebras. It is also a natural generalisation of the construction used by Victor Kac in his study of graded Lie algebras. In contrast to the Kac–Moody algebras, automorphic Lie algebras are quasi-graded and, in a certain sense, are deformations of the Kac–Moody Lie algebras. We discuss the progress in the classification problem for automorphic Lie algebras corresponding to finite groups of automorphisms (finite reduction groups).

Integrable systems related to quasi-graded Lie algebras are nonhomogeneous and are deformations of well known integrable systems in the graded case. The two dimensional generalisation of the Volterra chain is an interesting example of such a system. Its continuous limit is the famous Kadomtsev–Petviashvili equation. Using the dressing method we study exact solutions of the 2-d Volterra system. Apart of soliton-like solutions we have found exact solutions corresponding to wave fronts. Classification of soliton and wave front solutions is related to the Schubert decomposition of a Grassmanian.

Resonances in Quantum Chemistry: Complex Absorbing Potential Method for Systems

Michael Melgaard (DIT)

The Complex Absorbing Potential (CAP) method is widely used to compute resonances in Quantum Chemistry, both for scalar valued

and matrix valued Hamiltonians. In the semiclassical limit $\hbar \to 0$ we consider resonances near the real axis and we establish the CAP method rigorously in an abstract matrix valued setting. The proof is based on pseudodifferential operator theory and microlocal analysis.

Computation versus formulae for norms of elementary operators

RICHARD TIMONEY (TCD)

A long-standing problem was to find a formula for the norm of an elementary operator acting on a C*-algebra (or a matrix algebra). As with the best problems, the problem was quick to explain, although there was no conjectured answer. The solution which has been found involves concepts that are almost equally simple to explain, but the effective use of the formula is more subtle. It does lead to insights into the structure of the algebras. The techniques involved in justifying the formula are useful in practical and theoretical ways.

Portfolio optimisation in a volatile market

CONOR MASTERSON (RENAISSANCE REINSURANCE)

The recent financial crash has led to unprecedented volatility in the European Government bond markets, with downward pressure on interest rates, increased government borrowing to fund extensive public intervention in the banking system adding to concerns over the fiscal stability and the future of the Euro system. Core European countries including Germany have seen bond yields at record low levels resulting from the flight to quality, while peripheral countries, led by Greece have seen unprecedented high yields. This situation has posed major new challenges in the managing government bond portfolios, which are generally marked by steady returns and low volatility. This talk addresses the challenges of managing this type of portfolio and discusses quantitative methods used to manage risk and optimize returns in the current market.

Breathers, solitons and freak waves

FRÉDÉRIC DIAS (UCD)

The Peregrine soliton is a localised nonlinear structure whose existence was predicted over 25 years ago but which has not to date

been experimentally observed in any physical system. It is of fundamental significance because it is localised in both time and space, and because it defines the limit of a wide class of solutions to the nonlinear Schrödinger equation (NLSE). Here, we use an analytic description of NLSE breather propagation to implement experiments in optical fibre generating femtosecond pulses with strong temporal and spatial localization, and near-ideal temporal Peregrine soliton characteristics.

In showing that Peregrine Soliton characteristics appear with initial conditions that do not correspond to the mathematical ideal, our results may impact widely on studies of hydrodynamic wave instabilities where the Peregrine soliton is considered a freak wave prototype. (This is joint work with B. Kibler, J. Fatome, C. Finot, G. Millot, G. Genty, N. khmediev, J. M. Dudley.)

Mathematics support and other strategies for tackling issues in the third-level mathematics classroom

Eabhnat Ní Fhloinn (DCU)

Increasingly diverse student populations combined with large class sizes lead to serious challenges for many mathematics educators in third-level. As a result, various forms of mathematics support have been introduced in numerous third-level institutes. However, such support systems function most effectively when integrated into the overall mathematics education of the student, and are not a complete solution in themselves to the issues encountered in the third-level mathematics classroom. This talk will focus in particular on the experience of providing mathematics support in Dublin City University, largely through the form of a Maths Learning Centre, before moving on to address possible approaches to addressing challenges in the classroom.

A case history in interdisciplinary theoretical mechanics

Hilary Ockendon (University of Oxford)

The talk will describe a project from the defence industry which started as a student project and has developed into basic research into the modelling of materials under extreme stress. It will illustrate the breadth of knowledge required to attack such problems and will touch on ideas from fluid and solid dynamics, plasticity, Greens functions, inverse problems and hyperbolic PDEs.

The Hartogs phenomenon and holomorphic geometric structures

BENJAMIN MCKAY (UCC)

One can often find an open set in a complex manifold so that every holomorphic function on the open set extends to the manifold. This is called the Hartogs phenomenon. I will describe Hartogs phenomenon for many other complex analytic geometric objects on complex manifolds.

Markov and semi-Markov reward systems for patient care

SALLY MCCLEAN (UU)

Previously, (McClean 1976, 1980) we have developed Markov and semi-Markov models for a multi-grade system with Poisson arrivals. This system consisted of a number of transient states and an absorbing state. Joint distributions for the numbers in each transient state at any time were found in each case and the limiting distributions were shown to be independent Poisson. Such models have been applied to manpower planning (Bartholomew et al., 1991) and healthcare, for movements of patients through a hospital system (Taylor et al., 1998, 2000). Semi-Markov systems with constant size and known growth have also been discussed by a number of authors e.g. Papadopolou and Vassiliou (1990), Vassiliou and Papadopolou (1992), McClean et al., (2004).

More recently, results have been obtained for a reward (cost) model for a two (McClean et al., 1998a) and three (McClean et al., 1998b) transient state system. These models were applied to a hospital in-patient system where the"reward" are costs and the total cost for a group of patients moving through the system, with no new admissions, is known as spend-down. However, it is often required to identify daily and long-term costs of a system where, in addition to movements between the transient states, and departures to the absorbing state(s), there are arrivals to the transient states. These arrivals correspond to recruitment in a manpower system and admissions of new patients for the hospital patient system. A semi-Markov approach provides more generality that can serve to describe the complex semantics of such models; for example Bartholomew et al. (1991) describe the use of semi-Markov models for manpower systems while Kao (1978) has previously used a semi-Markov model

to describe patient stays in hospital. Results for a semi-Markov system, in which the total size of the system at any time are known, have been provided by Papadopolou (2001).

In McClean et al. (2004), we have thus developed a reward model for a discrete time homogeneous semi-Markov system with Poisson arrivals and, also, for the case where the system has grown by a known amount at each discrete time point. Results were obtained for the distribution, mean and variance of daily costs of such a system at any time, and in steady state.

For the healthcare application these models can also help us to assess the complex relationship between hospital and community care where there may be possible trade-offs between hospital treatment and community care costs (McClean and Millard, 2006). Stroke disease is particularly suited to our approach as patients that do not receive appropriate therapy or rehabilitation in a timely manner may subsequently buildup huge costs over time. Semi-Markov models can assess where and how patients should be treated. We have thus developed Markov and semi-Markov models to describe the whole integrated system of stroke patient care and facilitate planning of services. Based on data on stroke patient data from the Belfast City Hospital, various scenarios are being explored, such as the potential efficiency gains if length of stay in hospital, prior to discharge to a Private Nursing Home, can be reduced.

On Hopfian and co-Hopfian groups

Brendan Goldsmith (DIT)

The notions of Hopfian and co-Hopfian groups have been studied for a long time. The terminology "Hopfian" seems to have arisen from the fact that the topologist H. Hopf showed that the fundamental groups of certain closed two-dimensional manifolds have the defining property. In modern terminology we say that a group G is Hopfian if every surjection: $G \to G$ is an automorphism; it is said to be co-Hopfian if every injection: $G \hookrightarrow G$ is an automorphism. Finite groups are, of course, the prototypes for both Hopfian and co-Hopfian groups. Hopfian and co-Hopfian groups have arisen recently in the study of algebraic entropy and dual entropy. Despite the seeming simplicity of their definitions, Hopfian and co-Hopfian groups are notoriously difficult to handle and easily stated problems have remained open for a long time: if G is Hopfian, is the direct

product $G \times Z$ Hopfian; this is still an open question. Our interest in this paper shall be principally focussed on Abelian Hopfian and co-Hopfian groups. The assumption of commutativity does, of course, make the situation somewhat more tractable—for example, the problem above has a positive answer—but it by no means removes all the difficulties; we still appear to be a long way away from any satisfactory description of these classes of groups. In the case of Abelian p-groups, it is rather easy to bound the size of a group from either class and torsion-free co-Hopfian groups are countable and classifiable by a single finite cardinal invariant. It is easy to see that an Abelian group with endomorphism ring isomorphic to the ring of integers, is necessarily Hopfian and consequently, it follows from modern realization theorems that arbitrarily large torsion-free Hopfian groups exist. But beyond that, little by way of classification is known. It is still not known whether the direct sum of two co-Hopfian groups which are not torsion-free, is co-Hopfian. The talk, which shall be accessible to non-experts, will give an overview of recent developments based on the work of the author and his student, Ketao Gong.

Drumlins

ANDREW FOWLER (UNIVERSITY OF LIMERICK)

Drumlins pervade the Irish landscape, and also many other parts of the world where ice sheets were formerly present. They present an intriguing problem in pattern formation, and although they have been avidly studied for well over one hundred years, it is only recently that anything approaching a mathematical theory for their formation has been put forward. This talk will attempt to describe what drumlins are, what processes are involved in their formation, and how mathematical models are being developed to describe their origin.

The incentives of hedge fund fees and high-water marks

PAOLO GUASONI (DCU)

Hedge fund managers receive as performance fees a large fraction of their funds' profits, in addition to regular fees proportional to funds' assets. Performance fees are paid only when a fund exceeds its previous maximum—the high-water mark. The most common

scheme, dubbed Two and Twenty, entails performance fees of 20 percent of profits plus regular fees of 2 percent of assets.

We study the risk-shifting incentives created by such fees, solving the portfolio choice problem of a manager with constant relative risk aversion, constant investment opportunities, and a long horizon. The portfolio that maximizes expected utility from future fees is constant, and coincides with a Merton portfolio with effective risk aversion equal to the weighted average of the manager's true risk aversion and the myopic value of one, with the performance fee as the myopic weight. Moreover, the optimal portfolio coincides with that of an investor facing the constraint of a maximum drawdown less than one minus the performance fee, as a fraction of the last recorded maximum.

Since performance fees modify a manager's risk aversion, we investigate their potential as agency tools, solving a Stackelberg equilibrium between an investor and a manager. We find that an equilibrium exists only if both the manager and the investor have very low risk aversion. In all other cases, no equilibrium is consistent with positive performance fees. Joint work with Jan Obloj (Oxford).

Thanks from the Organisers

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