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Supersymmetric Quantum Stochastic Analysis

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This is an abstract of the PhD thesis *Supersymmetric Quantum Stochastic Analysis* written by Clodagh Carroll under the supervision of Dr. Stephen Wills at the School of Mathematical Sciences, UCC and submitted in June 2010.

$\mathbb{Z}_2$-graded quantum stochastic calculus is reformulated into a basis independent, infinite-dimensional calculus, expanding on the finite dimensional calculus developed by T.M.W. Eyre and R.L. Hudson. We follow J.M. Lindsay's synthesis, in the sense of Hudson and Parthasarathy, making use of the Hitsuda–Skorohod integral and Operator Space Theory to develop this calculus. $\mathbb{Z}_2$-graded integrals are expressed in terms of Bosonic integrals and the fundamental formulae and estimate of $\mathbb{Z}_2$-graded quantum stochastic calculus are derived, followed by their higher order analogues.

Two types of quantum stochastic differential equation are analysed; the first is the Hudson–Parthasarathy equation

$$dX_t = F_t \widehat{X}_t \, d\Xi(t), \qquad X_0 = \Psi \otimes I_{\mathcal{F}}, \tag{1}$$

$$dY_t = \widehat{Y}_t (L \underline{\otimes} I_{\mathcal{E}(\mathcal{S})}) \, d\Xi(t), \qquad Y_0 = \Phi \otimes I_{\mathcal{F}}, \tag{2}$$

with bounded generators F_t, L and bounded operators Ψ, Φ. Existence and uniqueness results for (1) and (2) are proved, with suitable local uniform boundedness and regularity conditions imposed. Necessary and sufficient conditions for isometric, co-isometric and unitary solutions are derived for each differential equation and the relationship between solutions of (1) and (2) is examined.

Existence and uniqueness of solutions of the quantum stochastic differential equation

$$dk_t = k_t \circ \phi \, d\Xi(t), \qquad k_0 = \kappa \otimes I_{B(\mathcal{F})} \tag{3}$$

is likewise demonstrated, once more, under appropriate regularity conditions. In addition, the question of which generators ϕ yield *-homomorphic solutions is addressed, where ϕ and κ are completely bounded maps. We demonstrate how a solution of (3) can be used to realise solutions of either of the differential equations (1) or (2) and vice versa.

For both types of quantum stochastic differential equation, we determine the necessary and sufficient criteria for homogeneous solutions and finally combine our results.

A Filtering Laplace Transform Integration Scheme For Numerical Weather Prediction

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This is an abstract of the PhD thesis *A Filtering Laplace Transform Integration Scheme For Numerical Weather Prediction* written by Colm Clancy under the supervision of Prof. Peter Lynch at the School of Mathematical Sciences, University College Dublin and submitted in November 2010.

A filtering time integration scheme is developed and tested for use in atmospheric models. The method uses a modified inversion of the Laplace transform (LT) and is designed to eliminate spurious high frequency components while faithfully simulating low frequency modes. The method is examined both analytically and numerically.

For the numerical studies, two atmospheric models are developed, based on the shallow water equations. The first uses an Eulerian form of the governing equations and is based on the reference Spectral Transform Shallow Water Model (STSWM). The second uses a semi-Lagrangian trajectory approach. The LT method is implemented in both models. The models are tested against reference semi-implicit models using standard test cases and perform competitively in terms of accuracy and efficiency. Like semi-implicit

schemes, the LT method has attractive stability properties. In particular, the semi-Lagrangian LT discretisation permits simulations with high timesteps, exceeding the CFL cutoff of Eulerian models.

There are a number of additional benefits. The LT scheme is proven to simulate accurately the phase speed of gravity waves. This is in contrast to semi-implicit methods, which maintain stability by slowing down fast-moving waves. This improved representation is shown both analytically and numerically in the case of dynamically significant Kelvin waves.

In addition, the semi-Lagrangian LT method has advantages for the treatment of orography. Semi-Lagrangian semi-implicit discretisations have been shown to generate a spurious resonance where there is flow over a mountain at high Courant number. It is demonstrated, with both a linear analysis and numerical simulations with the fully nonlinear shallow water equations, that the LT discretisation does not suffer from this problem.

Large Fluctuations of Stochastic Differential Equations with Regime Switching: Applications to Simulation and Finance

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This is an abstract of the PhD thesis *Large Fluctuations of Stochastic Differential Equations with Regime Switching: Applications to Simulation and Finance* written by Terry Lynch under the supervision of John Appleby at the School of Mathematical Sciences in Dublin City University and submitted in September 2010.

This thesis [1] deals with the asymptotic behaviour of various classes of stochastic differential equations (SDEs) and their discretisations. More specifically, it concerns the largest fluctuations of such equations by considering the rate of growth of the almost sure running maxima of the solutions. The first chapter gives a brief overview of the main ideas and motivations for this thesis. Chapter 2 examines a class of nonlinear finite–dimensional SDEs which have mean–reverting drift terms and bounded noise intensity or, by extension, unbounded noise intensity. Equations subject to Markovian switching are also studied, allowing the drift and diffusion coefficients to

switch randomly according to a Markov jump process. The assumptions are motivated by the large fluctuations experienced by financial markets which are subjected to random regime shifts. We determine sharp upper and lower bounds on the rate of growth of the large fluctuations of the process by means of stochastic comparison methods and time change techniques. Chapter 3 applies similar techniques to a variant of the classical Geometric Brownian Motion (GBM) market model which is subject to random regime shifts. We prove that the model exhibits the same longrun growth properties and deviations from the trend rate of growth as conventional GBM. The fourth chapter examines the consistency of the asymptotic behaviour of a discretisation of the model detailed in Chapter 3. More specifically, it is shown that the discrete approximation to the stock price grows exponentially and that the large fluctuations from this exponential growth trend are governed by a Law of the Iterated Logarithm. The results about the asymptotic behaviour of discretised SDEs found in Chapter 4, rely on the use of an exponential martingale inequality (EMI). Chapter 5 considers a discrete version of the EMI driven by independent Gaussian sequences. Some extensions, applications and ramifications of the results are detailed. The final chapter uses the EMI developed in Chapter 5 to analyse the asymptotic behaviour of discretised SDEs. Two different methods of discretisation are considered: a standard Euler–Maruyama method and an implicit split-step variant of Euler–Maruyama.

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p-adic Hypergeometric Series and Supercongruences

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This is an abstract of the PhD thesis *p-adic Hypergeometric Series and Supercongruences* written by Dermot McCarthy under the supervision of Dr. Robert Osburn at the School of Mathematical Sciences, University College Dublin and submitted in June 2010.

In examining the relationship between the number of points over $\mathbb{F}_p$ on certain Calabi-Yau manifolds and hypergeometric series which correspond to a particular period of the manifold, Rodriguez-Villegas identified 22 possible supercongruences. In this thesis, we extend Greene's hypergeometric series over finite fields in the p-adic setting. We prove various congruences between this new p-adic hypergeometric series and truncated generalised hypergeometric series. These congruences provide a framework for proving all 22 supercongruence conjectures of Rodriguez-Villegas. Using this framework we prove one of the outstanding supercongruence conjectures between a special value of a truncated generalised hypergeometric series and the p-th Fourier coefficient of a modular form. In the course of this work we also establish a relationship between this new series and the p-th Fourier coefficient of the modular form in question, and two new binomial coefficient–harmonic sum identities.

K-Theory of Azumaya algebras

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This is an abstract of the PhD thesis *K-Theory of Azumaya algebras* written by Judith Millar under the supervision of Dr. Roozbeh Hazrat at the Department of Pure Mathematics, Queen's University Belfast and submitted in September 2010.

Green et al. [1] proved that for a division algebra finite dimensional over its centre, its K-theory is "essentially the same" as the K-theory of its centre; that is, for a division algebra D over its centre F of index n,

$$K_i(D) \otimes \mathbb{Z}[1/n] \cong K_i(F) \otimes \mathbb{Z}[1/n].$$

By the Artin-Wedderburn Theorem, a central simple algebra over a field is isomorphic to a matrix over a division algebra. Central simple algebras over fields are generalised by Azumaya algebras over commutative rings. An Azumaya algebra A over a commutative ring R can be defined as an R-algebra A such that A is finitely generated as an R-module and $A/\mathrm{m}A$ is a central simple R/m-algebra for all $\mathrm{m} \in \mathrm{Max}(R)$.

For an Azumaya algebra A which is free over its centre R of rank n, we prove that the K-theory of A is isomorphic to the K-theory

of R up to its rank torsion; that is,

$$K_i(A) \otimes \mathbb{Z}[1/n] \cong K_i(R) \otimes \mathbb{Z}[1/n]$$

for any $i \geq 0$. This result appeared in [2, Thm. 6]. We observe that a graded central simple algebra, graded by an abelian group, is a graded Azumaya algebra and it is free over its centre. So the above result, from the non-graded setting, covers graded central simple algebras. For a graded central simple algebra A, we can also consider graded projective modules. Let $\mathcal{P}\mathrm{gr}(R)$ be the category of graded finitely generated projective R-modules and K_i, $i \geq 0$, be the Quillen K-groups. Then $K_i^{\mathrm{gr}}(R)$ is defined to be $K_i(\mathcal{P}\mathrm{gr}(R))$. We give some examples to show that the graded K-theory of A does not necessarily coincide with its usual K-theory. For a graded Azumaya algebra A, free over its centre R and subject to some conditions, we show that $K_i^{\mathrm{gr}}(A)$ is "very close" to $K_i^{\mathrm{gr}}(R)$. These results were published in [3].

We consider additive commutators in the setting of graded division algebras. For a graded division algebra D with a totally ordered abelian grade group, we show how the submodule generated by the additive commutators in QD relates to that of D, where QD is the quotient division ring.

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The Information-carrying Capacity of Certain Quantum Channels

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This is an abstract of the PhD thesis *The information-carrying capacity of certain quantum channels* written by Ciara Morgan under the supervision of Prof. Tony Dorlas at the School of Theoretical Physics, Dublin Institute for Advanced Studies and Prof. Joe Pulé

at the School of Mathematical Sciences, University College Dublin and submitted in January 2010.

In the thesis we consider the classical capacity of certain quantum channels, that is, the maximum rate at which classical information, encoded as quantum states, can be transmitted reliably over a quantum channel.

We first concentrate on the *product-state* capacity of a particular quantum channel, that is, the capacity which is achieved by encoding the output states from a source into codewords comprising of states taken from ensembles of non-entangled (i.e. separable) states and sending them over copies of the quantum channel. Using the "single-letter" formula proved by Holevo [1] and Schumacher and Westmoreland [2] we obtain the product-state capacity of the qubit quantum amplitude-damping channel, which is determined by a transcendental equation in a single real variable and can be solved numerically. We demonstrate that the product-state capacity of this channel can be achieved using a minimal ensemble of *non-orthogonal* pure states. We also consider the *generalised* amplitude-damping channel and show that the technique used to calculate the product-state capacity for the "traditional" amplitude damping channel also holds for this channel.

Next we consider the *classical capacity* of two quantum channels with memory namely, a periodic channel with quantum depolarising channel branches and a convex combination of quantum channels. The classical capacity is defined as the limit of the capacity of a channel, using a block of states which are permitted to be entangled over n channel uses and divided by n, as n tends to infinity.

We prove that the classical capacity for each of the classical memory channels mentioned above is, in fact, equal to the respective product-state capacities. For those channels this means that the classical capacity is achieved without the use of entangled input-states. We also demonstrate that the method used in the proof of the classical capacity of a periodic channel with depolarising channels does not hold for a periodic channel with *amplitude-damping* channel branches. This is due to the fact that, unlike the depolarising channel, the maximising ensemble for a qubit amplitude-damping channel is not the same for all amplitude-damping channels.

We also investigate the product-state capacity of a convex combination of two memoryless channels, which was shown in [3] to be given by the supremum of the minimum of the corresponding Holevo

quantities, and we show in particular that the product-state capacity of a convex combination of a depolarising and an amplitude-damping channel, is not equal to the minimum of their product-state capacities.

Next we introduce the channel coding theorem for memoryless quantum channels, providing a known proof [4] for the strong converse of the theorem. We then consider the strong converse to the channel coding theorem for a periodic quantum channel.

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Yang-Mills Instantons on the Taub-NUT Space and Supersymmetric $N = 2$ Gauge Theories with Impurities

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This is an abstract of the PhD thesis *Yang-Mills Instantons on the Taub-NUT Space and Supersymmetric $N = 2$ Gauge Theories with Impurities* written by Clare O'Hara under the supervision of Dr. Sergey Cherkis at the School of Mathematics, Trinity College Dublin and submitted in August 2010.

We write a formula for arbitrary charge calorons, instantons on $\mathbb{R}^3 \times S^1$, in terms of the Green's function of the Laplacian defined for the Nahm Transform, thus generalising the formula for the charge one caloron derived by Kraan and van Baal in [1]. The Laplacian is constructed from Nahm data. The usual approach to the Nahm Transform involves an integration over the interval on which the Nahm data are defined. By using Green's functions we avoid this integration and our formula is straightforward to use.

Using the same approach, we derive a formula for an $SU(2)$ instanton on the Taub-NUT space. Here, the Laplacian is constructed from Bow data that solve the Nahm Equations in the interior of the interval. The Bow data includes bifundamental data at the end-points of the interval.

We write the Lagrangian for the low-energy effective field theory on the D3-brane in a Chalmers-Hanany-Witten configuration of intersecting D3-, D5- and NS5-branes [2] [3], by adding bifundamental fields to the Lagrangian written in [4]. The low-energy theory on the D3-branes is described by $N = 2$ Super-Yang-Mills gauge theory with codimension one defects. The supersymmetric vacuum conditions for the gauge theory give the Bow data for an instanton on the Taub-NUT space.

We write an explicit expression for a charge one $SU(2)$ instanton on the Taub-NUT space, in terms of the Green's function values at jumping points and end-points of the Nahm interval. For the charge one instanton we find the Green's function explicitly.

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Convolutional Codes from Group Rings

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This is an abstract of the PhD thesis *Convolutional Codes from Group Rings* written by Jessica OShaughnessy under the supervision of Prof. Ted Hurley at the School of Mathematics, Statistics, and Applied Mathematics, National University of Ireland, Galway and submitted in April 2010.

Convolutional codes have been widely researched and have many practical applications. Convolutional codes from group rings have also been considered.

First, this thesis extends known group ring constructions and applies them to an interesting class of convolutional codes known as 'Quick Look In' (QLI) convolutional codes. QLI convolutional codes have previously been extended to turbo codes. QLI convolutional codes were proposed as an alternative to systematic convolutional codes. It is shown that all QLI convolutional codes can be constructed using units in the group ring $\mathbb{Z}_2C_2C_\infty$.

Some new QLI convolutional codes are also given. Simulations are provided comparing the new codes alongside existing QLI convolutional codes and existing optimal (2.1) convolutional codes. Next, group ring constructions are extended to a new group ring construction. This construction uses units in the group ring $\mathbb{Z}_2GC_\infty$ and the group ring matrices corresponding to these units. For this construction, the order of the group G must be even. Properties of the new construction allow for calculations of lower bounds on the free distances of the convolutional codes for some types of generators. The new construction can also be used in the construction of some optimal $(2, 1)$ convolutional codes up to degree 10. It is shown that all $(2, 1)$ systematic convolutional codes can be constructed using this new construction. Finally, LDPC convolutional codes using the proposed construction are considered.

Harmonic Functions on Quadrature Domains and Denjoy-type Domains

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This is an abstract of the PhD thesis *Harmonic functions on quadrature domains and Denjoy-type domains* written by Joanna Pres under the supervision of Prof. Stephen Gardiner at the School of Mathematical Sciences, University College Dublin, and submitted in February 2010.

The main purpose of this thesis is to investigate how the geometry of certain domains affects the boundary behaviour of positive harmonic functions. It also deals with related problems of integrability and approximation of positive harmonic functions.

Chapter 1 concerns problems related to quadrature domains. Let Ω be a bounded domain in $\mathbb{R}^N$ ($N \geq 2$) and let μ be a signed measure with compact support in Ω. We say that Ω is *a quadrature domain for harmonic functions with respect to* μ, if

$$\int_\Omega h(x)dx = \int h d\mu \quad \text{for all integrable harmonic functions } h \text{ on } \Omega.$$

It was shown by Gustafsson, Sakai and Shapiro ($N = 2$), and Gardiner and Sjödin ($N \geq 2$), that all positive harmonic functions are integrable on a quadrature domain. This leads to the significant conclusion that quadrature domains with respect to signed measures are, in fact, quadrature domains with respect to positive measures. In Chapter 1 we establish a corresponding result in the case of quadrature domains with weight. Counterexamples to a certain density question that arose in the above work of Gustafsson et al., are constructed in the second chapter of the thesis, where domains of Denjoy-type are investigated.

A domain Ω in $\mathbb{R}^N$ whose complement $\mathbb{R}^N \setminus \Omega$ is contained in the hyperplane $\mathbb{R}^{N-1} \times \{0\}$ is called *a Denjoy domain*. Benedicks established a harmonic measure condition that describes when Ω inherits the potential theoretic character of the half-space $\mathbb{R}^{N-1} \times (0, +\infty)$, in the sense that there is a minimal harmonic function u on Ω such that $u(x) \geq x_N$. (A positive harmonic function u on Ω is called *minimal* if any harmonic function v on Ω satisfying $0 \leq v \leq u$ is a constant multiple of u.) In Chapter 2 we investigate positive harmonic functions on a domain Ω whose complement is contained in the boundary of the infinite cylinder $U = B' \times \mathbb{R}$, where B' is the unit ball in $\mathbb{R}^{N-1}$. Let α denote the square root of the first eigenvalue of $-\Delta = \sum_{k=1}^{N-1} \partial^2/\partial x_k^2$ on B' and let ϕ be the corresponding eigenfunction normalized by $\phi(0) = 1$. We establish explicit criteria of Benedicks and Wiener-type that describe when Ω inherits the potential theoretic character of the cylinder U, in the sense that there exists a minimal harmonic function u on Ω such that $u(x', x_N) \geq e^{\alpha x_N}\phi(x')$ on U. We also provide illustrative examples and two applications. The first application is a quantitative version of a recent construction of Gardiner and Hansen concerning minimal harmonic functions associated with an irregular boundary point. The second application gives an answer to the approximation question that arose in the study of quadrature domains, by showing that there are bounded domains in $\mathbb{R}^N$ ($N \geq 2$) for which the positive L^p-integrable harmonic functions are not dense

among all positive harmonic functions for any $p \in (0, 1]$. The results of this chapter were published in *Positive harmonic functions that vanish on a subset of a cylindrical surface* [written jointly with Marius Ghergu, Potential Analysis 31, 147–181 (2009)].

In Chapter 3 we further investigate the boundary behaviour of positive harmonic functions in relation to the classical angular derivative problem. Suppose (a_n) is a strictly increasing sequence of non-negative numbers such that $a_n \to +\infty$ and $a_{n+1} - a_n \to 0$ as $n \to \infty$. We consider a domain Ω in $\mathbb{R}^N$ such that

$$\mathbb{R}^N \setminus \Omega = \bigcup_{n \in \mathbb{N}} (\mathbb{R}^{N-1} \setminus B') \times \{a_n\}.$$

In two dimensions domains of such a form are known as *comb domains*. We reformulate the angular derivative problem for comb domains in terms of harmonic functions. Although the concept of an angular derivative cannot be defined in higher dimensions in the classical sense, we show that it is possible to investigate its counterpart for harmonic functions. We thus characterize comb-like domains of the above form that admit a minimal harmonic function u such that $u(x', x_N) \geq e^{\alpha x_N} \phi(x')$ on U, in terms of the -spacing between the hyperplanes $\mathbb{R}^{N-1} \times \{a_n\}$.

High Frequency Elastic Wave Inversion

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This is an abstract of the PhD thesis *High Frequency Elastic Wave Inversion* written by Niall Ryan under the supervision of Dr. Clifford Nolan at the Department of Mathematics and Statistics, University of Limerick and submitted in November 2010.

High frequency elastic wave inversion is the problem of determining sharp, localised changes in the properties of materials beneath the surface of the earth using only measurements of reflected seismic waves taken at or near the surface.

The central objective of this thesis is to construct multiparameter inversion operators which map data from surface wave measurements into accurate estimates for high frequency perturbations in the elastic parameters of underground anisotropic inclusions; namely the density, ρ, and the 21 independent Hooke's tensor components,

c_{ijkl}. Using results from the field of microlocal analysis of Fourier Integral Operators, it is shown that asymptotically valid inversion operators exist which can invert all 22 independent elastic parameter perturbations directly, without relying on statistical estimates.

Building on work by Burridge and others in [1], and by Nolan in [2] and [3], the technique of using ensembles of linked seismic experiments is introduced and analysed in the context of a standard linearised single scattering model for elastic waves based on the Born approximation. This forward model gives an asymptotically valid representation for the data, $\mathbf{d}$, gathered by the ensembles, as a function of the 22 the elastic parameter perturbations, $\mathbf{c}^1$, being sought.

It is shown that the data gathered in ensembles of experiments is given asymptotically by the equations

$$\mathbf{d} = AW^T\mathbf{c}^1, \quad \text{and} \quad \mathbf{d} = \int e^{i\omega\Phi} AW^T\mathbf{c}^1 d\mathbf{x}đ\omega$$

in the cases of point and volume inclusions respectively. To show that the data can be inverted in principle, a theoretical framework is introduced which shows under what circumstances multiparameter inversion can be achieved; specifically showing which types of seismic ensemble setups and elastic wave modes permit non-singular W. These results will also show under which circumstances multiparameter inversion is not possible, in particular for the case of volume inclusions. To complement these theoretical results, an application of evolutionary algorithms is presented which is used to find practical seismic ensembles that allow inversion to be carried out feasibly.

Following this analysis, asymptotic inversion operators are constructed for invertible ensembles, and are shown to take the forms

$$\mathbf{c}^1 \cong B^+\mathbf{d}, \quad \text{and} \quad \mathbf{c}^1 \cong \int B^+ e^{-i\omega\Phi}\mathbf{d}\; d\mathbf{r}_0 đ\omega dt$$

respectively in the cases of point and volume anisotropic inclusions which are embedded in isotropic elastic backgrounds. It is also shown that these results can be extended to inversion in multiple types of elastic background materials.

The thesis also presents an introductory overview of the techniques of Fourier Integral Operators [4] and microlocal analysis, used to construct later inversion operators. Other more elaborate mathematical techniques used in the thesis are also introduced or expanded on in the appendices for the benefit of the general reader.

Finally, supporting lemmas in the appendices introduce a novel method for determining the dependencies of Hooke's tensor components in linear elastic materials with symmetries.

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Numerical Modelling of Industrial Processes Exhibiting Layer Phenomena

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This is an abstract of the PhD thesis *Numerical Modelling of Industrial Processes Exhibiting Layer Phenomena* written by Martin Viscor under the supervision of Prof. Martin Stynes at the School of Mathematical Sciences, University College Cork and submitted in September 2010.

Part I of this thesis examines the singularly perturbed degenerate parabolic problem

$$Lu(x,t) := \varepsilon u_{xx}(x,t) - x^{\alpha} u_t(x,t) = f(x,t,\varepsilon), \tag{1}$$

for $(x,t) \in (0,1) \times (0,T]$, with Dirichlet initial and boundary conditions. Here $\varepsilon \in (0,1]$ is a small parameter and $\alpha > 0$ is a positive constant. The parabolic differential operator L degenerates at the boundary $x = 0$ of the domain and consequently its properties are not encompassed by the standard theory of parabolic partial differential equations.

Existence of the solution u of (1) is established for $0 < \alpha < 4$ and bounds on the solution and its derivatives are derived. These clarify the interaction between the singularly perturbed nature of the problem ($0 < \varepsilon \ll 1$) and its degenerate character (vanishing

of the coefficient x^α along the boundary $x = 0$). Furthermore, a finite difference method to approximate u is developed. Using the bounds on the derivatives of u, convergence results are proved for this numerical method; these depend on the parameter α but not on ε. Finally, several numerical examples are provided to support the theory.

Part II of the thesis is devoted to the spray drying of a single spherical particle. A model is developed for this process, then attention is focussed on tracking the moving interface that separates the wet core of the particle from its dry crust during the drying process. The new aspect of our model, compared with existing models in the research literature, is that both temperature and moisture content are allowed to vary inside the particle. This results in a Stefan-type problem describing the location of the moving interface, but this problem is of a non-standard mathematical type that is difficult to solve numerically. A new numerical method is developed for its solution and numerical examples are provided to illustrate its accuracy.