The IMS September Meeting 2007 at UCD
Abstracts of Invited Lectures

Unfinished business:
Some open questions in classical analysis

DAVID ARMITAGE (QUEEN'S UNIVERSITY BELFAST)

Some easily stated open problems will be discussed.

(i) It is known (Zalcman, 1982) that the Radon transform is not injective: there exist non-trivial continuous functions $f \colon \mathbb{R}^2 \to \mathbb{R}$ with zero (proper) integral on every (doubly infinite, straight) line. All known examples of such functions have extremely rapid overall growth. Can such a function have slow growth, or even be bounded? Is it true that a continuous function on $\mathbb{R}^3$ with zero integral on every line must be identically zero?

(ii) Every polygonal domain D in $\mathbb{R}^2$ has the Pompeiu property (PP): if $f \colon \mathbb{R}^2 \to \mathbb{R}$ is continuous and $\int_{\sigma(D)} f(x)dx = 0$ for every rigid motion σ, then $f \equiv 0$. The corresponding assertion for functions on the sphere S^2 is false: there are infinitely many (non-congruent) regular spherical polygons that lack PP, and they can be characterised. But it still seems unclear whether, for example, all non-trivial regular spherical triangles have PP, and whether (up to congruence) the known example of a spherical square lacking PP is unique.

(iii) One formulation of the maximum principle asserts that if h is a non-constant harmonic function on a ball centred at the origin O in $\mathbb{R}^n$ and $h(O) = 0$, then h takes positive values and negative values on every neighbourhood of O. In the case $n = 2$, this can be quantified: it is easy to show that, with h as above, the subset of $\{x \colon \|x\| < r\}$ and the subset where $h < 0$ have roughly the same area. (The ratio of the areas tends to 1 as $r \to 0+$.) What can be said in the case $n \geq 3$?

Ideals of homogeneous polynomials
from a historical viewpoint

GERALDO BOTELHO (VALENCIA, SPAIN & UBERLÂNDIA, BRAZIL)

The main aim of this talk is to give an overview, from a historical viewpoint, of the study of ideals of homogeneous polynomials between Banach spaces. We start by describing, on the one hand, the beginnings of infinite-dimensional analytic function theory at the turn of the 20th century; and, on the other hand, the emergence of functional analysis in the 1920s and 30s. Our idea is to present the theory of polynomial ideals as a natural and unifying evolution, initiated in the 1980s, of developments which these two theories had undergone over the period 1950–70.

Nonpositive curvature on metric spaces

STEPHEN BUCKLEY (NUI MAYNOOTH)

We define various concepts related to negative or nonpositive curvature on metric spaces. We discuss how these concepts are related to each other, and how they lead to boundaries at infinity.

Exploring voting blocs in the Irish electorate:
a statistical modeling approach.

CLAIRE GORMLEY (UCD)

The electorate in any election is a heterogeneous population in that voters have different political and idealistic persuasions. A voting bloc is defined to be a group of voters who have similar voting tendencies. It can be assumed that the electorate consists of a finite number of voting blocs or 'expert networks'. Such a framework is known as a mixtures of experts model. Each voter has a probability of belonging to each of the voting blocs—the covariates of a voter determine these voting bloc membership probabilities through a multinomial logistic regression model. Interest lies in examining the Irish electorate as in Irish elections voters rank some or all of the candidates in order of preference. Rank data models are employed to model the votes cast by members of the electorate. This allows inferences to be drawn on the number of voting blocs present and on their characterising voting patterns. Thus rank data models are incorporated with a mixtures of experts model to provide a unique

exploratory tool for rank data. Model fitting involves the exploitation of properties of convex functions. The application of the model to the 1997 presidential electorate reveals that age and current government opinion were influential factors on voter preferences.

Calculating the weather:
The mathematics of atmospheric modelling
PETER LYNCH (UCD)

Over recent decades, weather forecasting has evolved from a mainly qualitative activity to a rigorous quantitative science. The accuracy of weather predictions has increased steadily, and continues to improve. The changing climate will have major implications for humanity. It is essential that we determine probable future changes with as much precision as possible. The computer models for modelling climate change rest on the same mathematical and physical foundation as the models used for weather prediction. As the range of prediction increases, the errors grow larger. For predictions beyond a few days, probabilistic prediction is more apposite than deterministic forecasting. This is achieved through the ensemble approach. In this presentation we will review the mathematical foundations of modern numerical weather prediction and climate modelling. We will describe the techniques used for assimilation of observational data and for integration of the partial differential equations governing the evolution of the atmospheric flow.

'Gauss' method for the determination of the
minimal polynomial of the Gaussian period
CIARAN MAC AN BHAIRD (NUI MAYNOOTH)

It is commonly believed that Gauss' method for the determination of Cyclotomic Numbers, and thus the determination of the minimal polynomial of Gaussian Periods, is unwieldy for the general case. See for example, the remarks of André Weil in *Numbers of Solutions of Equations in Finite Fields*, Bull. A.M.S., v. 55, 1949, pp. 497–508. The prevailing wisdom now seems to be that the determination of the minimal polynomial of the Gaussian Periods, using Jacobi Sums etc., should be done first and then the Cyclotomic numbers be determined as a consequence. In fact, this was suggested by Weil in the paper above. It appears that Gauss' original method has now

been abandoned. We have shown that Gauss' method leads to a series of functional equations. We then obtain necessary and sufficient conditions for these functional equations to have integer solutions. This leads to a finite Diophantine system—the number of equations is independent of the prime. We have shown that this purely Diophantine system has precisely $\phi(l)$ solutions which correspond to the Cyclotomic numbers of order l. This is, in fact, the first purely Diophantine characterisation of the cyclotomic numbers and the coefficients of the minimal polynomial of the Gaussian periods and the problem is solved for all orders. The bulk of the work involves proving that the Galois group of a related polynomial acts cyclically on its roots and therefore the polynomial is irreducible. It is then not too difficult to show that the polynomial is in fact the minimal polynomial of the Gaussian Periods. In view of Weil's belief, as alluded to above, it is of interest that Jacobi sums appear nowhere in our argument and furthermore, that the determination of the Cyclotomic numbers can be taken as a starting point for the determination of the Gaussian periods. This of course was Gauss' motivation for his work on Cyclotomic numbers of orders 3 and 4. There are other descriptions of the general Cyclotomic numbers which involved Diophantine systems, but these descriptions all employ a rejection criterion and so cannot be considered to be purely Diophantine. A purely Diophantine description has the advantage that if one obtains alternative formulae for the cyclotomic numbers, the result can be proved by simply verifying that these formulae satisfy the system of equations.

A multivariable Cayley–Hamilton theorem

MARTIN MATHIEU (QUEEN'S UNIVERSITY BELFAST)

The Weyl calculus for a pair $A = (A_1, A_2)$ of selfadjoint $n \times n$ matrices, due to H. Weyl, associates a matrix $W_A(f)$ to each smooth function f defined on R^2 in a linear but typically not multiplicative way. Letting $c_A(\lambda) = \det((A_1 - \lambda_1)^2 + (A_2 - \lambda_2)^2)$ for $\lambda = (\lambda_1, \lambda_2) \in R^2$ denote the joint characteristic polynomial of the pair A it is known, for $n \leq 3$, that $A_1 A_2 = A_2 A_1$ if and only if $W_A(c_A) = 0$. It is an open problem whether this is still true for $n > 3$. We shall discuss two new approaches to this problem: the role of the canonical order structure for selfadjoint matrices; and topological invariants arising from continuity properties of the non-linear map $(f, A) \mapsto W_A(f)$.

This is joint work with W. Ricker, Eichstätt, Germany to be published in Math. Proc. Royal Ir. Acad.

On spaces of rational curves and their cohomology

Anca Mustaţă (UCC)

Kontsevich and Manin have introduced the moduli spaces of stable maps to parametrize curves in complex projective manifolds. Cohomological computations on these moduli spaces have provided solutions to important enumerative problems regarding curves. In this talk I will describe a new construction of the Kontsevich–Manin spaces for rational curves, starting from the simpler example of the Grassmannian of lines in the projective space. This new construction allows us to understand the structure of their cohomology rings in a natural, geometric way.

This talk is based on joint work with Andrei Mustaţă.

Ramanujan, partitions, and overpartitions

Robert Osburn (UCD)

During his lifetime, Ramanujan made many beautiful discoveries ranging from elegant identities and formulas to work which influences modern number theory. In this talk, we discuss his work on the arithmetic of the ordinary partition function, Dyson's rank, and a generalization of partitions called overpartitions.

This is joint work with Jeremy Lovejoy (Paris).

The equation $x^p y^q = z^r$ in tree-free groups

Shane O Rourke

It is a classical result due to Lyndon and Schützenberger that in a free group , solutions of the equation $x^p y^q = z^r$ commute for integers $p, q, r \geq 2$. Groups that admit a free action (without inversions) on a Λ-tree for some ordered abelian group Λ so-called *tree-free* groups are a natural generalisation of free groups, and they satisfy many of the same properties as free groups. On the other hand this class properly contains fully residually free groups (called limit groups by Sela). In this talk we will discuss the extent to which the result of Lyndon and Schützenberger extends to tree-free groups.

This is joint work with N. Brady, L. Ciobanu and A. Martino.

How do students learn to understand formal mathematical concepts?

Keith Weber

(no abstract provided)

Large fluctuations in stochastic dynamical systems

Huizhong Wu (DCU)

Stochastic differential equations (SDEs) are commonly used in modeling the trajectories of processes whose motion is determined by random movements. In particular, we are interested in the application of SDEs to inefficient financial markets (IFM) in which investors take historical information into account when making their investment decisions. We study the almost sure asymptotic rate of growth of the partial maxima and minima of solutions of both linear and non-linear SDEs, as these represent the largest possible fluctuations in the price or returns of the asset.

Since in IFM, prices or returns exhibit short run positive autocorrelations (which mimic market bubbles), SDEs involving delay factors are investigated and compared with SDEs under the classical Efficient Market Hypothesis. We also develop Markov models driven by semi-martingales other than standard Brownian motion. While these semi-martingales preserve the size of large fluctuations of Brownian markets, they differ from Brownian motion by possessing dependent increments, and so can create the presence of non-trivial autocorrelations in security returns.

The work is joint with my supervisor Dr. John Appleby.