The Interface between Mathematics and Physics: A Panel Discussion sponsored by the DIT & the RIA. Academy House, 6th September 2005.

Panellists: Prof Sir Michael Atiyah; Prof Sir Michael Berry; Prof Luke Drury; Prof Arthur Jaffe; Prof Brendan Goldsmith (Chair).

Brendan Goldsmith: The interface between mathematics and physics predates the emergence of the separate disciplines of mathematics and physics, but for a long time the relationship was perceived to be a somewhat one sided relationship with mathematics providing techniques and justifications which enabled physicists to develop further their justifications and insights into our understanding of nature suggesting interesting areas in which to find mathematical problems. The most quoted examples of this are the interplay between the differential calculus and Newton's laws of motion, or Einstein's use of abstract concepts of geometry in his exposition of general relativity. There are of course many, many others. In more recent times, some would even say that situation has been dramatically reversed. For example, quantum field theory has had a significant influence in many areas of geometry from elliptic genera to knot theory and indeed Witten's work has provided direct connections between certain quantum field theories and topological theories in mathematics. And these developments continue apace. In some senses we are experiencing, really and truthfully the unreasonable effectiveness of mathematics in physics and equally the unreasonable effectiveness of physics in mathematics. Despite all this interesting and important collaboration, there are undoubtedly tensions that have surfaced. These are largely centred on questions of rigour, the nature of proof, philosophical questions concerning the very nature of mathematics, the social dimension of mathematics, the role of speculation etc, but these tensions of course are not new. One can think back for example to the early nineties when Paul Halmos wrote his very provocative article titled 'Applied Mathematics is Bad Mathematics' or indeed the much earlier article by Jack Schwartz at the beginning of the sixties; he wrote an article titled 'The Pernicious Influence of Mathematics on Science'. Add to this the growing influence of computing and the fundamental issues arising from, for example, proving correctness of programs and software etc. and it would appear that there are a great number of issues to be discussed.

Question: What do the panellists understand when they hear the words "Mathematician" and "Physicist"?

Arthur Jaffe: I'm often asked if I'm a mathematician or a physicist. I like to think of myself as a mathematician when I work with mathematicians and as a physicist when I'm with physicists. I'm not really sure what the difference is except that some years ago there wasn't such a distinction between the two. A set of cultures has grown up though where you get a degree in one subject or the other and yet the ideas as Brendan outlined cross the boundaries in absolutely wonderful ways so that there has been this revolution bringing the two subjects together, which I think is not only historical but will last for many more years into the future. So I would like to think of myself, in answering your question, as both.

Michael Berry: People occasionally ask me am I a mathematician or a physicist, I say yes. I've just learned a very nice expression this afternoon: I was reading the beginning of this nice book by David Wilkins on the correspondence between Tate and Hamilton, and in the very first letter from Tate, he states "I prefer to consider myself a mixed rather than a pure mathematician", and I think thats quite a nice expression. I'm paid as physicist, I don't prove theorems and I rather tend to define a mathematician as someone who proves theorems (maybe that's too old fashioned!) but if so, I'm not one. I was very generously described by Brendan as a mathematical physicist and I think that side of application is of people who prove theorems whereas theoretical physicists—I suppose this is what I would call myself—are people who use mathematical concepts and think about the world in mathematical terms but don't prove theorems. But it doesn't really matter actually.

Michael Atiyah: There is this question about defining the difference between a mathematician and a physicist which Michael referred to, and one definition is that if you look at the papers and the word theorem appears you're a mathematician and otherwise you're a physicist. Thats partially true, but I've recently found a better distinction. To a mathematician all constants are equally big or small, but to physicists—size matters. And that actually is quite significant. First of all, mathematics encompasses many things, some of which has nothing to do with physics and similarly physics has parts that are really only tenuously connected to mathematics. But there is a main part of physics and a main part of mathematics, which are very closely linked, and for anyone who works in this field, they really form a spectrum. There is no clear divide and you can choose your own definition and where to cut the cake. I recently tried to produce a spectrum illustrating this with peoples names and I normalised by putting Newton's name in the middle, saying that he is equally mathematician and physicist. Mathematicians like to call him a mathematician one of the inventors of calculus, and physicists undoubtedly think that he is one of the greatest physicists of all time. And then I had a scale; Hamilton I put as distinctly more mathematical than Newton and then below Hamilton and Newton, I thought that Dirac and Schrödinger would have been more physicist. Einstein was much more physicist than any of them, he wasn't much of a mathematician at all and at the top end of the scale, I put Ramanujan, who was a brilliant mathematician but no physicist. All of these were great men and you could divide it differently. You could say well Newton was more really this side or that side; I think the fact that it is a spectrum and continuous is the important factor, that there is no natural division and historically if you go back in the past, the people would have regarded themselves as indistinguishable—if you asked Newton if he was a mathematician or a physicist, he wouldn't have known what you meant, and I think Hamilton would have taken the same view. I think we are all one happy family.

Luke Drury: Well, I think I must be at the physical end of the spectrum. I did my undergraduate work in both pure mathematics and experimental physics but I have drifted more and more into the physical regime. I think the key distinguishing factor is really the nature of what you regard as evidence. In the physical sciences experimental evidence, testing by experiment is what determines what we regard as truth; in mathematics—it's logical proof. Well, it's not quite as clear cut as that but essentially that's to my mind, the key difference. It's essentially an epistemological one of what you regard as valid knowledge.

Question: What do you think of the American use of the term 'Applied Mathematician' particularly with regard to someone like the mathematician Gauss?

Luke Drury: Actually Gauss is interesting because he was both a very brilliant mathematician and an extremely able experimental physicist. His magnetic observations showed a deep appreciation of instrumental error, the need for proper analysis of observations, the need for rigorous observational procedures.

Michael Atiyah: I think I would put Gauss a little bit more on the mathematical side of the spectrum than Newton.

Luke Drury: In terms of his mathematical contributions yes, but he did have deep physical insight as well.

Michael Berry: It's good that you draw attention to these curious cultural differences between east and west. Some of my work was using singularity theory to understand aspects of optics, and to my physicist colleagues in Bristol where I worked, this was the farthest extreme of pure mathematics. I once read a Russian review article which was kind enough to mention my work and which spoke of me as the 'Experimentalist' Berry. So it really depends from where you're sitting.

Question: Are the concepts behind mathematics and physics actually the same and is it that you look at them in different ways, or is there an essential difference between what is a physics concept and a mathematical concept.

Michael Atiyah: That's a very important question. I think fundamentally on the conceptual level, there's a great deal of common ground, but when you spell these things out in detail: of course the mathematicians will write down definitions and formulas and the physicists will take measurements and do experiments; but on the level of ideas and concepts—if you think about things like space and time—the concepts are common to both physicists and mathematicians, even when you start thinking about particles moving around. I think the concepts are common but the mathematicians will apply to these techniques of mathematics and formulas, and the physicists on the whole will tend to do the experiments, but, of course, they will need the mathematical connection as well. But I think on the level of concepts, that it's possible to make a bridge between the two, because you can talk about ideas that transcend the technical details.

Arthur Jaffe: What makes it difficult is that the concepts are often the same but the language is different. The same concept can be referred to in words that may be opposite in one subject or the other. So in communication there can often be a difficulty.

Michael Atiyah: It does seem to me that there is a deep issue here though, which is the nature of what are mathematical objects and in what sense do they exist. It has always struck me that in many ways, although they may deny it, pure mathematicians are actually Platonists. Even the most formal of formalist mathematicians will always say that they discover a theorem, never that they invent a theorem. As if in some sense the mathematical objects have some objective external existence. Any of this is philosophically very naïve, but intuitively that is the way I believe that mathematicians think about mathematical objects as somehow existing in an ideal world.

Brendan Goldsmith: Certainly I recall Alain Connes saying exactly that; that mathematicians are in reality Platonists and that it's only when you push them to defend that position that they revert to being formalists, because they are not really able to defend it.

Michael Berry: I think this question is actually a very deep one and it goes to the heart of what we're discussing; it's to do with the nature of abstraction and how we do it; we abstract aspects of the world to make sense of the world and the purpose of abstraction is to connect things that superficially seem different and therefore make sense of them. Now, when I see a rainbow for example, a rainbow is a phenomenon to do with focused light that is a member of a general class of phenomena called caustics which includes tsunamis and the V-shaped waves found behind moving ships. Now if you look at the fine detail there's something called an Airy function, it's a solution of a certain differential equation. When I see a rainbow, I see an Airy function. OK, but of course it connects all these different things together conceptually and that's a good thing because it makes connections. However, as I have said before, I am not a pure mathematician and I don't think of these mathematical objects in terms of theorems, I think of them in a rather physical way but at second order, so when I think of these mathematical structures, these wave patterns, I think of them in a physicist's way but disembodied from their individual instantiations, whether they are rainbows or tsunamis or whatever. So one way I often put it, which usually just mystifies people, is to say that I study the physics of the mathematics of the physics, and that's very precisely the level of abstraction that I work in, but different people think differently and it's to do with abstraction and how one makes sense of the world.

Michael Atiyah: I don't think all mathematicians are as platonic as Alain Connes. He has taken an extreme point of view and so have some others, and I don't think there's a distinction between Platonists and formalists. I consider myself as a realist. I think the mathematics we use is derived from the outside world by observation and abstraction. If we didn't live in the outside world and see things, we wouldn't have invented things and thought of things as we do. I think much of what we do is based on what we see, but then abstracted and simplified, and in that sense they become the ideal things of Plato, but they have an origin in the outside world and that's what brings them close to physics. The idea that there is a pure world totally divorced from our experience, which somehow exists by itself, is obviously inherent nonsense; we are ourselves a product of evolution, the long development of the earth, we are part of nature, and our minds function according to laws of physics and biology. You can't separate the human mind from the physical world. And therefore everything we think of, in some sense or other, derives from the physical world. The extreme points of view of the formalists are really not totally coherent and some middle ground, which much more connects with observation, is really more to the point.

Luke Drury: Well I would agree with that, because I have always felt that the unreasonable effectiveness of mathematics derives precisely from the fact that it is abstracted physics.

Michael Atiyah: Of course the converse part in terms of the unreasonable effectiveness of physics is much harder to understand. That remains a bit of a mystery at the moment.

Michael Berry: Is it really? Isn't it that by abandoning rigour or not being sensitive to it, you can sometimes be a little bit bolder; you lose something of course, because you don't know precisely what it is you're talking about; it's a criticism mathematicians often make of us, but on the other hand you can go further.

Michael Atiyah: Well, without going into detail, recent applications of very abstruse areas of quantum field theory to parts of algebraic geometry is much more than just a question of using conceptually imaginative thinking to get round the barriers of rigour.

It's actually an enormous jump from totally different areas and an enormous surprise, because the kind of mathematics that have been used in physics are well understood and linked closely, but some parts seem so far away that when they were being developed, if it had been suggested by anyone that they had anything to do with physics, they would have been laughed out of court. The more you find out about it technically, the more it stands out as exceptionally striking. Of course, in a way you eventually understand things better, so we gain that perspective but at the moment it is a bigger mystery than the one that was referred to before, the effectiveness of mathematics in physics which is much older, better understood and has a long history.

Brendan Goldsmith: Can I just widen that and ask our panel, in some senses then, is it fair to say that mathematicians, and in particular pure mathematicians, are living in a sort of a dream world of their own, where they have an adherence to notions of proof that are really no longer viable, as shown for example by the complexity of proving even the simplest piece of software or the consequences of Gödel's theorem in logic. Is it time perhaps for the pure mathematicians to re-evaluate?

Michael Atiyah: We recently had a discussion at the Royal Society on just this issue and I think the situation really is that there is a spectrum involving proof. At one end you have the physicists who are happy with rather loose notions of proof and then you have more rigorous physicists who use mathematics more precisely, and then you have the pure mathematicians who try to prove things completely and then you have the logicians who go right off the far end of the spectrum and finally you have the computer scientists who try to put everything on a machine, but everybody recognises that even a mathematical proof that seems to be correct, and has been checked by everybody and is then published, can be wrong—mistakes can be made, particularly for very long proofs. Consider, for example, the proof for the classification of finite simple groups. I think there are 15,000 pages in that collected proof, and actually afterwards, when it was realised to be a marvellous achievement, a small mistake was discovered and rectifying that mistake took somebody a further ten years and another 1,500 pages. At this stage you begin to not have total confidence in the process. And so I think there is no such thing as absolute certainty and Gödel in some ways formalised that. We recognise that there are various levels of proof and should be happy, pragmatically, with the kind that suits our own work. If you're an applied mathematician you don't have to prove something; you do a calculation sufficiently proximate, it will work and you can check it out with experiment. Pure mathematicians don't have experiments to check it out and so they have to test it more carefully, but they can never be totally certain. They check it against the other mathematics that other people do—that they regard as consistency—but that isn't total proof. So I think you have to recognise that pure mathematicians haven't really—you're right they thought they were God, that they were above this stuff and what they did was totally, totally correct. Well, I think that they recognise now the problem with proofs like that (and also those proofs used in the Four Colour Problem); who can check all that computer software? More of that may come around; this is not a failure of mathematics, it's just a recognition of reality: mathematics of different kinds require different levels of proof. We do the best we can, and, you know, perfection is not on this earth.

Brendan Goldsmith: Perhaps Arthur would like to say something about this; he's been involved in this controversy I know.

Arthur Jaffe: Well, I would just like to comment on one thing that's become very popular, which is to prove things to a certain degree, to a certain probability. So if you can prove that if a number is prime to 99.9999% correctness, have you really understood things? With the Riemann hypothesis, we've computed on a computer 15 billion zeros of the zeta function and they all lie on the critical line, but is this enough to make it really true? I think that there is reason to search for mathematical proof in the classical sense because there are consequences of the Riemann Hypothesis for other things in mathematics while if you were trying to break a code, it might be sufficient to know that things are true up to a certain accuracy and therefore there is this spectrum of ideas of when proof is a valid concept to use. Mathematicians are thinking and talking about this a great deal, but classical proof will be with us for a long period of time.

Michael Berry: I agree with that, and on this particular question of the Riemann Hypothesis it's especially important there to have a proof. For people who don't know, this is a very important conjecture in mathematics related to prime numbers, which are themselves the atoms of arithmetic, so it's one of the central problems of mathematics. The question is whether certain mathematical objects will lie on a line. Now there are infinitely many of these objects—that's been proved—and I think some 50–80 billion of them have been numerically shown, not approximately but with the kind of numerics that lead to exact results, to lie exactly on the line. Then some people say "Why are you physicists convinced by this, after all it's only numerics?" Now my response is that I am a physicist and I am not convinced by the numerics. It's certainly interesting and reassuring for those who believe this Hypothesis is true, to find that 80 billion lie on the line, that infinitely many lie on the line and indeed that 30 or 40%, in some average sense, also lie on the line, but on the other hand why is it so hard to prove? One reason is that it might not be true! There are, in number theory, things that go wrong at extremely high values well beyond anything that we can compute, so I think I agree with Arthur that there are circumstances when proofs are important. On the other hand when I'm using Mathematica and I want to know if some large number is prime for a particular purpose where I can be satisfied with 99% probability, the fact its algorithms are probabilistic ones and not deterministic ones, gives me the freedom to go much higher than I otherwise would and that's useful for certain other purposes. So as Michael said, one needs to be sensitive to the context in which one asks these questions. So I think proof will certainly be with us for a long time.

Michael Atiyah: Can I just follow up by saying that proof is by no means the most important aspect in mathematics. I think the most important aspect is understanding, trying to understand things, why things are true, how they hang together. Insofar as you get a proof which contains within it an explanation that is coherent, then you've gained something. A proof that is seen to be rigorous, but involves vast amounts of checking things by hand or computer calculations may be satisfactory as a proof, but is not satisfactory to me if it does not explain in some sense why the result is true. So searching for proof is one thing, but searching for understanding is much more important and they are not quite the same.

Arthur Jaffe: I think that's why these connections between mathematics and physics are so amazing in recent years, because the way physicists perceive things and the way mathematicians proceed in the early stages, is often the same but bringing together these ideas and understanding could lead sometimes to proof; sometimes it does, sometimes it doesn't, but it gives tremendous insight.

Question: Would you encourage Math and Physics Students to talk to each other or would you be careful about it?

Michael Atiyah: Talking to anybody is a good thing. The exchange of ideas between mathematics and physics students is an excellent thing. You learn a bit more about the philosophy, the point of view that goes with the other side; that must be helpful. Of course if you're a student at the beginning learning mathematics, you really need to concentrate on mathematics and not diversify too far. If you start talking to everyone and don't write your thesis down, then you're not going to make progress, so on a purely practical level a supervisor may give some advice about being careful. However it's good to be exposed and the earlier you start the exposure the more likely it is that you will absorb it. It's not just a case of going in for one coffee morning and then coming out and saying "I've mastered physics"; it's a slow process and it's better to start young rather than starting when you're middle aged.

Michael Berry: I agree with Michael and as with all forms of fundamentalism this concern with purity and the avoidance of corrupting influences from other cultures is a minor psychological disorder. The more impurity the better, as I said earlier 'mixed mathematics'!

Question: It's very encouraging to hear so much communication between mathematicians and physicists and this seems to have taken place quite vigorously in recent years. I was wondering whether to any extent people, perhaps in the philosophy or the history of science or working on scientific method, were also engaged in this discourse and if so, do you consider that it would be helpful in promoting the public understanding of science as well as perhaps communication between mathematicians and physicists, where in some places they're not communicating too well already.

Arthur Jaffe: That's a very good point, to my knowledge there hasn't been such a great interaction with people in the history and philosophy of science. I think it would be a very good thing to have that. Some of the concepts they need to understand cover the frontiers of both subjects and therefore it's very difficult, but having people bring this to the public would be extremely constructive. I think that it also shows that it's very hard to predict what the best direction in research is, because if you asked twenty years ago if this tremendous coming together of these two subjects, which traditionally had been one, would happen, most people would have said 'of course not'.

Michael Berry: There begins to be a culture of historians and philosophers of science who know a great deal more science than their predecessors did. It's easy to disparage other cultures—I don't want to do it—but there were people who spoke about the philosophy of physics, but weren't very successful at actually doing it. That's changing, it's gradually changing. I have a little anecdote to report: some of the ideas which I have developed in asymptotics to do with divergent series which come out of physics, are very strongly related to the question of how one theory of physics reduces to another one at some limit; how quantum mechanics reduces to classical, or geometrical optics is a special case of wave optics and so on. It's a very difficult problem of asymptotics: wavelength is small, Planck's constant is negligible and so on and this bears on the philosophical question of reduction. Philosophers talk about this a great deal in words without realising there's a lot of mathematics behind it. Now I tried to put this view to a conference on the philosophy of science and it went down like a lead balloon. However, now there's one guy, Bob Batterman, who has written a book and takes this idea very seriously and understands the technicalities and so on. He's also a very good historian and philosopher of science and he has traced the idea back to where I certainly couldn't, to its historical roots and so on. I think this is happening more and more now. I have already mentioned David Wilkins's fine editing of the correspondence between Tate and Hamilton, so it's a golden age of communication between historians and philosophers and physicists. Historians have their own standards of rigour and we're terrible, we physicists, and I suspect mathematicians too, we have a kind of folk approach towards history: in a sense we treat anecdotes as factoids and then we don't really care if they're true, they ought to be true. For example Kac reportedly said to Feynman—surely the other end of the spectrum we discussed earlier!—"Surely mathematicians must have some value and you must agree that without mathematics the progress of physics would have been delayed", which drew the response "Well by a week or so". Now this is probably not a true story and you need the rigour of historians to distinguish between the factoids and the facts and so I think this is a good question. I'm in favour of these rapprochements, of these new standards that have come in.

Michael Atiyah: Could I just say that on the whole, and understandably, people in the history and philosophy of science are looking at the science of past eras, maybe people still study Newton and all that—they come to mind more easily—but now they study quantum mechanics which is nearly a century old, but they are all the time fifty years behind the front line because they're studying history. Now all of the exciting developments are much more recent—in the last 25 years. It's unreasonable to expect historians to have already focused in on that. Hopefully they will, but it takes time, partly because they're behind and working from a different timescale and partly because more new complicated technical ideas have arisen, which are not that easy to understand if you're not a technically trained mathematician or physicist. For a combination of reasons it's not happening, but hopefully it will and if this rapprochement goes on, it will have an impact and if we have meetings like this, and maybe there are in the audience people who are interested in the history and philosophy of science, they will be taken up, because they do raise fundamental questions about what is the nature of reality, what is the nature of mathematics and how is it related to experiment: these are difficult questions and new developments do shed light on that. So I think there are interesting new questions which do arise and should be studied.

Question: I'm interested that you both refer to rigour. Now rigour is the bugbear of many students. It can be quite a useful ladder to probe into the past. How much do you, as mathematicians and physicists, use rigour to get to your destination? How much do you just do a little leap or a big leap forward and subsequently try to make the little step ways of rigour to make other people understand it?

Michael Atiyah: Rigour is important in mathematics because it constitutes proof, the aim of mathematics finally. But I regard it as the last step of the process. The first step of the process, very early on, is the creative imagination when you try to search for ideas and think about things in some very vague way and then, well on in the process you begin to start focusing and defining the question and then you go about solving it; that's really when you start to speculate and try various ideas; the proof comes very far on, just the last bit of crossing the I's, dotting the T's. You don't start off saying "blank page, I'm going to prove some brilliant theorem", that's not the way anyone works and it's a mistake if students are taught to think that way. Mathematics is about proof, but proof is the end of the process, not the beginning!

Question: How do you train people for the beginning bit?

Arthur Jaffe: I would say that the physicist and the mathematician work exactly the same way in the early stages; the beginning part perhaps is the physics and the end result, the final proof, is the mathematician's part, the add-on at the end. The concepts of thinking of the people going along the same way are very similar.

Luke Drury: It's very hard to put your finger on it, but there is definitely such a thing as intuition, both mathematical intuition and physical intuition. All the great advances have been in some sense quite intuitive leaps into unknown territory; if you take for example the development of the calculus, Leibniz and Newton both instinctively saw how you had to handle varying quantities but to put that on a rigorous foundation took almost two hundred years. There is such a thing as physical intuition and it's not something that you can easily teach. It's something that you learn from being around people who have it.

Michael Atiyah: If you want to teach students how to do that, you do it on a small scale. You don't have an enormous ambition, you are trying to get them to a goal, you try to encourage them to think about it in a creative way and get going in a small scale. They should do miniature research at that level on a micro level and then they will get the ideas. That's the only way. If you are like a painter in the old days, you worked in Michelangelo's Studio and you studied the great master at work but he will have give you an exercise and said, "paint this little corner over here" and then you get to work on the detail. So you have to do a combination of copying everything of the great man, who is your mentor and trying your hand at a little bit of minor experiment/research on your own.

Question: I would like to ask maybe an unfair question about predicting the future. Brendan started out with some remarks about mathematics and physics, quantum field theory etc. Would the panel care to speculate on where the important breakthroughs would come in the future and maybe to be more specific whether they would fall on the mathematical or physical side?

Michael Atiyah: Can I answer that very quickly; it's important that just things are unpredictable and therefore you cannot predict them, end of question. If you know where things are going to go, it's interesting to carry out and do it but it's not so exciting. The really exciting things are the breakthroughs; the things which you can't predict, which no-one has thought about. Suddenly some inspiration comes and those are by nature unpredictable and hopefully we will have more unpredictable things happening in the future, but we can't predict them!

Arthur Jaffe: Maybe if you have bright people work on mathematical or physical problems then we can hope that by identifying the most talented people, they will produce something good in the future

Michael Berry: I agree exactly with Michael, you can't predict, end of answer.

Michael Atiyah: Let me sort of modify that extreme view. If you look at what's happening at any given moment, you can see the trends of where things are going, and you can try to extrapolate a little bit into the future. So halfway between predicting the obvious and speculating entirely on the unknown, there's a middle ground and you can sketch out some vague possibility and then you can make something of a guide which will steer you in a direction that you think might be productive. So maybe my view was a little extreme; I wanted to correct it a little bit.

Michael Berry: The question was about the major advances.

Michael Atiyah: Well, you can look at what's happening in some of the major advances, ask questions and pose problems, but it could just end up being idle speculation and anything you say could be totally worthless.

Question: Can I go on in a contradictory manner to wonder whether biology will be pulled into this; understanding how life might have evolved in the universe, is that not the next great undertaking?

Michael Atiyah: Yes of course, we understand life in some senses, but understanding, say, how the human brain works, these are enormously important problems. I should say, in general, that the role of mathematics in biology is still open; the role of mathematics in physics is quite clear, but whether mathematics has any fundamental role in biology is entirely an open question. Many biologists will say that biology, or rather, evolution, was a series of accidents that didn't follow any predetermined pattern laid down by God—there are no fundamental laws—you mathematicians are wasting your time. That's a point of view that you can't ignore. Within biology there are lots of sub-questions which are obviously very close to physics and mathematics, where mathematics can be very useful. And there is no question but that there are lots of areas which are currently using mathematics in biology; DNA analysis, the human genome and lots of other smaller things. Whether mathematics has a bigger role for example understanding how the human brain works, which is really the big question, whether the kind of models mathematicians might construct in the future and not necessarily now, might provide the kind of logical framework in which biologists could think and tie their experiments to, that could be a big question. If I had to speculate, I would say that mathematicians should at least try to get themselves involved in this with biologists, to see whether they can contribute.

Question: The really big question is surely whether one could see fundamental physics developing to the point where life is inevitable from the physical laws.

Luke Drury: In some sense it's already answered. In fact, that in principle self reproducing systems are possible was proven a long time ago, but how exactly is still open. To actually produce an example and show how it works in detail, and in practice, is another issue. A slightly related question is to what extent physics should be seen as a purely reductionist science and to what extent you regard complex phenomena as a valid discipline for physics. This is an interesting development. Traditionally physics saw its goal as being the reduction of all phenomena to a few very basic principles and that is a very powerful model which underlies a lot of theoretical physics. But there is also a school of thought which holds that there are valid areas of physics, which arise from inherent complexity of systems, turbulence for example, and that there are emergent phenomena that you can study in terms of physics, but are not simply reducible to a reductionist paradigm. Maybe I haven't explained that very well, but it should go some way to answering your question: could life be seen in that sense as a necessary consequence of a sufficiently complex system?

Question: I notice there was mention of one important obstruction between mathematicians and physicists, namely the difference of languages. I have always felt that whenever I talk to physicists, they often discuss exactly the same thing but use completely different words. As soon as I understood the translation it is then much easier for me to understand, so I still feel that is quite a severe obstruction and I was wondering what advice you would give to overcome this?

Michael Atiyah: I think it depends on the younger generation basically, with the older generation it's harder for them to understand the new language being used. With the younger generation they learn both old and new languages and they put them together and then a new fused language emerges and this is happening over the last 25 years. A new generation of physicists and mathematicians, who do understand each other very well and move across the frontiers, borrows from each world and a kind of hybrid language emerges. That's happening but it takes time.

Question: This is the year of Hamilton; I think last year was the year of Joyce so I'd like to treat you to some stream of consciousness. I started out in life as a very frustrated mathematician; let me explain, I studied maths/physics and I never understood the complete disregard for rigour. I think I almost lost my mind trying to understand quantum mechanics, to quote Feynman "that was the big mistake; one doesn't attempt to understand the subject". With mathematics the problem was that I enjoyed the aesthetic aspect of it, rings groups etc., but it was a sterile subject. Looking back there was no input of the personalities into it. And when I saw the flyer about the talk here today and I looked up the speakers and was aware of them from the 'music of primes', 'popular science' etc. I think that's the thing that inspired me first about mathematics was Bell's 'Men of Mathematics' and I think that's lacking in education. Where do the structures arise from in pure mathematics? They were presented to us as definitions, theorems and it was just sterile. To some extent if I draw an analogy with Gaelic Football—to liven things up, one starts a row. So I'm going to say that I'm glad I went away from mathematics into architecture. The questioning here was relatively staid, I think the replies, they're very staid, there's none of the lifeblood that I would associate with mathematics, which may have to do with aesthetics or your personalities, you must have been really invigorated when you found beautiful proofs etc, and I don't get any of that feel from the top table and of the questions being asked. I don't know if that's a fair comment. The other thing is the very first question that was asked, the difference between maths and physics, I thought Hardy had answered that and we are all familiar with his quip that "Mathematics is the subject we don't know what were talking about and care less and that we don't assign any values to variables", that it is kind of formulistic rather than we're looking for quantities to be verifiable. So rather than running the risk of getting thrown out, I'll stop.

Michael Berry: If I were the kind of person to get insulted I would be now, but of course I'm not. Well I'm driven crazy by journalists who come and talk to me and when you want to tell them something serious, they say "Oh yes, but all that's fine but what about the personality, are you not excited by things." They're asking us to say if we're human. Well we are! Of course we are, and of course we're excitable but if we're talking about it all the time, it gets boring. The subject itself is much more interesting. If you have somebody around to repair your plumbing, you don't want the life story about how exciting his plumbing is, you want him to get on with it and do it.

Brendan Goldsmith: I can also say that I had the benefit of hearing a session in the British Association yesterday where two of our panellists spoke. You certainly would have gotten some of the sense of excitement conveyed to you there.

Michael Atiyah: You said so many things that were stream of consciousness that I'm not quite sure where to begin. Let me go back to the exposure of undergraduate physics students to mathematics. I totally agree that mathematics, as presented to students, fails on two major grounds which you've pointed out and that it's taught in a very dry formal way without any explanation of the origins and the motivations of where things are coming from. That is a terrible mistake and shouldn't happen, but people are human. The second thing is I would have to disagree with the other Michael. I think knowing something about the personalities and history of mathematics is an interesting addition to your knowledge. It's nice to know that mathematics hasn't always been like that and that somebody created it and it's nice to know who created it and when and how. Some treatment of the history of mathematics is very important I think and part of that history is, of course, talking about the people and where they came from with their contribution and it also gives you a chance to explain the motivational origins—the roots if you want—and to follow these things back into the past. Of course the trouble is, curricula are large and you have to compress everything into three years and when you've done all that, there are exams; all this other stuff gets thrown out—a terrible mistake! We should be teaching a lot more about the origins, the history, and the people behind it and making it more interesting for the student. In the long run this would pay off because people would study more. Trying to force-feed them formulas, lemmas and theorems is a disaster. So I'm sorry you had to go into architecture but I will let you into a secret here; my first name is Michael but actually I was going to be called Michelangelo, only at the last minute did my parents change their mind.

Michael Berry: We all like our stories but don't want to go on about it!

Question: The aesthetic criteria of today seem to be the only way of judging aspects of theoretical physics that they are beyond experiment almost, with regard to size and cost. Is it the aesthetics which excite you?

Michael Berry: We've touched on this already, and part of this is the delight in abstraction and finding connections between very different areas, and that's part of it. It's not all of it, aesthetics are important.

Michael Atiyah: Many mathematicians have explained that an appreciation of beauty is an important part of mathematical truth. The reason for that is we try to and aim to produce and understand in an elegant and beautiful way. Elegance and beauty are a sign of success, they are not just an add-on extra. They are an essential part. It has to be beautiful, it has to be elegant or otherwise it fails its main task which is to unify explain and simplify. So we all find marvellous things which are beautiful in mathematics and which impress us aesthetically: different things, different levels for different people of course. All of us search for beauty in Mathematics, beauty which is not just skin deep, things that are real, and fundamentally beautiful for all sorts of reasons. Mathematics is like Architecture. We build beautiful buildings.

Question: I would like to ask the panel a question about mathematics and society at large. This was touched on in the last question indeed and replied to in some extent in an earlier question. We're looking here at the relationship between mathematics and physics and I wonder what the panel's view is about those two disciplines; both the challenges and the accomplishments that they have had in communicating the importance of each discipline to society at large, in particular as far as education is concerned and as far as government is concerned. So how do physics and mathematics relate in that general discourse?

Michael Atiyah: I'll start if you like, it's a very big question, and you did say several things all at once. You're interested in how mathematics and physics relate to the general society, to the public and also to the government. How do you explain the importance of these things? In doing that, of course, you may not be talking about the relationship of mathematics and physics alone. You explain mathematics by picking out examples of important applications of mathematics that the audience in front of you can understand. There are no shortages of these examples and similarly with physics. You would talk about the actual applications of physics in the real world, how we get our electrical light and so on. You illustrate all these things by suitably chosen examples, because examples are something that the other person can understand. The example is chosen for the level of the audience you are talking to. For a government you would talk about examples that would involve big money that would save them a lot of money. And for school children you would do things at a relevant level, so I think explaining to the general society as a whole we have to understand our own subject and what role it plays and to what extent it effects peoples lives in a concrete way which you illustrate by examples. If you can't do that, you've failed. And there are no shortages of areas where both mathematics and physics are enormously beneficially now, rather than in the past, and hopefully in the future which provide material for illustration but you have to do it in a vivid way that your audience understands. You can't just produce very general theories but ways that even the politicians can understand.

Luke Drury: I think there is also a very serious issue and one also has to address the unfortunate view that has been promulgated by some postmodernists that all knowledge is arbitrary and relative. But there is a very real sense in which physics does study the objective reality of the world. I mean aircraft fly, they fly because they are built according to our understanding of physics and you can't just say that this is a cultural construct; this is an argument which I think needs to be fought.

Brendan Goldsmith: Perhaps the chairman can indulge himself in putting one final question to the panel. It seems to me that a question we might well get asked if we had some politicians in the audience is "What about computing and how is computing going to interact with all this and does it overtake it all; doesn't computing do everything?" So there is a relationship there I think that we need perhaps to explore. So maybe I will just finish up by asking each of our panellists some of his views on this.

Michael Berry: Computers have transformed the way I do science; I didn't anticipate this at all. There are three types of activities in theoretical physics that computers have revolutionised; I foresaw one but I didn't foresee the others. One is just number crunching. A lot of theoretical physics that I do is finding consequences of laws that are already known. Some people try to seek fundamental laws that govern phenomena and regimes not previously reached, but most of us take existing formulations, quantum mechanics and so on, and extract the infinite wealth that they contain. Now some of that involves getting numbers out of equations and in the old days that was something that I employed research students to do. Not that I used them as drudges, but that I hated big computing, FORTRAN and all that. The moment personal computers came along with powerful software, instantly I did all my own computing and my research students are now free to be much more creative and this of course distinguishes the good ones from the bad. So number crunching is one aspect. Another aspect is Algebra. There's an area that I worked on in the 1990's which would not have developed without the ability of a computer to do algebra; this is the understanding of divergency and how to make sense of infinite series that diverge which come up all over the place in physics. This subject has a very long history but major advances were made in the 1990's because the enormous algebra that you need to illustrate and understand the way these formulations behave was then possible and it wasn't before. So that's a second area. And another one which I didn't anticipate was this. Much of what I do results in a picture. It's a truism, a cliché, to say a picture is worth a thousand words; certainly we all at this end of the table know that an equation is worth many thousands of pictures, but it's very hard to see sometimes what the equations contain. I've found that pictures are the right way to explore formalism beyond what I can understand analytically, and this has led to discoveries. If you use colours in the right way and you zoom in on a picture, you can find, for example, certain singularities that you thought were cones, are actually separated and later you understand why. In practice, I've found in these three ways that computers have become very useful; it has transformed the way I do science in the 15 years since we've had these personal computers. Other people may have different experiences and I certainly respect that, but to me it's been enormously important.

Luke Drury: I would agree with all of that. The one thing I would add is that it is a very bad mistake to think that you can just rely on computing without having some analytic understanding of what's going on. There are very many examples of where naïve use of computational models leads to disastrously wrong results. You have to understand what's going on, you may not be able to understand it in all details, but you can at least use analytic ideas to understand the results coming out of numerical models; if you don't do that then, frankly, you are walking through a minefield.

Brendan Goldsmith: There is a beautiful comment I remember from Hamming's book on numerical analysis to that effect. He had a wonderful quote that said "The purpose of computation is insight, not numbers."

Michael Atiyah: I agree! I won't elaborate more on the use that mathematicians make of computing, it's an enormously valuable tool that replaces graduate students and so on, and that will increase. I agree with Luke that it's very dangerous to assume that computers will replace mathematicians and we would then be out of work. A computer is a machine and will do what it's told, but you have to understand what you trying to tell it. You have to understand what the questions are and why you're asking them and then it will carry out tasks very efficiently, but it's not the primary source. The thing that determines what questions you ask, how you go about it, that's the really hard part. The computer frees us and mathematicians to think about the fundamental questions, the really important questions, which ones to ask and how to select them. As a good servant or a good researcher, it will do what it's told, but it needs to be told. That's our job and the more we have computers, the freer we are. I look to the future where mathematicians and physicists will have a marvellous time making fantastic speculations. As soon as they have a vague idea, they'll say "put it in the computer and try it out and let me know how it works". Then in 5 seconds the computer will tell them. In the old days you would tell a research student and after 6 weeks he would come back and tell you, so it is an enormous advantage to help the creative process. Of course it never replaces it. The message you have to get across to someone on the street or in government who says "close mathematics departments, we will just buy more computers" is that that is a disaster from all points of view. I totally agree with that and it also goes back to the question that it is our aim to understand things and producing answers, beautiful pictures or things are a guide to help you to understand or step towards it, but by themselves they don't substitute. Finally going back to the question before about the use of computers for proof, i.e., whether you can prove mathematical theorems in the future by computing and that's going to be the way we are going to go, again I revert to the point that we want to understand what the answers are and why they are true; if you hand it all over to the computer and say "You tell me whether it's true or not" and the computer comes back and says it is, what good is that to me. I think for all sorts of reasons we have to keep the computer in its place. It's an important place but we're boss!

Arthur Jaffe: I agree with what the other three panellists have said but maybe to add a couple of minor remarks; you certainly have to tell the computer what to do. I know that when the National Security Agency, which is the largest employer of mathematicians in the US, perhaps even in the world, wanted to effectively know the best way to break codes, they decided that their decision was not to employ just computer hackers, but it was better to employ mathematicians, because mathematicians could give the concepts that would enable this to be possible. I think the idea of understanding of knowledge is central. Most mathematicians also use computers for word processing. I also use computers to sometimes test an idea, to do a little experiment and follow that up to try to prove something, but I agree with the other panellists that there's a wide spectrum of mathematics and hopefully there will be much more mathematics in the future.