Book Review

James Joseph Sylvester
Life and Work in Letters

Karen Hunger Parshall

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Reviewed by Rod Gow

James Joseph Sylvester (1814-1897) was one of the foremost British mathematicians working in the 19th century. His name survives in Sylvester's law of inertia, proving the invariance of signature when a real quadratic form is diagonalized, and also in Sylvester's law of nullity, which gives an estimate for the nullity of a matrix product. He is also remembered for a method in elimination theory which gives a necessary and sufficient condition for two polynomials to have a root in common. Sylvester's mathematical output was prodigious and his collected papers fill four large quarto volumes (still available as an AMS/Chelsea reprint). Sylvester largely confined his research to algebra, especially invariant theory, a subject that he and his colleague Arthur Cayley (1821-1895) greatly developed over several decades, sometimes with the collaboration of George Salmon. It has frequently been related that much of the explicit computational invariant theory, in which Sylvester was a specialist, fell into disuse as more conceptual methods replaced it, and this may account for the comparative absence of Sylvester's name from the modern algebra curriculum. Cayley worked in several branches of mathematics, including invariant theory, algebraic geometry and elliptic functions, and his contributions have survived better into later mathematics (witness Cayley graph of a group, Cayley embedding theorem for groups, Cayley–Hamilton theorem, Cayley parametrization of the orthogonal group, Cayley-Salmon theorem on the 27 lines on a cubic surface). Cayley was even more prolific in his publications than Sylvester, despite full time employment as a conveyancer until 1863, and his collected mathematical papers fill fourteen quarto volumes.

The book under review presents 140 letters to and from Sylvester, written between 1837 and 1896, which form a significant fraction of about 1200 letters relating to Sylvester that exist in the libraries and archives of such institutions as St John's College, Cambridge (the principal repository) and Johns Hopkins University, Baltimore. As might be expected, the greater part of the letters (over 40) were written to or by Cayley, in a correspondence devoted to the technicalities of proof and conjecture. It is a pity that there is no index of the letters published in this volume, as it would help in locating and counting them. As is not uncommon in scholarly works of this nature, footnotes account for at least half the text. These footnotes provide detailed information about persons or mathematical theories described in the letters, and Parshall has taken great pains to be as informative as possible in her commentary. There is a certain amount of repetition in the footnotes, with complete titles of works being given several times over, but one is left with the overriding impression of an excellently researched work. Furthermore, the book is pleasantly produced from the author's own computer files and is remarkably free from typographical errors (this is surely the great advantage of allowing the author to typeset the work).

The letters provide numerous insights into Sylvester's life and help illuminate the way in which mathematics emerged as a subject for research by professional specialists. After a somewhat unsatisfactory series of appointments, including actuary to the Equity and Law Life Assurance Company and professor at the Royal Military Academy in Woolwich, Sylvester was recruited to the newly-founded Johns Hopkins University in 1876 by its president Daniel Coit Gilman. This was an ideal appointment for Sylvester, as he was expected to develop a research community–an idea totally new to America–in whatever way pleased him, and without the need for undergraduate teaching on his part. Sylvester seems to have been an indifferent lecturer, and he was swept along by his latest enthusiasm for a new idea. He did not work systematically, and was unable to provide research lectures in a sustained way on an agreed topic. Nonetheless, he succeeded in his mission of establishing a new research-led department, and provided inspiration for several young researchers. His lectures on constructive methods in partition theory were especially fruitful, and led to Fabian Franklin's famous proof of Euler's pentagonal number theorem for the partition function. Another student, William Durfee, introduced the idea of the Durfee square in the graph of a partition (Sylvester had a low opinion of Durfee and wrote "Durfee's *square* is a great invention of the importance of which its author has no conception.").

A letter of 1881 indicates that Sylvester attempted to persuade Cayley to accept a position at Johns Hopkins, and he seems to have obtained Gilman's approval for such action. While these efforts proved to be unsuccessful, he did manage to persuade Cayley to visit Baltimore in 1882 to deliver a series of lectures on theta functions. Eventually, however, loneliness, depression and uncomfortableness during the hot summers led Sylvester to apply for the Savilian Professorship of Geometry, at Oxford University, following the death of its previous holder, Henry Smith, in 1883. Previously, such positions in the ancient universities of England had been barred to Sylvester on account of his Jewish faith (he had already been denied a degree at Cambridge University, although finishing as Second Wrangler, as he could not subscribe to the Thirty-Nine Articles of the Church of England) but the religious tests had been abolished a few years earlier. As his friend Cayley was one of the electors to the professorship, Sylvester kept himself well informed of his chances of success (in March 1883, he wrote to Cayley "Do you think I am likely to be appointed?"). After initial enthusiasm following his appointment, Sylvester began to realize that the position involved substantial undergraduate teaching, for which he had little aptitude, especially on geometry, not his favourite subject. Furthermore, there was little interest in Oxford for his original research.

The letters give a good idea of Sylvester's working methods. There was a certain rivalry between the English school of invariant theorists, led by Cayley and Sylvester, and their German competitors, including Clebsch and Gordan. Sylvester was especially irked by a theorem of Gordan (1868), asserting that the number of covariants in a minimum generating set for a binary homogeneous form is finite. This theorem, proved in too geometric a manner for Sylvester's taste, contradicted an earlier assertion of Cayley. Sylvester tried for several years, without success, to find a more constructive proof of Gordan's theorem, although he was able to give explicit values for the minimum number of covariants in certain specific cases. In some ways, the failure of his explicit methods was a considerable blow to the philosophy espoused by Sylvester. There are also several instances of Sylvester's lack of rigour — even in his published work, he often checked that a theorem was true in a few small cases and then asserted that he was morally certain that the general case would follow along similar lines. On the other hand, he sometimes grasped at methods that would become dominant in later theories, for example, his use of Lie algebra methods, described in letter 92 of 1877.

Of interest to Irish mathematicians are various letters written by George Salmon to Sylvester. Salmon was in frequent correspondence with Sylvester in April 1852, at a time when invariant theory was being rapidly developed by Cayley and Sylvester, and five letters of his are printed here. They show how much Salmon's interest in invariant theory was motivated by his own work on algebraic curves and surfaces, rather than by algebraic considerations. In letter 20, Salmon wrote with surprising honesty:

> I have taken from you on trust & without proof all the leading propositions of the theory ... and were you to desert me I should be a very babe in the wood, although as long as I am sure of having you to set me right if I go astray, I can venture to wander to short distances from you in the search of the flowers which grow in the beautiful regions to which you have led me. ... A great part of every one of your previous letters was unintelligible to me. But since then the epistolary labors which you expend on me have been much less thrown away.

The culmination of Salmon's work on invariant theory during the 1850's, when he was in frequent contact with Cayley and Sylvester, was his book *Lessons Introductory to the Modern Higher Algebra* (1859). The dedication for the book shows how much he benefited from his correspondents:

To A. Cayley, Esq., and J. J. Sylvester, Esq., I beg to inscribe this attempt to render some of their discoveries better known, in acknowledgment of the obligations I am under, not only to their published writings but also to their instructive correspondence.

A later letter of Salmon's (April 1877) gives a valuable insight into his relationship to research-level mathematics following his election to the Professorship of Divinity in 1866:

...I am very glad that you have rehabilitated Cayley's method which I naturally dismissed in disgrace after it seemed to have broken down in his hands. ...But alas I am becoming very rusty; learning nothing new & forgetting half the old. I suppose you will be as little pleased with me for giving my time to to the study of Gnostic heresies as I was at your giving yours to translating Horace. I am sure it must do you good to be brought in contact with fresh minds. I think some Oxford men talk great nonsense about the endowment of Research. What security have you that the men you endow will research? But if you give your researcher a class of intelligent young men, you make sure of getting at least some good out of him in the way of teaching; the better man he is the more he will stimulate his class: and if you don't *over*burden him with teaching the class will stimulate him.

The comments about research are illuminating, as Salmon proved to be unamenable to plans for promoting research at Trinity College during his provostship.

The book by Karen Parshall is an excellent source of information about British and American academic life in the nineteenth century, seen through the eyes of a figure probably better suited to the twentieth century approach to the pursuit of new mathematical theories and truths. The review above has concentrated on a few topics that appealed to this reviewer and give no indication of how much of Sylvester is revealed through his correspondence and the attendant commentary. We recommend the book to anyone interested in the history of mathematics, and especially in the British school of algebraists.