

THE ROLE OF MATHEMATICAL MODELLING IN UNDERGRADUATE COURSES

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Abstract This paper examines the role of mathematical modelling in undergraduate courses. After setting modelling in the context of the three M's of mathematics-methods, models and modelling-and after describing the process of mathematical modelling, it suggests that mathematical modelling can help achieve the following aims in a course

- have a unifying effect
- create interest in novel applications
- develop interpersonal transferable skills
- develop deep learning
- develop self knowledge.

The implications for staff are that they will need to embrace radically new methods of teaching and assessing.

Introduction

It is my thesis that mathematical modelling is the way of life of a professional applied mathematician. More than that, mathematical modelling is a way of life, full stop; a way of life for everyone whether they realize it or not. So, it seems to me, every undergraduate student, of mathematics or of something else, and every pupil at school should meet the *concept* of a model, particularly a mathematical model; they should be aware of the *process* of modelling, and they should have some understanding of the *philosophy* of modelling. It seems to me that if there were a more widespread understanding and acceptance of the relationship between reality and models of reality and how they are created, then fundamentalists, both scientific and religious, would cause us all fewer problems.

While I would describe mathematical modelling as an art rather than a science because, at its best, it requires creative and imaginative thinking, that is not to say that the scientific method of investigating, conjecturing and proving is not valuable to the mathematical modeller. Of course it is, and, indeed, it is possible to take almost an algorithmic approach to modelling, but, as we shall see, even this includes the verb "create."

The three M's of mathematics

Mathematical Modelling is one of what I describe as the "three M's" of mathematics-Mathematical Methods, Mathematical Models and Mathematical Modelling. (The reader may be able to suggest some more.)

A mathematical model is a representation of some aspect of reality which, however complex, is necessarily a simplified representation. A model is used to describe and/or to predict the aspect of reality. A mathematical model consists of some mathematical entity such as an equation together with a statement of the simplifying assumptions that have been made in going from the reality to the model. Perhaps I should say, in going from our perception of the reality to the model. Thus Newton's second law of motion is a model of planetary motion. It is a differential equation which describes and predicts the motion of a planet and it embodies all sorts of assumptions such as the relationship between acceleration and force and the nature of the force of gravity as expressed in another model, Newton's universal law of gravity. It makes assumptions about planets being point masses in order to simplify the calculation. Increasingly mathematical models are being used in many walks of life and many academic disciplines.

Mathematical modelling is the activity of creating or modifying a model and using it. It is, I believe, the essence of applied mathematics. It requires a wide range of knowledge and skills. We must know something about mathematics before we can start, and similarly we need to know quite a lot about the aspect of reality we want to model. A knowledge and understanding of other models of this or other aspects of reality is also useful, as is an appreciation of how other modellers have worked in the past. It

is particularly useful if we can obtain some insights into the way they went about it, the ideas they had, the questions they asked, even the blind alleys they went up. And we need to develop the same skills—reading, questioning, conjecturing and, indeed, proving. We need at least to attempt to validate or justify the model we have created.

Mathematical methods are the tools we use to create models and to answer questions about them. Arithmetic, algebra, calculus, geometry and so on—all are useful and some one, for some aspect of reality, is necessary. We need to learn which tools to use; we need to know of their existence and their scope; we may have to invent new ones.

Learning methods, studying models and engaging in modelling should all be part and parcel of an undergraduate course in mathematics. Also they should be part and parcel of high school mathematics. Enterprising primary school teachers might be able to find ways of getting their pupils in on the act as well. But there needs to be an awareness on the part of the student or pupil as to what is going on. When they are learning new tools they need to know in what context this might be useful; when studying models they need to look out for the assumptions made, the questions asked, the methods used and the extent to which the model is valid; when engaging in modelling they need to be aware of what they are doing at any particular time, what they have already done and what still remains to be done. In other words, they need a methodology for modelling.

The process of mathematical modelling

I expect that this is generally well known. Let me summarize it as an algorithm.

- Study the aspect of reality
 - Identify main features
 - Define variables
- Label 1 Make simplifying assumptions
 - Create model
 - Establish relationships between variables
 - Use model to answer questions

Translate into mathematical problems

Use methods to find solutions

Interpret solutions

Attempt to validate solutions

If valid (or valid enough) finish cycle and report back

If not valid, revise model by returning to Label 1 or earlier.

To repeat what I wrote earlier, I believe it is important for modellers to know when they are making simplifying assumptions and to articulate them always. This, if you like, qualifies the answer they may give to a problem. It is important for them to be aware of the range of validity of a solution and to be able to interpret their solution in terms of the original aspect of reality. It helps if they can say where they are in the modelling cycle when they are engaged in any particular task.

The role of mathematical modelling

Mathematical modelling can contribute in a number of ways to achieving the aim of a course.

Unifying effect

Modelling has a unifying effect on an undergraduate course in applied mathematics. Whether the student is studying mechanics or its derivatives, statistics or operational research, the ideas behind *methods*, *models* and *modelling* can be applied in each of those subject areas. Pure mathematics may be taught to give a rigorous foundation to methods, or it may be taught for its own sake, and in this case it may not be possible to link it to the theme of modelling. But wherever possible, it is desirable to see a topic as a study of a model, or a method to solve a problem deriving from a modelling activity. The idea of learning about the way of life of an applied mathematician can be used to bring those diverse topics together.

Creating interest

When used in a novel situation, the theme of modelling can help stimulate interest in an application area. Of course, some students will not want to get involved with reality, but will want to stay within the safe world of mathematics that they know and love.



The trouble is that not very many people earn their livings as pure mathematicians and at some stage (and the sooner the better in my view) they will have to engage with problems out there in physics, or economics, or society, or wherever. If they can bring a trusted philosophy and methodology to a new situation, then it will not be such a daunting task. Modelling provides opportunities for the lecturer to introduce students to many areas of human endeavour wherein mathematics is useful.

Developing personal skills

In 1987 in the UK, the Department of Employment introduced the Enterprise in Higher Education Initiative, [1]. The term *enterprise* was defined widely and the proposed objectives of the initiative included the ideas that students should

- be more ready to be enterprising
- have developed personal transferable skills
- be better prepared to contribute to and take responsibility in their professional and working lives.

Personal transferable skills are "the generic capabilities which allow people to succeed in a wide range of different tasks and jobs" and include the development of

- group work skills-leadership and followership
- verbal communication skills
- written communication skills
- problem solving skills
- numeracy skills
- computer literacy
- the ability to achieve results
- self assessment skills.

Mathematical modelling can help develop these skills and turn passive receptors into active learners.

In industry, much work is accomplished by teams of people. The mathematician may be only one person of several working on a problem. Employers tell us that besides mathematical skills and knowledge, they look particularly for good interpersonal skills in those whom they employ. The ability to work with others, perhaps



as a leader, perhaps as a follower, the ability to communicate with others, especially "lay" men, the ability to write cogent and persuasive English-these skills are highly valued by employers, [1].

Mathematical modelling provides opportunities for students to learn and practise their skills. When students are involved in creative modelling, it is best they do so in groups. They can be taught about group dynamics and group work and they can practise it. Yes, it takes up some time, but it is a more valuable use of their time than studying yet another topic on an overlong syllabus.

Students can report their group project work to the rest of the class via a student-led seminar and this gives them experience and practice at making presentations using an OHP or even, nowadays, PowerPoint. They can also write up their work in a report which is also assessed. Recently I have been experimenting with student poster sessions, [2], wherein they present their work in a poster instead of a seminar. This introduces students to yet another aspect of professional life and it presents to a student different challenges from a seminar.

Developing learning

Mathematical modelling helps convert students from being passive receptors into active learners. It is all too easy for a student to attend class, take notes and submit homeworks. They may use the library only as a place to sit without ever opening any of its books or consulting any hyper-media instructional packages. Their conversation with their peers may not extend beyond football, beer and sex. They may pass their exams through having a good memory and through spotting questions. I know that this caricature is mythical to an extent, but the point I want to make is that students learn more thoroughly and with a deeper understanding if they are actively engaged with their learning and are prepared to take responsibility for all aspects of it, [3]. Talking with their peers about mathematics helps them express themselves more clearly; teaching others (or explaining things to them) encourages a deeper understanding (or else exposes their misconceptions, and this is also an important aspect of learning).

Through the requirement to carry out research, through the use of comprehension tests, students are encouraged to read mathematics. More than that, they are encouraged to *study* mathematics *independently*. They begin to learn to take control of their own lives and their own learning and this is a terribly important step on the way to maturity.

Moreover, students learn from one another and this informal peer tutoring is a valuable learning resource for students to have. Recently I have been experimenting with ways of using peer tutoring to help students learn methods and models as well as modelling, [4].

Developing self knowledge

*Had some power the gift to gie us
To see ourselves as others see us*

Burns.

The ability to know one's own capabilities and to assess one's own performance is an important one to develop, and modelling provides opportunities for engaging in self and peer assessment. Students will learn more and perform better if they know what the assessment criteria are. To start with, students are inexperienced assessors and so need to be taught how to construct assessment criteria and they need practice in applying this and in making judgements. As they progress, they will engage more and more in critical self-reflection (i.e. self assessment) before submitting work for summative assessment by their lecturers and so should perform better because they now have a better idea of what they are striving to achieve.

It is important to develop assessment criteria with students so that, as far as possible, they belong to the students in that they have made them their own, understand them and can apply them. This involves discussion and the use of exemplar material. Again, this is, in my view, time well spent. See [5] for examples of such assessment criteria.

Implications for staff

It is of course possible to introduce mathematical modelling to one or two modules of a course, but it is better if the whole course and

the whole course team embrace the philosophy and the unifying theme of modelling. This requires the head of department to embrace this idea and to exercise leadership. It is not enough for one or two enthusiasts at a junior level to get involved.

Radical changes are required

- from staff-centred to student-centred management of learning
 - from precise (!) methods to more fuzzy methods of assessment.
- Staff need to be prepared to "let go" to some extent and to function more as enablers of learning than as central performers. This is, for many, a journey into uncharted waters, although they can of course read the adventure tales of those who have explored these ways before them. It requires people to do things differently and to be prepared to embrace new ideas and methods.

Staff also have to get involved in assessing oral and written work. Colleagues in the humanities have been doing this for years, but it requires us to accept a greater measure of fuzziness in our marking than before. Until people have had experience of assessing oral and written work, they will find it hard to agree, as they can do now, that a particular piece of work is worth (exactly) a mark of 61, say, out of 100.

Accordingly, staff development is a necessary precursor to the introduction of modelling on a widespread basis. Staff must themselves engage in the same activities they are planning for their students. They must learn how to develop assessment criteria and to apply them consistently. Above all, they need an enthusiasm for the job.

Conclusion

In this paper I have described the concept of mathematical modelling and the role it can play in undergraduate courses. Modelling can have a unifying theme and can create interest in novel areas of application; it can develop interpersonal, transferable skills and it can encourage deep learning and self assessment.

The implications for staff who wish to introduce modelling are that they must be prepared to engage in continuing professional development of their teaching and to adopt a new, different role in relation to their students.



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AN APPROACH TO THE NATURAL LOGARITHM FUNCTION

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1. Introduction

In many, if not all, modern calculus texts, the logarithm function is usually defined, and its properties developed, following a discussion of the Riemann integral. It seems to me, however, that, for many students of the physical, engineering and biological sciences, this is much too late, and a *careful* treatment of the elementary functions should be given much earlier in any course aimed at such students. The emphasis here is on the word 'careful': I mean that every effort should be made to keep the technicalities to a minimum, without sacrificing rigour, even if this means that some results may have to be stated without proof. Instead, the utility and importance of these should be pointed out at every opportunity.

This note, then, is a contribution to the ongoing debate on what material should be taught in a modern calculus course, how it should be treated and at what stage it should be presented. Its main purpose is to outline an approach to the natural logarithm function that can be adopted in any a course that treats sequences and series early on in a serious manner, starting with a discussion of the completeness axiom for the real numbers. Its main novelty is that it deals with sequences which are indexed on the dyadic

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