Book Review

Mathematics—The Music of Reason

Translated from the French by J. Dales and H. G. Dales

J. Dieudonné

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Reviewed by Robin Harte

Here is a master of exposition at the peak of his form - the interpreter and expositor of Grothendieck's theories offers us a dancing run over the surface of modern mathematics, carrying us from astronomy in the ancient world to Gödel, "independence" and Cohen "forcing". As we might expect, the perspective throughout is very "Bourbaki". After two more or less introductory chapters on "Mathematics and Mathematicians" and "The Nature of Mathematical Problems", each chapter is addressed to non-specialists and then furnished with an Appendix for the professionals. Thus we have "Objects and Methods in Classical Mathematics", with an Appendix ranging from ratios à la Euclid to limits via exhaustion, "Some Problems of Classical Mathematics" with an Appendix covering prime numbers and the Riemann zeta function, "New Objects and New Methods", whose Appendix is about Galois Theory and the foundations of metric spaces, and finally "Problems and Pseudo-problems about Foundations", with an Appendix about surface geometry and models of the real numbers. The translation, by J. and H. G. Dales, is uniformly excellent: only the typeface seems to have an old fashioned air about it.

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Solutions to the Problems of the 36th IMO

1. First solution. Let DN meet XY at the point R. The triangles RZD and BZP are similar and hence $RZ/ZD = BZ/ZP$. Thus $RZ = BZ.ZD/ZP = ZX^2/ZP$. If S is the point of intersection of AM and XY, then a similar argument proves that $SZ = ZX^2/ZP$. Thus the points R and S coincide and the result follows.

Second solution. Choose coordinates so that the line $ABCD$ is the x-axis with Z as origin and XY is the y-axis. Let the coordinates of A, B, C, D and P be $(a,0)$, $(b,0)$, $(c,0)$, $(d,0)$ and $(0,p)$, respectively. The problem can now be solved using routine calculations.

2. The expression on the left hand side of the inequality can be made a little more friendly by letting $a = 1/x, = 1/y$ and $c = 1/z$. Then $xyz = 1$ and the inequality to be proved is:

$$\frac{x^2}{y+z} + \frac{y^2}{z+x} + \frac{z^2}{x+y} \geq \frac{3}{2}.$$

If S denotes the left hand side, then

$$\begin{aligned}2(x+y+z)S = [(x+y)+(y+z)+(z+x)]S = \\ [(\sqrt{x+y})^2 + (\sqrt{y+z})^2 + (\sqrt{z+x})^2]\times \\ [(\frac{z}{\sqrt{x+y}})^2 + (\frac{x}{\sqrt{y+z}})^2 + (\frac{y}{\sqrt{z+x}})^2] \geq (z+x+y)^2\end{aligned}$$

by Cauchy's inequality. But the arithmetic-geometric mean inequality gives $x+y+z \geq 3$, since $xyz = 1$. Thus

$$2(x+y+z)S \geq 3(x+y+z).$$

Hence $S \geq 3/2$ and the result is proved.

3. If A_1, A_2, A_3, A_4 are the vertices of a square of unit area and if $r_i = 1/6$ for $i = 1,2,3,4$, then the triangle $A_iA_jA_k$ has area $r_i + r_j + r_k$ for each triple i, j, k $(1 \leq i < j < k \leq 4)$. So the result holds for $n = 4$.