positive metric, although it is not the boundary of a parallelizable manifold. Hence the converse to Theorem B is false.

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THE 36TH INTERNATIONAL MATHEMATICAL OLYMPIAD

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The 36th International Mathematical Olympiad took place in York University, Ontario, Canada, on 19th and 20th July 1995. 412 students participated from 73 countries and Ireland was represented by Robert Hayes (St Thomas Community College, Bray), Gavin Hurley (Coláiste an Spioraid Naoimh, Cork), Brian Jones (Gonzaga College, Dublin), Peter McNamara (East Glendalough School, Wicklow), Deirdre O'Brien (Mount Mercy College, Cork) and Gregory Wall (St Mary's Academy, Carlow). The team consisted of the top six performers in the Irish Mathematical Olympiad, which took place on 6th May 1995. The University of Limerick hosted a three-day intensive training session for the team from 5th to 7th July. The training involved staff from UL, UCC and UCD and some previous Irish Olympiad team members. The team leader was Gordon Lessells of UL and the deputy leader was Eugene Gath, also of UL. I accompanied Gordon as an "observer".

Gordon and I flew to Toronto on 12th July, whence we were taken to the University of Waterloo. The first job of the team leaders was to select six problems, from a short list of 28, which would form the two papers of the IMO exam. A local committee had chosen the short-listed questions from a total of about 200 problems which were submitted by the participating countries. The leaders and observers were free to work on the problems for 24 hours, without the distraction (?) of having the official solutions! I felt a bit disappointed because I only managed to solve four of the problems in that time, but felt a little less chastened on discovering that many of the team leaders had similar success! One of the questions submitted by Ireland was shortlisted and was in contention for a long time, before being eliminated from consideration. The problem subsequently appeared on this year's Iranian Mathematical Olympiad. The problem, which was composed by Tom Laffey, is as follows:

At a meeting of $12k$ people, each person exchanges greetings with exactly $3k+6$ others. For any two people, the number who exchange greetings with both is the same. How many people are at the meeting? Prove that such a meeting is possible.

The team, accompanied by Eugene Gath, arrived in Toronto on 15th July and were taken to York University. A lot of social activities were organized for them and there was plenty of time to establish friendships with students from other countries. For two days after the exams the leaders and deputy leaders were fully involved in marking the students' work and agreeing marks with the Canadian problem coordinators – this was an acrimonious business, at times. Gold, silver and bronze medals were awarded to the top performers - not more than 50% of the students can get medals. The performance of the Irish team was a bit disappointing. They won no medals, although Deirdre O'Brien and Peter McNamara got an "honourable mention" for their solutions to Question 1. Overall the team came 61st out of 73 countries.

Before the exam took place, some team leaders were vociferous in their claim that the exam was too easy - one leader going so far as to claim that each member of his team would get full marks. However, once the exam had taken place, no more such remarks were to be heard, although it was generally agreed that Question 1 was a bit too easy. China took first place, with Romania second and Russia third. The United States was very disappointed with 11th place, particularly after their unique performance in Hong Kong in 1994, when all their students got full marks. Questions 2 and 6 turned out to be very difficult for most students, although one Bulgarian student was given a special prize for the elegance of his solution to Question 6 (the second solution given on p.75).

Here are the questions.

First Day

1. Let A, B, C and D be four distinct points on a line, in that order. The circles with diameters AC and BD intersect at the points X and Y. The line XY meets BC at the point Z. Let P be a point on the line XY different from Z. The line CP intersects the circle with diameter AC at the points C and M, and the line BP intersects the circle with diameter BD at the points B and N. Prove that the lines AM, DN and XY are concurrent.

2. Let a, b and c be positive real numbers such that $abc = 1$. Prove that

$$\frac{1}{a^3(b+c)} + \frac{1}{b^3(c+a)} + \frac{1}{c^3(a+b)} \geq \frac{3}{2}.$$

3. Determine all integers $n > 3$ for which there exist n points A_1, A_2, …, A_n in the plane, and real numbers r_1, r_2, …, r_n satisfying the following two conditions:
(i) no three of the points A_1, A_2, …, A_n lie on a line;
(ii) for each triple i, j, k $(1 \leq i < j < k \leq n)$ the triangle $A_iA_jA_k$ has area equal to $r_i + r_j + r_k$.
Time Allowed – $4\frac{1}{2}$ hours.

Second Day

4. Find the maximum value of x_0 for which there exists a sequence of positive real numbers x_0, x_1, …, x_{1995} satisfying the two conditions:

$$\text{(i) } x_0 = x_{1995} \text{ (ii) } x_{i-1} + \frac{2}{x_{i-1}} = 2x_i + \frac{1}{x_i} \text{ for } i = 1, 2, ..., 1995.$$

5. Let $ABCDEF$ be a convex hexagon with

$$AB = BC = CD$$
$$DE = EF = FA$$

and

$$\angle BCD = \angle EFA = 60^\circ.$$

Let G and H be two points in the interior of the hexagon such that $\angle AGB = \angle DHE = 120°$. Prove that

$$AG + GB + GH + DH + HE \geq CF.$$

6. Let p be an odd prime number. Find the number of subsets A of the set $\{1, 2, \ldots, 2p\}$ such that
(i) A has exactly p elements, and
(ii) the sum of all the elements of A is divisible by p.

Time Allowed– $4\frac{1}{2}$ hours.

The solutions to these problems are on pp.69-76.

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Research Announcement

OPTIMAL APPROXIMABILITY OF SOLUTIONS OF SINGULARLY PERTURBED DIFFERENTIAL EQUATIONS

R. Bruce Kellogg and Martin Stynes

Using the theory of n-widths, the approximability of solutions of singularly perturbed reaction-diffusion and convection-diffusion problems in one dimension is quantified. Full details appear in [1].

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