Book Review

QUADRATIC AND HERMITIAN FORMS OVER RINGS

Grundlehren der mathematischen Wissenschaften 294

Max-Albert Knus.
Springer-Verlag, 1991,
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Reviewed by David W. Lewis

The theory of quadratic forms was traditionally regarded as a part of number theory until the work of Witt in the 1930's. His work paved the way for the algebraic theory of quadratic forms over arbitrary fields, a branch of algebra involving a mixture of linear algebra, ring theory and field theory. Witt's work lay more or less dormant until the 1960's, when the work of Pfister demonstrated that there was a rich theory to be explored, and from there the subject really took off. Developments in topology (calculation of surgery obstruction groups), algebraic K-theory, and algebraic geometry led some mathematicians in the 1960's and onwards to examine quadratic and hermitian forms over various kinds of rings. Before that, the only work on forms over rings was of a number-theoretic nature, involving rings of integers.

This book introduces the reader to the theory of quadratic forms over commutative rings in a general setting. The author, M.-A. Knus, has been one of the principal researchers in this area over the last two decades or more. The book is suitable for graduate students, and for mathematicians working in other areas who wish to learn something of the subject. The reader is assumed to have a knowledge of the usual basic results in algebra, including some homological algebra. Unproved theorems are always quoted, unless they are basic results. For the latter part of the book, a

familiarity with some algebraic K-theory and algebraic geometry is helpful.

Chapter 1 introduces the basic definitions and terminology of forms, and develops tools which are used later. Chapter 2 deals with the general theory of forms in categories. There is considerable overlap here with Chapter 7 of the book of Scharlau [1]. Chapter 3 is entitled "Descent theory and cohomology". It introduces the technique of faithfully flat descent, and the notion of twisted forms. Chapter 4 lays the foundations of the theory of Clifford algebras for quadratic forms over rings. The Clifford algebra is used to define the discriminant, the Arf invariant, and the Witt invariant of a quadratic space. Chapter 5 describes the classification of quadratic spaces of low rank (specifically rank ≤ 6) via invariants such as the above. An interesting and surprising by-product of the work on forms of rank 6 over arbitrary commutative rings is a result about involutions of orthogonal type on rank 16 Azumaya algebras. A criterion for the decomposability of such an involution is obtained, utilizing an invariant called a Pfaffian discriminant. The involution decomposes if and only if the Pfaffian is trivial. The surprising thing about this result is that it was not observed at all in the special case of algebras over fields. Thus the more general setting of rings can sometimes yield results that have passed unnoticed for fields. Chapter 6 contains splitting, stability and cancellation theorems for unitary spaces. These are unitary versions of theorems of Bass, Serre and Vaserstein in algebraic K-theory, and are quite technical. Chapter 7 deals with polynomial rings, and is again fairly technical, utilizing some of the results of the previous chapter. Finally, Chapter 8 is concerned with the calculation of Witt groups of real affine curves and surfaces, an area in which there is currently a lot of research going on.

The book is well organized, clearly written, and seems to have few typographical errors. It is a welcome addition to the literature, and I warmly recommend it to those who wish to learn more about the general theory of quadratic and hermitian forms over rings.

Reference

[1] W. Scharlau, Quadratic and Hermitian Forms, (Grundlehren der mathematischen Wissenschaften 270). Springer-Verlag: Berlin, 1985.

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Book Review

CLASSICAL CHARGED PARTICLES

(Advanced Book Classics)

F. Rohrlich

Addison-Wesley, 1990, 305pp.
ISBN 0 201 51501 6

Reviewed by László Fehér

This book was originally published in 1965 as part of the Addison-Wesley Series in Advanced Physics. It is quite a unique text on the fundamental-theoretical aspects of the classical theory of charged particles. The author pays special attention to the logical structure of the subject and to properly placing it in the net of the bordering physical theories, such as special and general relativity, classical and quantum mechanics and quantum electrodynamics. The student, or indeed the researcher, has much to gain from the lucid exposition of the general structure of physical theory offered in this book through an example. The historical and philosophical aspects are also exhibited as an integral part of the theory.

The book consists of nine chapters, the first three of which deal with the philosophical and historical aspects of its subject matter and with the foundations of classical mechanics. Chapter 4 gives a detailed exposition of the Maxwell-Lorentz field equations, their solutions and symmetry properties, which form the basis for treating the theory of electromagnetic radiation in the next chapter. The central part of the book is Chapter 6, which deals with the equation of motion of the charged, classical elementary particle, given by the Lorentz-Dirac equation together with the asymptotic conditions. The derivation of the equation of motion on the basis of the Maxwell-Lorentz equations and the