



in chapter 3 are deduced using the ideas (in particular a Dieudonné type inequality) of this section. The final chapter in this book examines iteration methods. This leads naturally to a discussion of intervals of stability for numerical solutions of ordinary differential equations. Hubbard and West finish with a very brief discussion of iteration in one complex dimension.

Overall this book provides an interesting and enlightening introduction to the theory of ordinary differential equations. However in teaching a course on this subject I feel that the book would be more suitable as a supplementary or reference text. The reason for this is that certain sections would have to be optional reading and therefore the main text would only cover about one third of a course. Hence another book would be required and this is far too costly to the student! The book is surprisingly free of typos; the few I did find are hardly worth mentioning.

Hubbard and West's book will be of interest to those who at some stage were influenced, motivated, frustrated or even baffled by the theory of differential equations. *Differential Equations: A Dynamical Systems Approach* will provide some of the answers. Moreover it will motivate one to explore more; it is certainly a credit and does credit to the elegant theory of differential equations. If the reader receives even half the pleasure this reviewer obtained from this book then he or she will have purchased wisely. The book is that good.

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## Book Review

### FUNCTIONAL ANALYSIS AND LINEAR OPERATOR THEORY

Carl L. DeVito

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Reviewed by A. Christofides

This is a book with a strong personal flavour and a clear sense of purpose. The author explains in the preface that it is based on courses he gave over the years, which were designed not only for students of mathematics, but also for advanced engineering and science students. As one reads, one soon realises that there is a central theme. This theme is the spectral theory of linear operators—the whole book is designed to lead rapidly to the description of this theory and to the formulation and proof of its main theorems. The mathematical prerequisites are, a sound knowledge of basic analysis and linear algebra and familiarity with the elements of general topology. With such equipment, progress will be swift, and one will soon be immersed in the main topic. Some of the material that one might perhaps expect in a general introduction to functional analysis is left out, while other topics, such as fixed point theorems, are treated parenthetically, as illustrations, rather than for their own sake. Naturally, a lot of important basic material is necessary in order to understand spectral theory and this is discussed both carefully and concisely.

A long first chapter covers all this introductory material: We are introduced to normed vector spaces, Hilbert spaces and, in particular, to  $L^2[a, b]$  and to  $l^2$ . Here there is no compromise with regard to precision, and the definition of  $L^2$  is preceded by a brief section on Lebesgue outer measure and Lebesgue measurable sets.

Most of the results on Lebesgue measure are stated without proof, but one or two simple key results (such as the fact that a countable set is measurable and has measure zero) are proved. There is even an attempt to motivate Carathéodory's mysterious definition of a measurable set and though the section does not seek to provide a full introduction to measure theory, it succeeds, I think, in demystifying the subject sufficiently to allow an uninitiated student to proceed, without feeling an attack of panic whenever it is subsequently mentioned. This chapter also contains a concise treatment of Fourier series of functions in  $L^2[a, b]$ , Féjer's theorem and Weierstrass' approximation theorem.

The second chapter introduces bounded linear operators and their norms. Shortly afterwards, the spectrum of a linear operator is defined and two interesting examples are carefully described: We encounter the shift operators on  $l^2$  and the multiplication operator on  $L^2[a, b]$ . The spectra of these operators are worked out in detail. By the end of the chapter the reader is aware that the spectrum of a bounded linear operator is a closed bounded set, has seen examples of spectra, and realises that a real number can be in the spectrum of a linear operator without being an eigenvalue. Here again, precision has not been sacrificed in any way, yet the flow of the narrative has only barely been interrupted—once in order to prove a necessary point about closed subsets of normed vector spaces and, another time, to state the open mapping theorem—without proof.

Subsequent chapters follow one another with perfect logic: We encounter first the Riesz theory of the spectrum of a compact operator on a Banach space, then the spectral theory of a compact Hermitian operator and that of a compact normal operator. Then we have the spectral theory of general bounded Hermitian operators and a chapter on unbounded operators. A final chapter, on  $L^2[\mathbb{R}]$  and the related problems of Fourier analysis on the real line, illustrates some of the results of previous chapters and provides interesting examples of linear operators.

Many interesting topics are encountered on the way. The more "geometric" chapters, which deal with the spectral theory of compact Hermitian operators on a Hilbert space, include a com-

prehensive review of the spectral theory in the finite-dimensional case. There is a good, clear account of the adjoint of a bounded linear operator on a Hilbert space, with a complete proof that the adjoint of a compact operator on a Hilbert space is compact. This, naturally, involves the weak topology on a Hilbert space, which, like everything in this book, is treated carefully, but with minimum fuss. The same can also be said of the complicated spectral theory of non-compact Hermitian operators. As has already been indicated, new concepts are often introduced with the help of a key example. One such example, the Hilbert-Schmidt operator on  $L^2[a, b]$ , first makes its appearance in the second chapter, and recurs throughout the book. The related topic of integral equations is repeatedly used to illustrate the theory. The lists of exercises, at the end of each section, are well chosen to complement the text and the occasional comments, concerning more advanced topics of the theory and some of the famous problems that have been preoccupying the experts, help to bring the subject to life.

Maybe a list of topics that are not covered might be of interest. None of the so called "main theorems", such as the Hahn-Banach theorem or the open mapping theorem, are proved, but those that are needed in the text are carefully stated and references are given, indicating where one can find a proof. There is no treatment of  $L^p$ -spaces, or  $l^p$ -spaces, for  $1 < p < \infty$  and  $p \neq 2$ . Also, the exercises and examples are deliberately and strictly mathematical. The author explains his attitude on this matter in the preface: "My students", he writes, "told me that they want to know that what they are learning has applications but they don't want to see the details. To do so would mean learning the concepts and terminology of the application's subject." He feels that students are not sufficiently interested in each others' disciplines to justify the inclusion of examples from non-mathematical areas. Be that as it may in the case of experimental scientists, I must confess that, as a pure mathematician, I find the application of subtle mathematical concepts to the physical sciences very exciting. However, here, as in other matters, the author had to be selective, and his choice of exercises is a perfectly valid one.

The introductory remarks at the beginning of each chapter are useful and illuminating. The proofs are very complete and none of the details necessary to understand a proof are omitted. There is however a certain clumsiness of presentation, which I shall come to in a moment. Another feature, that I have already mentioned, is the practice of introducing concepts early on and returning to them repeatedly. In the preface, the author refers to this as the "spiral approach". The following extract from the preface gives an interesting insight in to the author's pedagogical method and to the objectives that he has set himself:

"The style in writing mathematics for mathematics students is to say something once and only once and we train our students to be aware of this. This book, however, is written for students who are primarily interested in using mathematics. As important as mathematics is to their course of study, they forget material that hasn't been discussed for awhile and appreciate a brief review. So I do repeat myself and go over some topics more than once."

On the whole, this method seems to work quite well. Combined with the single-minded pursuit of a central theme, it adds to the cohesion of the book and provides a link between new material and old, so that, when a new topic is introduced, this often throws further light on already familiar concepts. The effect is also reassuring, like seeing a familiar face in strange surroundings. There is, however, a negative aspect to this approach. The mathematical practice of saying something "once and only once", for all its drawbacks, does have the effect of making the reader particularly alert to the introduction of new concepts. There is a tendency, in this book, for concepts to drift in quietly unannounced and, as with familiar faces, one sometimes finds it difficult to recall the circumstances of one's first encounter. The problem is heightened by the rather indifferent system of cross referencing, and this takes me to the book's most serious defect, which has to do with typesetting, presentation and typographical accuracy.

The first half of the book is literally peppered with typo-

graphical errors of all kinds. I have counted as many as thirty-four in the first five chapters. Most of them are perfectly harmless, but some, which occur in displayed formulae and in mathematical symbols, result in statements that are either untrue or meaningless as they stand. In an exercise on page 19, the reader is asked to prove that, in an inner product space, elements  $u$  and  $v$  satisfy  $\langle u, v \rangle = \|u\| \|v\|$  if and only if  $u$  and  $v$  are linearly independent. Further examples of confusing mistakes will be found on pages 10, 19, 21, 22, 23, 41, 69, 90, 105, 108, 138, 184. The number of misprints falls off in the second half of the book, but some still occur. There is also a certain clumsiness and lack of consistency in notation. We find vectors being denoted by the symbols  $u$  and  $v$  and also by  $\vec{u}$  and  $\vec{v}$  within the same section of a chapter, (Section 1.6). On page 42, a trigonometric polynomial is denoted by  $\varepsilon(x)$ , and this gives rise to the following rather strange formula:

$$\|f - \varepsilon\|_{\infty} < \varepsilon.$$

Again, expressions such as  $\|Tx - Tx_0\| < \varepsilon$ , or  $|\sigma(x, y)| \leq \|A\| \|x\| \|y\|$  are not easy on the eye, while some readers might find it a little unnerving to be asked to note, in the closing sentence of a proof, on page 77, that the notation "has changed slightly" and that " $\mu$  is now  $\frac{1}{\mu}$ ."

I mentioned that the author's "spiral method" can give rise to difficulties. Here is an example of what I mean. The following operator on  $L^2[a, b]$  is defined on page 92:

$$P(f) = tf(t).$$

Leaving aside the rather pedantic objection that the argument  $t$  appears only on one side of this equation, one reads on, to find a clear and useful discussion which, three pages later, concludes with the theorem that the spectrum of the multiplication operator on  $L^2[a, b]$  is  $[a, b]$  and that this operator has no eigenvalues. The fact that this multiplication operator is our friend  $P$ , is not made clear when  $P$  is first introduced. Instead, half-way through the discussion, the author begins to refer to  $P$  by name. The index, which on the whole is good, is of no use in



this instance, since it lists eight references to the multiplication operator, but not the one where the term is first encountered.

Life is also made harder than necessary by the absence of Q.E.D. signs, or some equivalent indication of where proofs end. Chapters are referred to as sections, making it difficult to distinguish between chapters and sections within a chapter, numbered equations and results are sometimes referred to by the wrong number, unnumbered equations by number, and various typographical styles of numbering are used.

Throughout the book one is struck by the contrast between the content, treatment, and organisation of material, which are excellent, and these shortcomings in presentation. One sometimes has the impression that, what we have here is an excellent set of notes, which were rather hastily brought out in book form. In spite of its drawbacks, this is a very good introduction to the spectral theory of linear operators and a new, more careful, edition is bound to be popular.

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## Book Review

### AN INTRODUCTION TO ALGEBRAIC TOPOLOGY (Graduate Texts in Mathematics 119)

Joseph J. Rotman

Springer-Verlag, 1988, 433pp.  
ISBN 0-387-96678-1

Reviewed by Graham Ellis

This is a well written, often chatty, introduction to algebraic topology which "goes beyond the definition of the Klein bottle, and yet is not a personal communication to J. H. C. Whitehead." Having read this book, a student would be well able to use J.F. Adams' *Algebraic Topology: A Student's Guide* to find direction for further study. The book begins with a sketch proof of the Brouwer fixed point theorem: if  $f: D^n \rightarrow D^n$  is continuous, then there is an  $x \in D^n$  such that  $f(x) = x$ . Functorial properties of homology groups imply that the sphere  $S^n$  is not a retract of the disc  $D^n$ , and then a simple argument by contradiction shows that  $f$  must have a fixed point. This illustrates the basic idea of studying topological spaces by assigning algebraic entities to them in a functorial way. There follows a rigorous account of the singular homology of a space which assumes only a modest knowledge of point-set topology and a familiarity with groups and rings. The account includes the Hurewicz map from the fundamental group to the first homology group, and ends with a proof of the Mayer-Vietoris sequence. By p. 110 a complete proof of Brouwer's theorem has been given. Singular homology is good for obtaining theoretical results, but not so good for computations. So simplicial homology is introduced in Chapter 7, and used to compute the homology groups of some simple spaces such as the torus and the real projective