

MATHEMATICAL EDUCATION

Mathematics at Third Level — How We Teach

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Courses combining mathematics, statistics and computing at the University of Ulster are described. Innovative methods of teaching, learning and assessment are discussed.

Introduction

This paper describes degree and sub-degree courses combining mathematics, statistics and computing at the University of Ulster and discusses the non-traditional approaches taken to teaching, learning and assessment. It has been written in response to a recent article in the Bulletin by Maurice O'Reilly [1] which questions how we teach mathematics at third level.

The University is required by its Charter to offer courses at degree and at sub-degree level. Hence we offer an honours degree, an ordinary degree and a Higher National Diploma (which is validated by the University and by the Business and Technician Education Council). These are linked courses which means that students enter the course most appropriate to their entrance qualifications (usually A levels, but about 10% offer the Irish Leaving Certificate) but can subsequently transfer between courses if their progress so dictates.

The suite of courses was designed only after an extensive market research exercise. All U.K. firms known to advertise regularly for graduates of combined degrees in mathematics, statistics and computing were contacted in an attempt to assess their needs as regards knowledge and skills, the desirability of a sandwich year, and the potential availability of sandwich placements. Detailed information was received from 100 of the 246 firms contacted.

Each course comprises applicable mathematics, statistics, computing and operational research, and aims principally to produce graduates (or diplo-

mates) prepared for a mathematical career in industry, commerce or the public sector. The topics studied are there because they are useful in the solution of practical problems. Students are encouraged to look as much at the problem and its context as at the mathematical methods they will use to solve it. The non-traditional methods of teaching, learning and assessment which the courses embrace include:

- (i) an emphasis on the development of enterprise,
- (ii) group project work,
- (iii) a unifying theme of mathematical modelling,
- (iv) an industrial placement year.

These are discussed below together with details of the course units in which they especially feature.

1 Enterprise

Business today calls increasingly for enterprising employees, that is, lively, resourceful, adaptable people able to recognise and exploit opportunities, to take risks and respond to challenges, to innovate, to communicate well, to work effectively either alone or as a member of a team, and to organise and motivate others.

This suite of courses aims to develop these personal and interpersonal skills through the various group project based units and in the case of the degrees through conventional final year individual projects and through sandwich placement. Industry and commerce clearly make a major contribution to this process during the placement year. From October 1990 financial support from the U.K. Enterprise in Higher Education Initiative will enable them also to make a contribution during the college-based years of the degree courses. Each year students will attend about a dozen seminars given by visiting industrialists. These will both address selected aspects of enterprise in business and serve to introduce particular firms to prospective placement students and prospective graduate employees.

2 Group Project Work

Group project work plays a key role in the following units of these courses.

- (i) A workshop unit, (HND Year 1).
- (ii) An introductory unit in mathematical modelling and models, (common to Year 2 of the HND and Year 1 of both degrees).
- (iii) A more advanced unit in mathematical modelling case studies, (common to Year 2 of both degrees).
- (iv) A final year group project unit, (HND Year 2).

The first three of these units share a common organisational feature as regards group project content. The class is initially divided into small groups, (four students per group seems to work best), and each group is assigned a project involving the solution of a practical real-life problem. Different groups are usually, but not necessarily, assigned different projects. The members of each group then have a few hours per week for a few weeks in which to investigate the background to the problem, analyse the problem systematically, find one or more solutions, and then present their results in the form of individually or jointly written reports to the supervisor and (usually) a seminar delivered jointly to the rest of the class. They are assessed on their technique, results and reporting. Marks are awarded typically for method and organisation, content, understanding, oral and written clarity and presentation, and initiative. The reports are expected to exhibit conclusions in a form comprehensible by non-specialists. The class is then reorganised into different groups of four and the process is repeated using a different set of projects. One or more further reorganisations and sets of group projects then follow.

A final written examination is involved only in the case of the introductory unit in mathematical modelling and models.

2.1 HND Workshop Unit

This unit comprises only group project work, albeit very closely supervised. It aims to integrate all components of the HND course, that is, analytical and numerical mathematics, statistics, computing and operational research. Computing plays an important role in every project and practical guidance

is given in the use of mainframe and microcomputers in the solution of real-life problems. However, only later will the students be introduced to the methodology of mathematical modelling, and so here the emphasis is on the systematic analysis of the problem rather than a detailed investigation of its background. Each student participates in several projects but only delivers one (joint) seminar. The audience for this usually includes a visiting industrialist.

2.2 Introductory Unit in Mathematical Modelling and Models

This unit looks at the philosophy and methodology of mathematical modelling and is taught by a single member of staff. It is essentially a unit about applying mathematics. After an introductory lecture on the modelling process each student takes part in several group projects designed to emphasise and give practical experience of this process. The problems posed can be tackled reasonably satisfactorily using the students' existing mathematical knowledge. Each student writes an individual report and jointly delivers a seminar on each project undertaken.

The "models" section of this unit is a study of some standard mathematical models such as Newtonian mechanics and population dynamics. Emphasis is placed on the development and understanding of the models. This section is assessed both by coursework and by a final written examination.

Students are encouraged to read mathematical articles and comprehend them to the extent that they can satisfactorily explain them to others and answer questions about them. To help achieve this objective an article on some aspect of applied mathematics is copied and given to the students in advance of a timed written comprehension test relating to it. An example of such a test is given by Houston [2]. (It is worth noting that in Northern Ireland a comprehension test also forms part of the assessment for a Further Mathematics A-level examination [3].)

2.3 Mathematical Modelling Case Studies

This unit aims further to develop interpersonal skills and expertise in mathematical modelling, and to integrate the various components of the degree courses. It consists almost entirely of group project work. The problems differ from those of the introductory modelling unit in length and difficulty, in

the breadth of skills needed to solve them, in the standard of reporting required, and in the extent to which the groups are expected to work without close supervision. By the end of this unit the students should be confident public speakers and ready within reason to make worthwhile contributions to their sandwich placements no matter what tasks they are given.

The unit commences with a short lecture course in communication skills. This includes, for example, instruction in the preparation of formal written reports and a detailed comparative study of well written and poorly written articles taken from mathematical research journals.

Each student then participates in three group projects in mathematical modelling. The first of these requires only first year mathematical methods, the second is a case study in applied statistics, and the final one is based on operational research and computer simulation. Different groups are always assigned different projects. Each problem is posed by a different member of staff who both plays the role of an industrial client and acts as supervisor and assessor. Most probably the case-study has been designed by him. He simulates problems of communication, (for example by feigning a total ignorance of mathematics), and probably withholds some essential facts initially. It is up to the group to seek further information from him as they think necessary. The group then has one full day per week for four weeks in which to solve the problem and produce a single written report. Throughout this period the supervisor monitors progress and intervenes with guidance if necessary but otherwise the group is left to its own devices. Each case-study involves about the same amount of work as a conventional final year degree project. This places fairly severe pressure on each group as regards time, and forces the four members properly to plan their schedule of work and share out the various tasks. After submitting their written report the group members deliver a joint seminar on the project during which they are subjected both to a viva-voce examination of content and to a critical appraisal of their oral and visual communication skills, the rest of the class being encouraged to contribute to both of these exercises. Finally the group members meet with the supervisor to receive detailed feedback on all aspects of their performance.

Two important questions have arisen in connection with the organisation of this unit.

- (i) How does one distribute the students between groups, (and later redistribute them twice), so that all groups have as nearly as possible the same overall academic ability?

- (ii) How does one design and implement a monitoring, assessment and marking scheme which is as objective as possible, which properly discriminates between different members of the same group, and which ensures uniformity of marking of case-studies of differing length and difficulty by different assessors?

Both these matters are the subject of ongoing research, the final results of which will be reported in a subsequent article. To date two quite different assessment schemes have been tried. Neither has proved entirely satisfactory and a third will be used in the academic year 1990-91.

2.4 HND Final Year Group Project

Final year HND students are unlikely to have fully developed enterprise skills. So rather than undertaking an individual final year project each student takes part in a group project. This lasts for the entire academic year, but in all other respects is similar to the group projects discussed above. The unit aims to ensure that HND students, particularly those without relevant work experience, are given the opportunity to learn how theory is applied in the solution of realistic large-scale problems.

3 Industrial Placement

Both degree courses are of four years duration. Years 1, 2 and 4 are spent on campus and Year 3 in industrial placement. The aims and advantages of sandwich placement as regards both students and employers are too well known to need reiterating here. Nevertheless, placement is still very much a non-traditional component as far as combined degrees in mathematics, statistics and computing are concerned. This is possibly due to three widely held beliefs.

- (i) Employers place no value on the sandwich element when hiring graduates of such degrees.
- (ii) Undergraduates of such courses are very hard to place.
- (iii) Those placements which are available involve experience only in computing.

Our experience at the University of Ulster seems to dispel these beliefs except for the case of employment in the finance and commerce sector.

Overall, a majority of the firms who responded to our market research survey placed great value on the sandwich component and were keen to consider students of degrees such as these for placement. But in the finance and commerce sector, (about 32% of the firms contacted and 33% those who responded), the opposite was true and a majority never accepted students for sandwich placement.

At the time of writing of this article our first cohort of 23 students are just completing their sandwich year. All were placed with relative ease and all have reported in glowing terms on the value of the experience obtained. Of these 23 placements: 4 (17%) were in finance and commerce, 8 (35%) involved extensive use of mathematics, 10 (43%) involved extensive use of statistics, and 7 (30%) involved extensive use of operational research. All 23 placements naturally involved some computing, but in only 6 cases (26%) was it the principal element.

4 Conclusion

In his paper O'Reilly [1] poses 22 questions. We hope that our experiences at the University of Ulster, described in this paper, will help him find answers.

References

- [1] Maurice O'Reilly, Mathematics at Third Level - Questioning How we Teach, Bulletin Irish Math Soc, No 22, (1989), 50-54.
- [2] S. K. Houston, Mathematical Comprehension, in M S Arora, F. Mina and A Rogerson, editors, Mathematical Education: The Present State of the Art, to appear (1990).
- [3] J. R. McCartney, A Comprehension Paper in A-level Mathematics, Teaching Mathematics and its Applications, 9, No 1 (1990), 6-14.

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Use of MACSYMA and MAPLE in Mathematics teaching in UCG

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Introduction

Several years ago the Honours Mathematics program in UCG began to undergo major reform. At the centre of the changes which our courses are still undergoing was the introduction of modern computing techniques, and in particular the use of symbolic computation software. There were a number of reasons for this reform; two in particular stand out — a desire to reverse a trend of falling numbers of good Mathematics students, and a process of reeducation among members of the Mathematics Department themselves.

The number of students taking honours degrees in Mathematics in UCG had been decreasing through the late '70's and early '80's. Departments like Electronic Engineering and Medicine were full of frustrated mathematicians, but it was not just the distorting effects of the points system that robbed us of students. There was — and still is — an image problem associated with Mathematics. This problem has several facets. On the one hand, most school leavers could picture themselves in the role of an Engineer or Accountant or Solicitor, but few could imagine themselves as a Mathematician. Also, the divorce between Mathematics and Computer Science meant that many Mathematics programs were frozen in outdated modes of content and presentation, lacking the vital interplay with computing which would have ensured growth and change.

The second factor was the changing attitudes of the members of the Mathematics Department. Of course, computers had been used, and taught, for the past 25 years or so, but "Pure" Mathematics had remained largely untouched. Then, over the course of a few years, people became acquainted with Pascal, Lisp, electronic mail, TeX, CAYLEY, REDUCE, MACSYMA, and so on. The