Report on the Participation of the Irish Team in the 31st International Mathematical Olympiad

Fergus Gaines

The International Mathematical Olympiad is an annual mathematics competition, the most prestigious of its kind, for pre-university students, and in 1990 it was held in Beijing, China. This was the third year that Ireland was involved in the competition, after Australia in 1988 and West Germany in 1989 and it was by far our most successful performance. The competition consists of two four-and-a-half hour examinations, with three problems to be solved in each examination. Each country taking part is invited to send a team of up to six students, each student competing as an individual. The benefits of the competition are many; it enables talented students to test their skill against the best people of their own age in the world; it stimulates an interest in problem-solving; it enables budding young mathematicians to make contacts with their peers in other countries, contacts that, in many cases, are maintained over the years; it reflects the trends in mathematical education worldwide.

Choosing the Team

Last November letters were sent out to most secondary and vocational schools in the catchment areas of the training centres in University College Dublin, University College Galway and Limerick University inviting them to send their three most mathematically talented students in the post Intermediate Certificate classes to the training sessions in the three centres. It was intended to have training sessions in University College Cork also but due to unforseen circumstances they did not take place. In the case of a number of schools mathematically talented students had already been identified from information supplied by the Department of Education, from the results of the Irish National Mathematics Contest, etc., and these were invited to take part. About three hundred students were involved in the initial stages.

Training sessions were held in the three venues — as well as discussing techniques of problem-solving it was also necessary to teach some areas of mathematics that are not covered in the school syllabus — number theory, aspects of combinatorics and aspects of Euclidean geometry. Because the work of training is purely voluntary it was necessary to have some elimination tests which, as well as whittling down the numbers, also helped to identify the most talented of the students. It was noticeable that students who had been involved in the training in previous years had a distinct advantage over newcomers. Even students who were eliminated at the earlier stages were exposed to (for them) new mathematics which will serve them well when they compete next time, as well as stimulating their interest in mathematics.

The Third Irish Mathematical Olympiad took place on 5 May 1990. The top scorers in this competition were, in order,

1. Aidan Hollinshead, Blackrock College, Dublin.

2. Cian Dorr, Ashton School, Cork.

3. David Galvin, St. Fintan's College, Ennis.

4. Andrew McMurray, The High School, Dublin.

5. Patrick Connolly, Blackrock College, Dublin.

6. Julian McCrae, The High School, Dublin.

7. Stephen McInerney, Gonzaga College, Dublin.

8. Eoghan Burns, Blackrock College, Dublin.

9. Adrian Colley, O'Connell School, Dublin.
   Ken Humborg, CBS, Roscommon.

10. Edmund Roche Kelly, Ard Scoil Rís, Limerick.
    Charles von Schmieder, The King's Hospital School, Dublin.

The first three candidates were invited to take part in the IMO in Beijing, with Fergus Gaines of UCD as leader of the team and Gordon Lessels of Limerick University as deputy leader. At this point there was only sufficient sponsorship money to fund a team of three. Subsequently, further sponsorship was obtained and the candidates in the fourth, fifth and sixth places were invited to join the team. All six candidates accepted the invitation.

As there were insufficient funds to run a final training session the Dublin based students came to three afternoon sessions in UCD and Gordon Lessels gave some further training to David Galvin in Limerick.

The Competition

The team, accompanied by the leader and deputy leader, left Dublin on Sunday 8 July, and arrived in Beijing on the morning of Monday 9 July, after a ten-hour flight from Frankfurt. On arrival they were met by representatives of the organisers and by Mr. Éamonn Robinson of the Irish Embassy. The students were then interviewed by Chinese television. The team leader, F. Gaines, was taken to a hostel on the outskirts of Beijing, while G. Lessels and the students were accomodated more centrally. The leaders from all 54 countries taking part formed the jury whose task it was to select the six final questions, from those shortlisted by the organisers from the questions submitted by the countries concerned. Two of the questions submitted by Ireland were shortlisted, but did not make it to the final six. The work of selection, rewording and translation into the various languages took a little over two days. The formal opening took place on the afternoon of Wednesday, 11 July.

The first examination took place on Thursday, 12 July and the second one on Friday the 13th. F. Gaines and G. Lessels marked each of the Irish students' work, according to the marking scheme laid down by the hosts. On the Saturday and Sunday they went to a team of Chinese "coordinators", a separate team for each question, to agree the final marks that each student should get.

From the comments of the leaders of the teams taking part it was felt that the competition was one of the more difficult IMO's and thus the relatively good performance of the Irtish team was all the more commendable. Andrew McMurray won a bronze medal, a magnificent achievement, as it was the first time Ireland had won a medal in the IMO. Cian Dorr was one point short of a bronze medal and this, in its own way, was quite remarkable as Cian was not involved in the training programme at all — he was specially invited to take part in the Irish Mathematical Olympiad on the basis of his getting first place in the Irish National Mathematics Competition. When the team scores were totted up the final (unofficial) placings, by country, were: first, China, second, the Soviet Union, third, USA, and ... fortieth, Ireland. Out of 308 competitors 23 won gold medals, 56 silver and 76 bronze.

The questions in the competition were graded by the organisers, "easy", "moderate" and "difficult". (These are purely relative terms!). Question 1 was judged to be "easy", but proved very difficult for many of the students. This, perhaps, reflects the modern trend away from Euclidean geometry. It was interesting to note that two of our students had exactly the same idea for doing this question: pick AB and CD to be perpendicular diameters, work out the required ratio in this case, and this gives the correct answer! — this got 4 marks out of 7. Three of the students also had the same idea for doing Question 2 — they considered a maximal "bad" colouring. Question 5 proved our most successful question, as it did for most countries, and Question 4 our worst. By far the hardest question on the exam was Question 6. Thus, for example, one of the Chinese students who has a perfect score (7) on each of the first five questions, could only manage to get one point on Question 6. Thus Cian Dorr's score of 5 on this question was all the more commendable.

Acknowledgements

The Trainers

The mathematicians who helped with training were:

In UCD:
F. J. Gaines, T. J. Laffey, M. Ó Searcóid, R. M. Timoney (TCD).

In UCG:
G. Ellis, J. J. Ward.

In Limerick:
Mark Burke (University of Limerick), Gerard Enright (Mary Immaculate College), Alan Hegarty (University of Limerick), John Kinsella (University of Limerick), Jim Leahy (Thomond College), Gordon Lessels (University of Limerick), Marian Morrin (Ballynanty National School), Eamonn Murphy (University of Limerick), Pat O'Sullivan (Mary Immaculate College).

The Chairman of the Irish Participation Committee is Mr. C. C. Ó Caoimh of the Department of Education.

The Results

Each question scores 7 points and thus the maximum possible score is 42. Only four students in the whole competition scored 42. The top Irish scores were Andrew McMurray, 16; Cian Dorr, 15 and Julian McCrae, 13; The team scored a total of 65 points, which was almost double the previous year's score.

To put this in perspective, China scored 230, the USSR 193 and the USA 177 to get first, second and third places respectively.

The Problems

FIRST DAY
Beijing, July 12, 1990

1. Two chords AB, CD of a circle intersect at a point E inside the circle. Let M be an interior point of the segment EB. The tangent line at E to the circle through D, E, M intersects the lines BC, AC at F, G respectively. If $\frac{AM}{AB} = t$, find $\frac{EG}{EF}$ in terms of t.

2. Let $n \geq 3$ and consider a set E of $2n - 1$ distinct points on a circle. Suppose that exactly k of these points are to be coloured black. Such a colouring is "good" if there is at least one pair of black points such that the interior of one of the arcs between them contains exactly n points from E. Find the smallest value of k so that every such colouring of k points of E is good.

3. Determine all integers $n > 1$ such that $\frac{2^n + 1}{n^2}$ is an integer.

TIME: 4.5 Hours
Each problem is worth 7 points.

SECOND DAY
Beijing, July 13, 1990

4. Let $\mathbf{Q}^+$ be the set of positive rational numbers. Construct a function $f : \mathbf{Q}^+ \to \mathbf{Q}^+$ such that $f(xf(y)) = \frac{f(x)}{y}$ for all x, y in $\mathbf{Q}^+$.

5. Given an initial integer $n_0 > 1$, two players A and B choose integers n_1, n_2, $n_3, \ldots$ alternately according to the following rules.

   Knowing n_{2k}, A chooses any integer n_{2k+1} such that $n_{2k} \leq n_{2k+1} \leq n_{2k}^2$. Knowing n_{2k+1}, B chooses any integer n_{2k+2} such that $\frac{n_{2k+1}}{n_{2k+2}}$ is a positive power of a prime.

   Player A wins the game by choosing the number 1990, player B wins by choosing the number 1.

   For which n_0 does

   (a) A have a winning strategy,

   (b) B have a winning strategy,

   (c) neither player have a winning strategy?

6. Prove that there exists a convex 1990-gon with the following two properties:

   (a) all angles are equal,

(b) the lengths of the sides are the numbers

$$1^2, 2^2, 3^2, \ldots, 1989^2, 1990^2$$

in some order.

TIME: 4.5 Hours
Each problem is worth 7 points.

Subscription, IMTA Reciprocity

New subscription rates for IMS members come into effect in from 1990/91 (the standard rate is now £10).

In addition, the reciprocity agreement with the Irish Mathematics teachers Association has been renegotiated. The new agreement provides for members of each Society to join the other at half price. The previous system where IMS members paid their reduced IMTA subscription through the IMS treasurer (and vice-versa for IMTA members) has been discontinued, and subscriptions will be now be paid directly to each society.

CONFERENCES

1991 September Meeting

It will be held at University College Galway on September 5th and 6th. The details of the programme are not yet fully settled.

Operator Theory and Operator Algebras

The fifth international conference in the Cork series will be held at University College Cork from Wednesday May 15 to Friday May 17, 1991. The principal speaker will be Professor R.G. Douglas (SUNY, Stonybrook) and his title will be *Operator theory and algebnaic geometry*. Further information from G.J. Murphy, Mathematics Dept., UCC.

European Mathematics Congress

The European Mathematical Society (EMS) was recently inaugurated at a meeting in Warsaw, and the Irish Mathematical Society will become a corporate member. Individual members of the IMS will have the possibility of becoming EMS members through the IMS, but detailed arrangements have not yet been made for this. B. Goldsmith is the Society's representative for EMS business.

The EMS is organising a congress in Paris in 1992, probably July 6–10, 1992.

13th IMACS World Congress on Computation and Applied Mathematics

It will be held at Trinity College, Dublin July 22–26, 1991. Further details from JJH Miller, IMACS '91, 26 Temple Lane, Dublin 2.