

## BOOK REVIEWS

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STATISTICAL INFERENCE FOR SPATIAL PROCESSES  
by Brian D. Ripley. Cambridge University Press, 1988, viii + 148pp. \$34.50

In their quest to isolate signal from noise, statisticians know that a good statistical procedure depends not only on the data but also on assumptions they make to account for information they have concerning the mechanism generating the data. Depending on these assumptions, they then adopt a purely data-analytic angle, or attack the problem from a classical, Bayesian, robust, non-parametric, or other inferential approach. The strength of the results obtained will depend on the validity of the approach used and on the criterion of performance adopted. Because of special peculiarities associated with spatial processes, the above standard scientific process leads to severe difficulties in the analysis of spatial data. This scholarly essay by Ripley is an excellent attempt to describe and accommodate these difficulties.

Chapter 1 explains a number of problems that arise in statistical inference for spatial processes, including the major difficulties caused by edge effects and long-range dependence. The inappropriateness of time series models for spatial processes is highlighted, along with certain inadequacies of procedures based on the likelihood function of the data. These latter inadequacies are explored in more detail in Chapter 2 where the author explores an autoregressive Gaussian process as model.

Moving away from Gaussian processes, Chapter 3 examines spatial point processes employing a geometric approach. The author examines the behaviour of certain statistics that are based on distances between points and which accommodate edge effects. The asymptotic results presented are useful for detecting departures of the model from the simple binomial or Poisson process, but their complexity and corresponding incompleteness force a comparison of various estimates only in terms of their first two moments. Continuing in a similar vein, Chapter 4 deals with Gibbsian models, which, as the author notes, have their origin in statistical physics and account for interaction between points. Here estimation procedures are approximate and an

interesting feature is the incorporation of Monte-Carlo methods for portions of the estimation.

Chapters 5 and 6 involve the highly important study of digital images. Chapter 5 accommodates background knowledge of the signal in the form of a prior distribution, and hence involves Bayesian inference procedures. On the other hand, Chapter 6 belongs more to the area of (exploratory) data analysis, the methods discussed being useful from a descriptive viewpoint, in addition to permitting one to suggest a suitable model and allow appraisal of the procedures of the previous chapter.

Because of the inadequacy of time series models and the limitations of standard likelihood methods, the reader may feel despondent upon reading the first two chapters. This however should not be the case, because the author clearly demonstrates that the problems are not insurmountable. True, the theory is somewhat inconclusive, but this reflects only on the difficulties involved, and there is no doubt that the author has achieved his objectives of describing the state-of-the-art and offering much food for thought. It is this reviewer's opinion that while asymptotic theories have in recent years been developed by probabilists for dependence structures as severe as those implied in 'strong mixing sequences', the state of probability theory is not yet at a level appropriate for deriving a totally definitive theory for spatial processes. For this and other reasons, the author and others in his area deserve high praise for their efforts. From a purely statistical viewpoint, I note that while asymptotic procedures are at the core of much of statistical inference theory, spatial processes beg the question: 'asymptotic in what?' As pointed out by the author, asymptotic results differ according to whether we fix the region  $E$  (within which observations will be taken) and let the sample size increase or, as essentially done by Mardia and Marshall (1984), let  $E$  expand.

This timely essay is a must for specialists in the area of spatial processes who desire to keep abreast of recent developments in the area. Theoretical statisticians too will welcome this well-written addition to the literature, for it abounds in open research problems. For example, there is plenty of room for the development of procedures that are robust against, e.g., misspecification of the functional form of certain models and against forms of dependence among the observations other than those forms accommodated in this essay. (Needless to say, the mathematics will be enormously involved in such projects.) A valuable graduate course for students could be based on this book along with Ripley (1981). One caveat to the reader: the writing style is lucid but, as befits an essay, highly terse, and the author wastes no time in re-

viewing background probability or statistics. For such a preparation, it is not necessary that the reader be familiar with theory and techniques at the level of Billingsley(1968) and Serfling(1980), but be aware that the author liberally sprinkles measure-theoretic concepts and non-elementary limiting techniques throughout the essay! An understanding of the ideas in Ripley(1981) is also highly desirable for appreciating the many elegant ideas in this outstanding essay.

## References

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- [2] Mardia, K.V. and Marshall, R.J.(1984). *Maximum Likelihood Estimation of Models for Residual Covariance in Spatial Regression*. *Biometrika* 71,135-46.
- [3] Ripley, B.D.(1981). *Spatial Statistics*. New York: John Wiley.
- [4] Serfling, R.J.(1980). *Approximation Theorems of Mathematical Statistics*. New York: John Wiley.

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STATISTICAL MECHANICS: RIGOROUS RESULTS  
 by David Ruelle, Addison-Wesley (1969,1989)

Addison-Wesley have reissued Ruelle's famous book as part of a new series *Advanced Book Classics*. This book is a landmark in modern statistical mechanics. The basic concept of the book is the use of functional analysis as a foundation for statistical mechanics, and this idea is behind much study in the past three decades. Not only have the techniques of functional analysis provided insight upon physical problems, but standard methods of statistical

mechanics form the underlying basis for the *theory of large deviations*, which is of considerable current interest to researchers in probability theory.

During the twenty years since this book was first issued, the problems it approaches have become clarified, and some have been solved, but most remain as an open challenge. Mathematicians with an interest in functional analysis may wish to have a go. This book is a good place to start, but they will not find the going easy: Ruelle packs a lot into 200 pages.

The most interesting aspects of statistical physics involve phase transitions. For the standard models, phase transition does not occur with finite systems, so one must start with infinite systems which are the limits of finite approximations. One such is a Newtonian system of infinitely many point particles, but the simpler model of an infinite lattice where each lattice point must be in one of a finite number (usually 2) of states is also studied. The continuous and lattice systems can be considered as classical or quantum. Thus there are several stages of increasing difficulty, from the classical lattice to the quantum continuous systems.

The first step is to deal with the limit of finite systems in such a way that for energy considerations the *boundary* of the finite approximation can be ignored. The assumptions on the strength of the interactions are those needed to make the limiting process work. It has since been discovered the a slightly more restricted family of interactions has much nicer properties concerning phase transitions. For continuous systems a rather special class of interactions is considered. The important case of the Coulomb interaction is not treated in this book.

One area in which reasonably satisfactory results obtain is that in which the interactions of the system are sufficiently weak. In this case one can prove that the behavior is quite close to that of non-interacting systems. For slightly stronger interactions, even in the classical lattice model, one has the presence of several phases in the sense that the infinite limit with different boundary conditions yields different states.

The case of stronger interactions is more interesting and more difficult. Limited progress has been made in this case. The book also deals with various probabilistic, group theoretic, algebraic and functional algebraic methods of treating statistical mechanical systems.

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