analysis course. Perhaps it could be used as the introductory part of a course on numerical methods for the more mathematically mature engineers, but I think it would give a poor foundation for further study in analysis, differentiable manifolds or topology.

The other two books under review are much more serious books from the point of view of the honours mathematics program. The excuse for this review is that Copson is now available in paperback, but I find the book alarmingly old-fashioned in its approach — so much so that it must have been old-fashioned even when it first appeared in 1968. The book is written for those whose education was based on the classic 'Pure Mathematics' by Hardy and the first 20 of its 143 pages are devoted to background information including sets, set notation, equivalence relations and functions. Most of this introductory section (except possibly for some material on the axioms for the real numbers and sequences) is inappropriate now. The definition of a function is introduced gradually by recalling the notion of conformal mapping! Worse than that we are subjected to a further section on *Functions defined on an abstract set* over half way through the book.

By contrast I find Sutherland's first chapter *Review of some real analysis* to be written in the lively style which persists throughout the book, even though the chapter does really just rehash things that belong in a prerequisite course on analysis. Sutherland's style is more relaxed than Copson's throughout. When Copson gets around to the definition of a metric space, there is a surprising feature. He gives the 'wrong' definition! Well, of course it is not actually wrong, but decidedly unusual. Left out are the requirements that $\rho(x,y) = \rho(y,x)$ and $\rho(x,y) \geq 0$ — these are deduced from a slightly contorted version of the triangle inequality. I dislike also Copson's approach to examples. He leaves the examples till a few sections after the definition and starts with the discrete metric. Perhaps this was due to the effect of Bourbaki (who might have started with the empty metric space?).

Both Copson and Sutherland treat the examples of ℓ^p spaces early on, but I think they are misguided in never really treating them as normed spaces. In fact both of these books assume quite a degree of maturity on the part of the reader and reach more or less equal sophistication — the Baire category theorem, solutions of differential equations via the contraction mapping principle and the Arzelá-Ascoli theorem are treated. An early section on normed spaces would fit in well.

Copson is slightly more complete in some respects, but the main difference in content is that Sutherland launches into general topological spaces more or less immediately after the definition of a metric space and the examples. Sutherland does use open sets and continuity as a springboard for general topology with the result that functions appear earlier than in Copson where completeness, connectedness and compactness are studied before functions appear. On balance, this is a reasonable place to contemplate topological spaces if one wants to do so in the course, but it may slightly affect those who only have time to cover basic metric space concepts.

What's lacking in these books? There is probably scope for more pictures. Copson has none at all, Sutherland has a few and Bryant has the most. None of the books considers algebraic topological ideas (like the fundamental group) although Sutherland shows us a trefoil knot as an example of a homeomorphic embedding of the circle in space and also deals with Möbius bands as quotient spaces of rectangles.

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AN INTRODUCTION TO HILBERT SPACE
by N. Young, Cambridge University Press.

In the very interesting "afterword" to "An introduction to Hilbert space", Nicholas Young quotes the following passage from G.H. Hardy's "A Mathematician's Apology":

> "I have never done anything useful. No discovery of mine has made, or is likely to make, directly or indirectly, for good or ill, the least difference to the amenity of the world."

This statement, which, as we shall see, is contradicted in this book, is interesting not so much for what it says about Hardy's attitude to mathematics; I am quoting it here at third hand and out of context. What is important about it, and other statements like it, is that they were interpreted in a particular way and had a profound influence on the teaching of mathematics in these islands. One of the consequences has been the traditional undergraduate textbook in pure mathematics which I call a grammar. The grammar launches without warning into a description of a given mathematical idiom, duly listing rules, declensions, exceptions and irregularities. It ends as abruptly as it has begun, leaving the unfortunate student with a strong feeling of kinship with the Scholars of Minoan Linear B who, having brilliantly deciphered that impenetrable script, found almost nothing to read in it.

The present book is in a very different tradition and is inspired by a totally different conception of the nature and role of mathematics. This is no grammar, but a literary work with a strong narrative line, inspired by two unifying themes. By the time one has reached the rather impressive conclusion, one is in no doubt that the subject has substance, that one is not dealing with an empty formal shell of theorems and corollaries but rather with a fascinating aspect of reality.

The book succeeds in giving a concise and lucid account of the elementary theory of Hilbert spaces. This is done very economically (the whole book is less than 240 pages long) and with modest technical equipment. For instance, Lebesgue integration is not assumed and, although it is mentioned from time to time, it is not essential to the understanding of the text. But there is much more. As already mentioned, there are two important unifying themes, that give the book a sense of purpose. Early on, at the end of a first chaper of only ten pages, we meet one of these, in the form of an inner product space of complex valued rational functions, analytic on the unit circle. We are shown an elegant connection between the inner product of two such functions and their poles in the unit disc.

For a while these functions drop out of the picture and the second theme is developed, in conjunction with the familiar material on orthonormal sets, Fourier series, functionals, duality and linear operators. This second theme is Sturm-Liouville systems and linear partial differential equations. Their description is interwoven with the general properties of Hilbert spaces and interest is kept high by the many examples, problems and exercises. Finally a very satisfying synthesis is achieved between orthonormal systems and solutions of second order partial differential equations.

The focus then returns to the first theme and the early example is expanded into the theory of Hardy spaces. In spite of Hardy's own passionate profession of uselessness, an interesting discursive chapter, an interlude in the author's own words, describes an application of these spaces to engineering problems of automatic control.

Multiplication operators are introduced and described clearly and economically, so that, within a matter of pages, one becomes familiar with Toeplitz and Hankel operators. The book closes with a series of very fine approximation theorems with a strong geometric flavour. We learn about best possible L^∞-approximation of complex valued functions bounded on the unit circle by fuctions analytic in the unit disc (Nehari's problem); and of rational complex valued functions bounded on the unit circle by functions meromorphic in the unit disc (Adamyan-Arov-Krein theorem).

I hope that this book will set a trend in mathematical text books. One of the abiding difficulties of introducing students to advanced mathematical topics is to find an exposition which is complete and at the same time does justice to the subject. This book is an excellent example of how this can be done.

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