Forward implication in (6) is now induction on the number k of distinct eigenvalues λ_j; for if $T: X \to X$ is diagonal on each of its invariant subspaces $S^{-1}(0)$ and $R^{-1}(0)$ then it is diagonal on their direct sum $S^{-1}(0) \oplus R^{-1}(0)$. On $S^{-1}(0)$ the operator T coincides with the scalar $\lambda_j I$; on $R^{-1}(0)$ T has only $k-1$ eigenvalues.

This theorem is not new, and can be found for example in Jacobson [4]. We believe our direct deduction from the Euclidean algorithm has some charm; the same argument gives, with no assumptions about T, the "primary decomposition"

$$X = \sum_{j=1}^{k} (T - \lambda_j I)^{-\nu_j}(0) = \bigoplus_{j=1}^{k} (T - \lambda_j I)^{-\nu_j}(0).$$

An alternative version of the argument, passing through the medium of "Taylor invertibility", is given by Gonzalez [1].

When an operator $T: X \to X$ is "reduced" in the sense of (5) then its eigenvectors can all be obtained without solving any more equations: with $S = T - \lambda_j I$ and R as in (12), the first part of (13) says that the eigenspace corresponding to λ_j is the range or "column space" of the matrix R built out of the remaining eigenvalues. Of course in practice it will usually be easier and pleasanter to solve the equations $Sx = 0$ than to compute the matrix R.

References

[1] M. Gonzalez, *Null spaces and ranges of polynomials of operators*, Pub. Math. U. Barcelona **32** (1988), 167–170.

[2] R.E. Harte, *Almost exactness in normed spaces*, Proc. Amer. Math. Soc. **100** (1987), 257–265.

[3] R.E. Harte, *Invertibility and singularity*, Marcel Dekker, New York, 1988.

[4] N. Jacobson, *Lectures in abstract algebra II*, Van Nostrand, New York, 1953.

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BOOK REVIEWS

METRIC SPACES: ITERATION AND APPLICATION
by Victor Bryant, Cambridge University Press (1985), STG£5.95 (paperback).

METRIC SPACES
by E.T. Copson, Cambridge Tracts in Mathematics number 57, Cambridge University Press (1968), STG£22.50 (hardback), STG£7.95 (paperback).

INTRODUCTION TO METRIC AND TOPOLOGICAL SPACES
by W.A. Sutherland, Oxford University Press (1981), STG£10.95 (paperback).

Of the three books, I like the one by Bryant the least. It claims, with some justification, to make the subject interesting. But the result is a book which might be more appropriately described as an introduction to iteration and fixed point theory that includes a little on metric spaces. To be somewhat objective, the book does touch on many of the basic concepts (limits of sequences; closed, complete, compact and connected sets). The applications include the existence and uniqueness of solutions for ordinary differential equations. But my basic objection is the second class treatment given to open sets and the less than enthusiastic treatment of continuity. On page 35, having introduced closed sets via limits of sequences, we are told that open sets are not really necessary because "all theorems about open sets can be stated in terms of closed sets". While this is undeniable, most textbooks do not take such an upside down view, and I do not consider that one can be said to have learned 'metric spaces' without being comfortable with the notion of open set. Who would like to volunteer to rewrite a standard text on multivariable analysis (never mind ones about complex analysis, functional analysis or elementary manifolds) mentioning only closed sets? The last chapter (marked optional) of Bryant's short book does make some amends by looking into continuity (even uniform continuity and the fact that the continuous image of a compact set is compact) and defining open sets.

This brings us to the question of what the book sets out to achieve. It claims to be intended for courses for engineering or 'combined honours' students, or really for those who have taken but not grasped a single variable analysis course. Perhaps it could be used as the introductory part of a course on numerical methods for the more mathematically mature engineers, but I think it would give a poor foundation for further study in analysis, differentiable manifolds or topology.

The other two books under review are much more serious books from the point of view of the honours mathematics program. The excuse for this review is that Copson is now available in paperback, but I find the book alarmingly old-fashioned in its approach — so much so that it must have been old-fashioned even when it first appeared in 1968. The book is written for those whose education was based on the classic 'Pure Mathematics' by Hardy and the first 20 of its 143 pages are devoted to background information including sets, set notation, equivalence relations and functions. Most of this introductory section (except possibly for some material on the axioms for the real numbers and sequences) is inappropriate now. The definition of a function is introduced gradually by recalling the notion of conformal mapping! Worse than that we are subjected to a further section on *Functions defined on an abstract set* over half way through the book.

By contrast I find Sutherland's first chapter *Review of some real analysis* to be written in the lively style which persists throughout the book, even though the chapter does really just rehash things that belong in a prerequisite course on analysis. Sutherland's style is more relaxed than Copson's throughout. When Copson gets around to the definition of a metric space, there is a surprising feature. He gives the 'wrong' definition! Well, of course it is not actually wrong, but decidedly unusual. Left out are the requirements that $\rho(x,y) = \rho(y,x)$ and $\rho(x,y) \geq 0$ — these are deduced from a slightly contorted version of the triangle inequality. I dislike also Copson's approach to examples. He leaves the examples till a few sections after the definition and starts with the discrete metric. Perhaps this was due to the effect of Bourbaki (who might have started with the empty metric space?).

Both Copson and Sutherland treat the examples of ℓ^p spaces early on, but I think they are misguided in never really treating them as normed spaces. In fact both of these books assume quite a degree of maturity on the part of the reader and reach more or less equal sophistication — the Baire category theorem, solutions of differential equations via the contraction mapping principle and the Arzelá-Ascoli theorem are treated. An early section on normed spaces would fit in well.

Copson is slightly more complete in some respects, but the main difference in content is that Sutherland launches into general topological spaces more

or less immediately after the definition of a metric space and the examples. Sutherland does use open sets and continuity as a springboard for general topology with the result that functions appear earlier than in Copson where completeness, connectedness and compactness are studied before functions appear. On balance, this is a reasonable place to contemplate topological spaces if one wants to do so in the course, but it may slightly affect those who only have time to cover basic metric space concepts.

What's lacking in these books? There is probably scope for more pictures. Copson has none at all, Sutherland has a few and Bryant has the most. None of the books considers algebraic topological ideas (like the fundamental group) although Sutherland shows us a trefoil knot as an example of a homeomorphic embedding of the circle in space and also deals with Mōbius bands as quotient spaces of rectangles.

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AN INTRODUCTION TO HILBERT SPACE
by N. Young, Cambridge University Press.

In the very interesting "afterword" to "An introduction to Hilbert space", Nicholas Young quotes the following passage from G.H. Hardy's "A Mathematician's Apology":

> "I have never done anything useful. No discovery of mine has made, or is likely to make, directly or indirectly, for good or ill, the least difference to the amenity of the world."

This statement, which, as we shall see, is contradicted in this book, is interesting not so much for what it says about Hardy's attitude to mathematics; I am quoting it here at third hand and out of context. What is important about it, and other statements like it, is that they were interpreted in a particular way and had a profound influence on the teaching of mathematics in these islands. One of the consequences has been the traditional undergraduate textbook in