MATHEMATICAL EDUCATION

Mathematics at Third Level
Questioning How We Teach

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This essay comes in four parts: context, focussing questions, philosophies and questions for exploration.

Contexts

What I have to say arises from three contexts: (a) my/our own personal experience, (b) the constraints of the educational system in which I/we operate, and (c) the last two meeings of the Society.

On the first, each of us has his/her own 'teaching CV'. I myself have been on the staff of Dundalk RTC since September 1981 teaching courses at National Certificate and National Diploma level in the Science, Business Studies and Engineering Schools. Latterly, my work has been with students of computing, science and marketing. The Leaving Certificate grades in mathematics of student entering Dundalk RTC may range from D in the lower course to a good honour (in the higher): an indication that students' formation, and perhaps their capabilities, in Mathematics can vary greatly.

The second context has to do with such issues as institutional goals, educational resources, physical space, class sizes, class contact hours and course objectives. All of these are characterized by elements of structure rather than experience.

The third context is one which has drawn attention to at least two important areas in the teaching of Mathematics: (i) the low numbers who choose to follow mathematics courses (perhaps indicative of a flaw in the popular

perception of the subject), and (ii) the challenge posed by the increasing use of computers in every domain of mathematics teaching.

It is my belief that when we talk about teaching mathematics, we need to go behond merely prescribing courses. We need to consider questions of *how* we teach whatever-it-is that we teach. It is perhaps remarkable that in, for example, Ralston & Young's interesting study [5] on the future of college mathematics, there is nothing to be found on this issue. For sure, the content of courses is important, but not to the absolute exclusion of considering how we spend our time in the classroom (I use 'classroom' to include lecture theatre!). And, after all the talk, then there's the doing ...

Focussing Questions

To focus attention on the issues, I invite the reader to reply to the following questions:

1. What do I teach?
2. Where is my attention when I teach?
3. What different modes (methods) of teaching do I use as a mathematics educator?
4. How much time during a typical scheduled class am I silent?
5. Under what circumstances do I have my most fruitful pedagogic insights?
6. How do I cope in teaching when under (severe) time pressure?
7. To what extent are students actively engaged in my classes? Is this the same for all my classes?
8. To what extent do students know *in advance* what to expect in my class? Not only in terms of content, but also in terms of process?
9. What do students appreciate in a good lecture?
10. What do I consider important other than *content* in my teaching?

It is tempting to ask for answers to be shared, but when it comes to *how* we teach, our 'academic objectivity' tends to wear thin and unhelpful comparisons between approaches may ensue! Academics are sometimes identified as possessing a certain arrogance (if discreetly expressed!), or at least conceit, about the importance and validity of their work. Perhaps we mathematicians have developed this conceit to a fine art, since it is our practice, not only to proclaim our truths, but also to *prove* them! This is all very appropriate in a mathematical context, but what happens when it slips into our teaching methods too?

Philosophies

Perhaps there are two areas of attention in learning mathematics: theory and practice. At first, students usually perceive theory as a body of knowledge to be taken down in their notes. It is important that these notes be coherent so that they can be consulted usefully at a later date. It is only after intelligent study, involving practice, review of theory, more practice, etc., that understanding grows.

In [7], Sheffield identified the most important aspect of lecturing as 'to stimulate students to become active learners in their own right'. This might be said of all teaching!

The key question an educator of mathematics (or indeed any subject) can ask is how can I provide the best range of opportunities for my students to learn?

There are various modes of teaching which provide different opportunities for students' learning. These may be characterised by the proportion of participation/control assumed by the lecturer as opposed to the student. The spectrum includes traditional lectures, facilitated group work, private study, etc.

It is often the case that the educator spends most of his/her teaching time operating in just one mode. Likewise, he student may remain in the rut of just one learning mode.

Much of the above question can be restated as: How can I ensure that I use the appropriate mode (and variety of modes) in a particular teaching circumstance?

In an environment where many (most?) weak students are struggling even to begin to grasp our subject, surely it is up to us to develop *process* in our mathematics teaching?

The 'traditional' process in mathematics teaching is linear with emphasis on content and is supported by an exclusively logical structure. To become mathematicians, we have thrived in such an environment, and now, as teachers, we perpetuate it. Yet we know from our *research* work that intuition plays a vital role in doing mathematics. Where is intuition in our teaching? Is Poincare's essay 'Mathematical Discovery' [4], better known among psychologists than among ourselves? Hadamard [2] has said that 'logic merely sanctions the conquests of the intuition'. (An update of Hadamard's work is found in Muir [3].) Why is mathematics so often presented in a state of over-rumination: chewed beyond flavour?

Our own research involves exploration: exploration which is rooted in experience. What is the analogy of this in our teaching? Let me put it this way: Suppose E_{ij} is the experience of student i in topic j. How can we benefit from $\bigcup_i E_{ij}$ rather than merely $\bigcap_i E_{ij}$?

Before offering some (more) open-ended questions, a few cautionary words. Our inertia in improving our teaching can operate in subtle ways. One such way is to allow oneself to be side-tracked into proclaiming the superiority of one's own methods over those of others. Another way is to dismiss a method glibly by insisting, often inaccurately, that one already applies a particular recommended method in one's teaching. It is hoped that the following questions may stimulate honest, self-critical and constructive exploration of how we teach methematics.

Questions for Exploration

1. What is he best question I can ask to motivate topic, T?
2. How can I best spend time with my students in subject, S?
3. When is it desirable to alert students to my teaching mode?
4. How can I encourage students to engage more actively in my classes?
5. How can I encourage students to learn mathematics *intuitively*?
6. How can I encourage a healthy dynamic between theory and practice?
7. How can I encourage a spirit of mathematical confidence and independence in my students?

8. If I have new ideas about teaching mathematics, how can I be sure that I succeed in implementing, evaluating and extending them?
9. How can assessments be designed in order to complement good teaching so as to promote further opportunities for learning?
10. Are problem-solving and theorem-proving the only relevant elements in assessment?
11. What contribution could 'academic councils' make to the development of effective teaching?
12. What structures are necessary in my institution to support and promote effective teaching?

References

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[3] A. Muir, *The psychology of mathematical creativity*, Mathematical Intelligencer, **10** No. 1 (1988).

[4] H. Poincare, *Mathematical discovery in science and method*, Nelson, 1914.

[5] A. Ralston & G.S. Young (Eds.), *The future of college mathematics*, Springer Verlag, New York, 1983.

[6] C. Rogers,*Freedom to learn for the 80's*, Charles E. Merril Pub. Co., Columbus, Ohio 1983.

[7] B. Samples, B. Hammond & B. McCarthy, *4Math and Science: Towards wholeness in Science education*, Excel Inc., 1985

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Linking Mathematics with Industrial Problems

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There is a growing interest in establishing links between University Mathematics Departments and industrial and commercial organizations in order to identify industrial problems amenable to mathematical analysis. There is a variety of reasons for this including:

1. the desire of mathematics faculty to contribute to the solution of real life problems;
2. the desirability of involving graduate students with such problems;
3. offering industry the opportunity to view the useful mathematical expertise of graduates with possible resultant job offers;
4. the desire by industry to create links with mathematics faculties to avail of faculty expertise and to aid in student recruitment for the company;
5. the desire by industry to avail of technical expertise in areas of shortage of such expertise in the company.

As a result, a number of Universities in different parts of the world have established such links. The type of link varies somewhat from place to place. Probably the oldest continuing link scheme (running for more than fifteen years) is operated at the Mathematics Institute, University of Oxford, U.K. where a one week study group is held annually involving Oxford faculty members (augmented by invitees from other Universities) graduate students and industrial participants to discuss and hopefully outline solution paths to industrial problems. With initial help from Oxford faculty a similar one week study group is now held annually at both Rensselaer Polytechnic Institute in the USA and at C.S.I.R.O. in Australia. A different type of process (also running for more than fifteen years) is operated at Claremont Colleges, California, USA, where the postgraduate education of mathematics students is through involvement with industrial problems funded by industry and identified by