Remarks It is interesting to contrast and compare MP, DP, CP and TP for a general invariant P. For example, if P is "connected", MP is true (since, as is well known, the maximal connected subspaces are the connected components), DP is false (since, as is well known, the closure of a connected subspace is connected), CP is false (since each connected subspace of a disconnected space, being contained in a component, is therefore disjoint from any other (closed) component) and TP is false (since each connected subspace of a locally connected disconnected space, being contained in a component, is therefore disjoint from any other (open and closed) component).

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HISTORY OF MATHEMATICS

The Culmination Of A Dublin Mathematical Tradition

On The Maxwellian Struggle For A New Mathematical Physics And The Birth Of Relativity

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This paper is to celebrate the centenary of the Hertz Exporimatum Crucis that proved the FitzGerald electromagnetic theory of radio transmission.

Fitzgerald And The Electromagnetic Description of Light Propagation.

FitzGerald's chosen field of study in Dublin University for his Fellowship examinations in the period 1871–1877 was MacCullagh's mathematical researches. This study perhaps uniquely prepared him to comprehend the full significance of James Clerk Maxwell's development of an electromagnetic theory of light in 1865, which had until 1879 remained largely ignored, except for a handful of "electricians" from outside of the establishment of science and engineering.

FitzGerald and his uncle George Johnstone Stoney in Dublin, were the first mathematicians from the established universities to see Maxwell's work as the departure point for a programme of mathematical researches that would provide a unifying theory for physics. If successful of course, such a unified theory would have also established Cambridge and Dublin at the unchallenged head of developments in British science. There was at the time a determined ideological challenge to the scientific leadership based on the mathematicians in

This article is an abridged version of a longer, fully referenced article. Copies of the latter can be obtained from the author.

the established universities by "the practical men of science", and this threat was particularly keenly felt by FitzGerald in Trinity's Engineering School [1]. The Dublin Cambridge alliance had been developing since 1827 and this link was strengthened by Stoney's work in the 1860s, before Maxwell moved to Cambridge, in applying his kinetic theory to optical, astronomical and thermodynamic problems.

FitzGerald realised, perhaps in 1876, that MacCullagh's mechanical equations were the key to the problems of electrodynamics and worked on those also of Cauchy, Neumann and Green. It was the great treatise of Maxwell which sparked FitzGerald's first significant work. FitzGerald realised that MacCullagh's mechanical equations could be transposed into the new electrodynamic form to provide a full theoretical description of the reflection and refraction of light at a boundary, leading as did MacCullagh's equations earlier, to Fresnel's law for both polarizations and to the MacCullagh's equations for the amplitude of the refracted and reflected rays on refraction.

FitzGerald in 1879 produced three important papers on "Electromagnetic Theory of the Reflection and Refraction of Light" published by the Scientific Transactions of the Royal Dublin Society. These papers were significant in a number of ways. FitzGerald here produced what were the first real applications of the new Maxwellian theory. The papers were subsequently rewritten for the Proceeding of both the Philosophical Transactions of the Royal Society and Part II in the Philosophical Transactions of the Royal Society. This work establish FitzGerald as an important mathematical physicist in Britain. Finally, it was Maxwell himself who refereed the papers.

It is worth noting that FitzGerald's papers did not really require any great new development of mathematical method, as he explained himself: "Following a slightly different line from his (Maxwell) I obtained the same results as to Wave propagation, reflection and refraction as to those obtained by MacCullagh" [2]. FitzGerald had however been forced to pay careful attention to the physical interpretation of the mathematical equations. He had clearly been helped in this respect by the heroic, but tragic, efforts of MacCullagh to find a physical interpretation for his mechanical equations that required a very unusual medium, the ether, through which the wave could propagate. Unfortunately for MacCullagh, this ether, if mechanical in its operation, had to be an elastic solid of a type physically unknown, in that it had a strain energy dependent on the rotation of the volume element, rather than that observed in solids, in which the strain energy is dependent upon the deformation of the volume element. MacCullagh had introduced into the mathematical description of light the curl, by in essence, solving the problem of the form of the equations for light in a medium. For the new electrodynamic theory such physical requirements could be explained without contradiction, and perhaps it was this contradiction which led to MacCullagh's suicide.

FitzGerald's methods in this work were essentially mathematical and he inclined in any case strongly to the view at this time that a description of optical and electrical phenomena in terms of an elastic solid ether was inconsistent with physical requirements, since this would need to be soft enough to allow the free motion of planets yet be "The means by which tramcars are driven by shearing stresses" [3]. On the other hand the ether could not be "as thin as jelly "as it is the possession of properties analogous to rigidity that require explanation" [4]. Significantly, it was at this time when FitzGerald was unfettered with any of the burden of model building that he made his greatest mathematical advances, which began with this work of subsuming MacCullagh into the body of Maxwellian theory, while he was in revolt against "the thraldom of the material ether" [5]. He never was however able to completely break with the mechanical notions.

The Maxwellian Programme in Dublin

For the Maxwellians to establish themselves at the leadership of world science, first and foremost they required a consistent mathematical theory, which could address any practical problems in electrodynamics or optics. They also sought to extend their domain into the whole body of physics and chemistry, and in particular in this respect, thermodynamics. It is essential to understand these points, if the Dublin mathematicians concern with ether modelling is to be fully appreciated. It is also necessary to see that Stoney and FitzGerald in the 1880s inaugurated an entire programme of research and educational reform.

Stoney's mathematical researches really prepared the ground for the development of the Maxwellian programme in Dublin and he was a major influence on his younger relative FitzGerald. Stoney's early researches integrated into the Dublin programme, once this took final shape after 1879. Stoney had begun in 1861 a geometrical study of the examination of the conditions of propagation of undulations of planewaves in media. This interest in producing MacCullagh type geometrical procedures to generalize the treatment of optics, continued through his long and active life, with particularly important developments of these methods as late as 1897. Stoney's objective here was incredibly ambitious, since he sought to produce an analytical method for optics by which he resolved any wave front into flat wavelets, and thereby to do for optics what the calculus had done for geometry. In his last really major paper in 1896, Stoney extended his methods by resolving the wavefronts into spherical wavelets, to demonstrate the mechanism of image formation by interference.

The mathematical techniques evolved by Stoney in his optical researches, were subsequently applied to his major work on spectroscopy. His other major interest of thermodynamics began by his application of Maxwell's estimate of mean free path of a molecule, to obtain an estimate of the number of molecules present in a unit volume, and then in 1858 he demonstrated that inherent in Boyle's law was a model of a gas as an assembly of particles in constant motion and that such a gas could not be a continuous homogeneous substance. A decade later Stoney demonstrated that the motion of the gas molecules was related to the emitted radiation wavelengths using a comparative study of the relative magnitudes of the mechanical and optical components of the kinetic model. He then applied the kinetic theory to the interpretation of emissions from the sun and stars. and also to the atmosphere of planets and satellites. These researches aimed to integrate optical and thermodynamic principles, the essential foundation for a unifying mechanistic ether theory.

Stoney and FitzGerald both saw the discovery of the Crooke's force as a vital experimental test of theories linking radiation and kinetic theories. Stoney developed a kinetic-radiation theory of Crooke's radiometer in 1876 and two years later FitzGerald improved the mechanical theory of Crooke's Force. In these researches Stoney showed there was a distinction between the translatory motion of the molecule, that determines its temperature, and other internal motions within the molecule to "occasion the spectral lines". Stoney's "molecular" model of 1868, was of course non-electromagnetic and entirely mechanical, with the action resulting from mechanical vibrations of molecules producing a series of waves in the ether. Stoney extended this model in 1871 to propose an explanation for the harmonic sequences of spectral lines. Stoney subsequently applied his theory to hydrogen molecules, to obtain good theoretical match with Angstrom's measurements.

Both Stoney and Fitzgerald carried out an impressive range of thermodynamic researches, which are outside the scope of this discussion, except to point out that they related in the most part to attempts to establish the relation of kinetic principles to the spectroscopic and other optical and electromagnetic phenomena such as Fluorescence.

The reforming Maxwellians of course required to present a rationalized new view of science and there is considerable evidence that Stoney, and subsequently FitzGerald with Trouton, spent considerable efforts in attempts to introduce radical reforms in scientific terminology and concepts. Stoney was a member of the BAAS 1862 Committee to consider the standard of resistance, and this committee eventually sat for fifty years with Stoney being a member for all but the last ten. Almost from the outset in 1863 Stoney took a lead with radical proposals suggesting a qualitative vocabulary for dimensional analysis of lengthine, massine, timine, forcine, velocitine and so forth, and very significantly for charge, electrine. In 1873, when the BAAS committee recommended a complete new system of units, Stoney submitted a minority report. Stoney proposed a totally radical unified electrostatic and electromagnetic system of units. This proposed system was based upon fundamental quantities in nature. He consequently proposed for the unit of charge, the "atomic" charge of the electron, which Stoney named and was the first to obtain a value for this fundamental charge. He brilliantly derived the value of charge from Faraday's Law of electrolysis. Stoney's system aimed to remove the necessity of establishing connecting co-efficients between quantities in a system of units.

Trouton and FitzGerald carried out studies on Ohm's Law in Electrolysis between 1886 and 1888 for the BAAS and this work made the distinction between ionization of solutions, in which electrical seperation of charges occurs completely, and ordinary dissociation, in which it does not. It was in this work FitzGerald introduced the term *ionization*. FitzGerald's notebooks are indeed quite full with his notes on ideas for systems of units and nomenclature. His grasp of these questions led to FitzGerald's suggestion one month after J.J.Thomson's discovery of the Cathode ray particles, or corpuscles as he termed these, that the cathode particle was in fact a free electron. Furthermore he proposed the very useful, but unused term *electronization*, for the process of molecular decomposition involved in the formation of cathode streams.

The use of the term electron, itself inexplicably had only been adopted by the Maxwellians in 1894, following the growing collaboration of FitzGerald, Lodge, Heaviside and Larmor, in which they employed rather indiscriminately the term "ion" in their correspondence. This situation changed from 19th July 1894 when FitzGerald wrote to Larmor, "Johnstone Stoney was here just now and he will send a copy of his paper on the double line and c, Stoney was rather horrified at calling these ionic charges 'ions'. He or somebody called them 'electrons' and the ions is the atom and not the electric charge".

The discovery and naming of the electron [6], was according to Joly [7] the most important service Stoney rendered science. Stoney had another claim to fame, which is most appropriate for the man whose vision presaged the modern SI system. Stoney introduced the term "oscillator frequencies" in his pioneering work on spectroscopy [8], which were the reciprocal wavelength and became known in due course as wavenumbers.

FitzGerald's interest in mechanical models no doubt can be originally traced to Stoney's passion for molecular and atomic theory. FitzGerald, certainly did his most important work from 1879 using only mathematical paradigms. FitzGerald's soft quarto notebook contains work on ether modelling and it appears such models assumed some importance in his thinking from 1881. This notebook is undated until the 1887 Honours Lectures, but the first notes in the volume concern calculations on the electromagnetic action of charging spheres. This would date his first serious studies in ether modelling about 1881, and demonstrate that in this book, these attempts continued in a substantial way up until 1887(?), by which time he had converted to the liquid vortex-sponge ether model, which had first been proposed by Kelvin in 1867.

From about 1884 FitzGerald's mind was crystallizing with respect to his prefered model of the ether, he knew that he was against a solid ether, the jelly ether, and the stagnant ether, which was wholely unable to account for electromagnetic phenomena. He had been impressed by J.J. Thomson's 1883 study which had shown that the simple vortex theory predicted that the enertia of atoms ought to increase, and their velocity increase, as their temperature rises, a fact at variance with observations. FitzGerald was to declare that "to suppose atoms to be simply ring vortices in a perfect liquid can hardly be an adequate theory". He was inclined towards a moving liquid model , and in 1885 he published his well known "bands-and-wheels" model of the ether. FitzGerald's rectangular array of spinning wheels on vertical axes connected to their neighbours by rubber bands around their rims in his ether model, did impressively mimic Maxwell's electromagnetic field. Light propagation was demonstrated by an impulsive angular displacement given to a wheel which caused the disturbance to move out from this centre as a wave of tightened stretched rubber bands. The purpose of this model, was explained in an unpublished paper , but its main purpose was to show how Maxwell's "electric displacement" could in fact be represented by a change of structure (tightening bands) and thus to demonstrate that Maxwell's theory was not "unmechanical". This point was of particular importance to FitzGerald, who despite his philosophical idealism was in physics a convinced materialist. From 1885 FitzGerald became increasingly convinced of the truth and reality of the

vortex-sponge model, which he believed was a "likeness" of the ether and not simply an analogue like his model. For FitzGerald ether modelling was a search for the ultimate questions of science for he declared, "with the innumerable possibilities of fluid motion it seems almost impossible but that an explanation of the properties of the universe will be found in this conception" [9].

The ultimate development of the ether model produced by FitzGerald was perhaps that given in his letter to Heaviside on the 23 August 1893: "If the ether is ultimately an incompressible liquid, and I can't conceive any simpler hypothesis, there must be actions between different things (whirls, vortices, what not) in it like the action between two vortex rings in a perfect liquid. Effects propagated but immediately presents each vortex ring in itself really infinite, each atom of matter is infinite, the most probable. Thus gravity." This model was really close to Hamilton's Boscovichean atomic model [1] and had developed a considerable distance from those FitzGerald had inherited from MacCullagh.

The most important Dublin atomic model was the famous Stoney Atomic Model, which he proposed in his classic paper *An Analysis of the Spectrum of Sodium* in 1891. This was an energy model of the atom, in which the accelerated motions of the electrons in the atom or molecule, was resolved by Fourier's theorem and resulted in the splitting of the spectroscopic lines into doublets and triplets. This was a Maxwellian electromagnetic atomic model. The idea of the model that the electron from its own elliptical (apsidal) motion would produce emissions with split states. These ideas are very close to the modern concept of two "spin" states of the electron, while the idea of apsidal motion was of course later taken up by Sommerfeld and the Irishman Orr. Also, it may be noted, Larmor later developed his concept of precession from his development of Stoney's ideas.

There were other aspects of the Maxwellian programme in Dublin which deserve a mention. FitzGerald in his central position in Irish science at Trinity, was able to establish a dominant influence in an Irish Astronomical Network [11], which carried out very important observational, photographic and photometric astronomical measurements. FitzGerald also inspired and supervised the production of two classic Maxwellian textbooks , by his former student Thomas Preston *On Light* and *On Heat*. Stoney inspired the first textbook *On the Electron Theory*, and a number of others such as Minchin in the Dublin circle produced impressive mathematical reform texts. The exposition of the electromagnetic paradigms was vital contribution of the Dublin Maxwellian programme.

The Dawn Of The Theory Of Relativity

In the years 1881 to his death in 1901, FitzGerald's fertile mind was working almost incessantly on the conundrum of the ether, which of course as we now know with the benifit of hindsight, was a struggle to establish a relativistic basis for physics. The ideas which dominated his thinking were profound and most contradictory to common sense, and classical physics. FitzGerald's inner mental struggle disrupted his sleep and he was tormented by insomnia. He appeared to the contemporary scientific community as an electromagnetic crank, but he was a prophet of a new but only partially crystallized world view.

In the light of this situation, and the fact that FitzGerald started to teach the contraction hypothesis in Dublin University about 1881, it was quite significant that he composed a verse for the BAAS Meeting that year on J.J. Thomson which ends on very personal note, "Feels that fools be but am willing to play the part".

It was at these yearly meetings in particular that Fitzgerald found a forum to sound his bizarre ideas. Larmor was to explain, "he was the life and soul of the debate, he was always ready with some semi-paradoxical but wholly suggestive idea" [11]. The most outlandish of these ideas was the contraction hypothesis.

FitzGerald's electromagnetic departures, which led to his great discovery had began with the reworking of the MacCullagh equations, that contained an inversive geometrical analysis of space. This work was done using Hamiltonian algebra that itself incorporated a vectorial view of space and time. Trinity also produced during the second half of the nineteenth century, a number of important geometers, but in particular George Salmon's researches were of importance to the theories of FitzGerald. Salmon worked principally on the theory of invariants and covariants of algebraic forms to the geometry of curves and sufaces, and in this research collaborated with the "invariant twins" Cayley and Sylvester in Cambridge.

Hamilton had produced a non-commutative algebraic description of space involving a time dimension. The formal break with Euclidean geometry was however left to Riemann to achieve in 1854. In 1868 Plücker published *A New Geometry of Space*, which took up Hamilton's idea of space being defined by a set of lines, "a cosmic haystack of infinitely thin, infinitely long straws". Later Lie and Klein extended and unified these two mathematical developments and showed that Euclidean space is related to four dimensional space by transformational groups.

FitzGerald was attempting to produce an ether theory which would unify these geometric theories and electromagnetism. Following Stoney's discovery of the electron this modelling centred on the electron theory. With the discovery by spectroscopists of intra-atomic processes, FitzGerald had to refine his conceptions as he explained most candidly to his collaborator Heaviside in 1889, "I admire from a distance those who contain themselves till they worked to the bottom of their results but as I am not in the very least sensitive to having made mistakes I rush out with all sorts of crude notions in the hope that they may set others thinking and lead to some advance" [14].

The epoch making breakthrough in thought had been made by FitzGerald's introduction of the first *relativistic principle*, but he failed to identify this as the key to progress on the exposition of space-matter theories in physics. This failure caused him to historically lose the credit for his great discovery and this misfortune was compounded by some very harsh quirks of fate, as will be explained below.

FitzGerald's involvement with the Dutchman Heinrich Antoun Lorentz (1853–1928) arose from his first major paper *The Electromagnetic Theory of the Reflection and Refraction of Light*. In the review of this paper by Maxwell for the Philosophical Transactions the great man noted that the paper related to the work of Lorentz. It was in 1882 that FitzGerald first published a paper *On Electromagnetic Effects due to the Motion of the Earth*, in the Transactions of the R.D.S., and showed that he had at this early date developed a theoretical Maxwellian based notion of this important question, which had a central bearing on the famous Michelson-Morley experiment (1887).

It seems that the later development of Relativity has distorted the perception of the contemporary importance of this experiment, but FitzGerald certainly believed, as has been fully explained in the ether model discussion, that "gross matter" was held together by electrical forces and was to be explained by ether theories. FitzGerald had discussed this hypothesis with Lodge in 1892 and his tentative claim to priority has recently been dramatically borne out [15], and his priority established by the discovery of his lost publication in the journal Science. The FitzGerald-Lorentz correspondence which is preserved in the Lorentz collection at the Algemeen Rijksarchief, The Hague, reveals the events which unfolded in 1894. Lorentz wrote on November 10th, 1894 to FitzGerald mentioning Lodge's comments in Aberration problems of FitzGerald's hypothesis on the negative result of Mr. Michelson's experiment. Lorentz sent him a number of the proceedings of the Dutch Academy of Sciences (1892), in which he considered the subject of aberration on the basis of his development of the theory of the refraction of light and he asked, "you would oblige me very much by telling me if you have published your hypothesis. I have been unable to find it, and yet I should want to refer the reader to it". FitzGerald's reply dated 14th November 1894, is centrally important and will be quoted in full,

> My dear sir,
>
> I have been for years preaching and lecturing on the doctrine that Michelson's experiment proves, and is one of the only ways of proving, that the length of a body depends on how it is moving through the ether. A couple of years after Michelson's results were published, as well as I recollect, I wrote a letter to "Science" the American paper that has recently become defunct, explaining my view, but I do not know whether they ever published it, for I did not see the journal for some time afterwards. I am pretty sure that your publication is then prior to any of my printed publications for I have looked up several places where I thought I might have mentioned it but cannot find that I did. I certainly never wrote any special article about it as I ought to have done for the information of others besides my students here. I am particularly delighted to hear that you agree with me, for I have been rather laughed at for my view over here. I could not even persuade my own pupil Mr. Preston to introduce this criticism into his book on Light published in 1890 although I pressed upon him to do so and it was only after reiterated positiveness that I induced Dr. Lodge to mention it in his paper; but now that I hear you as an advocate and authority I shall begin to jeer at others for holding any other view. Thank you very much for your papers. I can make out their general drift and wish I were able to reciprocate by replying to you in Dutch.
>
> Yours most sincerely,
> Geo. Fran. FitzGerald.

Lorentz generously gave FitzGerald credit for independently establishing the hypothesis in his 1895 paper, and it was from this that Einstein developed the terminology "Lorentz-FitzGerald contraction", which was the only reason that the Irishman's name became associated with this relativistic hypothesis. The 1889 letter to Science, however, properly establishes FitzGerald's priority

as this sets forward clearly this hypothesis and provides a physical meaning of the Michelson-Morley experiment. Lorentz's contribution to this contraction idea however is real and significant, because he set out to discover the conditions that the Maxwellian laws of Electromagnetism should be invariant, that is, have the same form in a moving and stationary frame to use the modern relativistic concept, and it was he therefore, and not FitzGerald, who generalized Hamilton's algebra of time and set Einstein on his path of discovery.

In 1881 J.J. Thomson began a new departure in theoretical electrodynamics in his investigation which aimed to determine directly the effects produced by moving charged bodies, by means of Maxwell's equations of the electric field combined with the appropriate conditions at the surface. The theory probed the behaviour of a charged body as to whether it carried along with it an electric field. An important result of the paper was that the magnetic field thus produced, possessed a kinetic energy which carried along by an electron involves an addition to its effective mass. In his paper on J.J. Thomson's experiment FitzGerald pointed out that the analysis offered did not give the correct magnetic force and that Thomson had added a term to make a vector potential circuital and, "thus on closer examination, each portion of the electric charge is found to act independently; and so far from being able to exclude the electric charges from view by merging them in interfacial conditions, it turns out that their convection is the sole cause of phenomena of the electric field. When considered for the point of view of the aether, the hypothesis is that the total current is circuital, which lies at the very root of Maxwell's theory, involves and is equivalent to the magnetic influence of moving charges, which was verified experimentally before this time by Rowlands, though doubt still occasionally arises on the part of unsuccessful experimenters" [16,17].

This statement is extremely important in understanding the relationship between FitzGerald's work and that of Stoney. It is also the basis of Larmor's claim that MacCullagh originated electronic theories. MacCullagh worked back from the experimental description of "crystalline reflexion" by Brewster and Seebeck to discover his function of the Lagrangian type, but he was unable to conceive of any kind of material elastic medium, as ordinarily understood, whose properties were represented by his equations, because internal stress forces could not produce the unbalanced torque his treatment demanded, and "that a mechanical theory of this type can subsist only if there are polar forces (e.g. quasi-magnetic) capable of compensating the torque, or if there is a kinetic reaction torque arising from a distribution of gyrostatic rotations forming a part of the constitution of the medium" [17].

FitzGerald did not for a long period do any further work on this question, as he was occupied in other related studies of the generation and detection of electromagnetic radiation. In 1893 he produced a significant paper *On the Period of Vibration of Disturbances of Electrification of the Earth.* Then in 1900 with the growing acceptance of his theories, he was inspired to write four papers on the topic and he suggested an experiment to test the nature of the ether. This experiment, like the Michelson Morley experiment before it, yielded a null result. The experiment was shown eventually by Lorentz to be equivalent to the earlier null result, but this was after FitzGerald's death.

Summary

Stoney and FitzGerald were both leader of the Unionist faction of the Irish scientific community whose power base in Dublin was the R.D.S. The great Protestant mathematical tradition in Dublin, which culminated in a glorious fashion with their work, was eclipsed following the victory of Nationalism early in our own century. Their influence continued least in part, in the work of other Irish mathematicians working in the Royal College of Science, the National University and the Queen's Colleges, and more recently the Dublin Institute for Advanced Studies.

FitzGerald was the theoretical father of modern radio and the initiator and prophet for a new relativistic electromagnetic world view. Stoney was the theoretical father of electron theories, and therefore of modern theoretical chemistry. The Dublin mathematicians were consequently the most significant axis of the Maxwellians, in their very significant theoretical battle for the new physics and chemistry.

The very important scientific developments described above can only be properly understood in the context of Irish history, but only then within the general context of the international developments of mathematical physics.

FitzGerald philosophically was an idealist and follower of George Berkeley. This Trinity philosophical commitment was a considerable inspiration to him in proposing his non-common sense theories. His famous hypothesis was not in any way a flight of absolute idealism, and we can see now with historical research that this was no ad hoc hypothesis, as so frequently suggested by those ignorant of the facts. FitzGerald, paradoxically for an absolute idealist, in his work in physics, demonstrated theoretically the inextricable relationship between theory and practice, and the impossibility of ever absolutely separating the action of the mind from the material world.

Stoney's discovery of the electron inaugurated a new era in theoretical chemistry and atomic physics, but his identification of its charge as the first quantized quantity in physics marks the experimental birth of quantum theory. Stoney's consequent exposition of electron theories prepared the way with FitzGerald's discoveries, for our own modern quantum mechanical view which has arisen in our own century to replace older philosophical tenents such as those of Berkeley.

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NOTES

Diagonalising a Real Symmetric Matrix and the Interlacing Theorems

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In most linear algebra textbooks the diagonalisation of a real symmetric matrix is accomplished by first proving that the eigenvalues are real and then proceeding to the orthogonal diagonalisation. Anton's book [1] notes in the preface that the first part of this can require an excursion into the theory of complex vector spaces. The purpose of this note is to show that a more direct route is possible if one proves the realness of the eigenvalues and the orthogonal diagonalisation simultaneously.

In itself this would be of very little interest, at least for mathematics students, who usually handle $\mathbf{C}^n$ as readily as $\mathbf{R}^n$. However what one is led to, is something much more, namely the Cauchy inequalities, between the eigenvalues of any finite dimensional self adjoint operator and its compressions [3], and also to the Courant-Fisher min-max formulae for the characteristic numbers [2[, and these are topics not usually found in Linear Algebra textbooks. So, many mathematicians must be unaware of them. Yet the interlacing that one finds is both elegant and useful. For example it gives in several lines, the proof, that a symmetric matrix is positive if the principal minors are all positive.

I find linear operators a better setting for diagonalisations than matrices, so we work with them. Recall that if U is any subspace of an inner product space V, and $T : V \to V$ is a linear operator then the compression of T to U is just the operator $P_U T : U \to U$, where P_U is the orthogonal projection onto U. For students who prefer matrices, if an orthonormal basis $u_1, ..., u_r$ for U is expanded to an orthonormal basis $u_1, ...u_r, u_{r+1} ..., u_r$ for V then the matrix for $P_U T$ is just that block of the matrix of T whose entries are in both