- [56] W. Scharlau, Quadratic and hermitian forms, Springer-Verlag, Berlin, Heidelberg, New York, Tokyo (1985).
- [57] W. Scharlau and A. Tschimmel, On the level of skewfields, Arch. Math 40 (1983), 314-315.
- [58] C.L. Siegel, Darstellung total positiver Zahlen durch Quadrate, Math. Zeit 11 (1921), 246-275.
- [59] E. Spanier, Algebraic Topology, McGraw-Hill New York (1966).
- [60] T. Szele, On ordered skewfields, Proc. Amer. Math. Soc. 3 (1952), 410-413.
- [61] Olga Taussky, Sums of squares, Amer. Math. Monthly 77 (1970), 805-829.
- [62] C. Tsen, Zur Stufentheorie der Quasi-algebraisch Abgeschlossenheit Kommutativer Körper, J. Chinese Math. Soc. 1 (1936), 81-92.
- [63] T. Tsuzuku, On a conjecture of Kaplansky on quadratic forms, J. Math. Soc. Jpan 6 (1954), 325-331.
- [64] B.L. van der Waerden, Problem, J. ber. D. Math. Verein, 42 (1932).
- [65] C.T. Yang, On the theorems of Borsuk-Ulam, Kakutani-Yamabe-Yujobo and Dyson I, Annals 60 (1954), 262-282.
- [66] C.T. Yank, On the theorems of Borsuk-Ulam, Kakutani-Uamabe-Yujobo and Dyson II, Annals 62 (1955), 271-283.

Mathematics Department University College Dublin Dublin 4.

A Sociological Question

A.G. O'Farrell

Write on John H. White's theory of "open" and "closed" Catholicism, in the context of religion in modern Irish society.

- from a Maynooth BA exam paper.

Let OP be the set of all possible opinions. When endowed with Archdeacon Wellbeloved's aggiornamento topology (the topology of substantial agreement on the broad fundamentals of the question), OP becomes a completely regular connected Hausdorff topological space. Regrettably, OP satisfies neither the first nor the second axiom of countability, and hence is non-metrizable, but then you can't have everything. The space OP contains non-contractible loopy sets of opinions, and hence is not simply-connected. The problems this poses may sometimes be overcome by passing to the universal covering space, the space of all idee-fixed homotopy classes of circular arguments, also known as full socio-loopy space.

A person is a set-valued function p, defined on the set R of all real numbers, with values in the power set of OP. The majority of persons ordinarily encountered have the additional property that p(t) is empty before an initial conception-time, depending on the person (depending on some other persons, too, who enjoy it a lot more). It is also usually found that p(t) remains constant once t exceeds about 15 years after conception-time. The technical term for this is that p(t) has been set in concrete.

Let N denote the set of propositions contained in the Nicene Creed. Let A denote the set of propositions contained in the Apostle's Creed. Let I denote the singleton: { The Pope is tops }.

Definition A person p is a catholic at time t if and only if p(t) contains the union of N, A, and I.

Evidently, a catholic is open at time t if it holds a neighbourhood of each of its opinions. It is closed if it holds an opinion x whenever it holds opinions arbitrarily close to x.

Theorem 1 All open-closed catholics are loopy.

Proof The space of opinions is connected, because some people clearly find it possible to agree with everybody. Thus an open-closed catholic must hold all opinions, including inconsistent pairs of opinions. QED

Thus, at any given time, a loopless catholic may be open or closed, or neither, but not both.

Lemma 1 The Pope is a closed catholic.

Proof That the Pope is a catholic is well-known (cf. Mahoney, Acta Apost. Sedis, 1(33)1). If an opinion y is close to an opinion x, but not equal to x, then the Pope holds at most one of them, by the Infallibilitatsatz. Thus the Pope's opinions are discreet. Oops, discrete. QED

Corollary 1 If a catholic has trivial fundamental group, then it is closed.

Proof Let c(t) be a homotopically-trivial catholic. Then c(t) is loop-free, and hence agrees with the Pope, period. The result then follows at once from the lemma. QED

It should be noted here that Lefebvre, in his article on the application of sheaf-theory to the exegesis of the Reaper-parable, has established that coherent analytic catholics are bl—dy fascists. This result is, of course, a simple consequence of the above corollary, as is seen by referring to the pontifical exact spectral sequence.

This leaves us with only one class of catholics to consider: the loopy, non-open-closed class. Passing to full socio-loopy space, we observe that the Pope remains discreet, when lifted, because the covering map is a local homeomorphism. All catholics become loopless when raised, and hence closed. We may not conclude from this that the unlifted catholics are closed, or even Suslin-analytic, since the spaces involved are not Polish. We have proved:

Theorem 2 All uplifted catholics are closed. Some loopy ones are also open.

This brings us to cardinals. A *cardinal* is a catholic who is discrete, compact, totally-ordered and well-ordered by inclusion, and humble beyond belief. Thus, if x is a cardinal, then it holds each of the opinions of all its predecessors.

Theorem 3 There are no inaccessible cardinals.

Proof Since cardinals are discrete and compact, they are automatically finite, hence do not go beyond the opinions of their predecessors, and thus they are accessible. QED

The results may be applied to modern Irish society in a subsequent paper, unless the author is silenced.

St. Patrick's College Maynooth, Co. Kildare, Ireland