

Every chapter is followed by a set of problems and answers to all the problems are provided. The book is remarkably devoid of misprints, in fact the only nontrivial one I encountered occurs in the second of Problems 5, page 77, which deals with the generating function for $J_n(x)$. I feel that the author has succeeded admirably in his intention of producing an elementary text which is accessible to any undergraduate student with fairly basic mathematical education and I should have no hesitation in using this book should the occasion arise.

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"POLYHEDRAL COMBINATORICS AND THE ACYCLIC SUBDIGRAPH PROBLEM"

By M. Junger (Research and Exposition in Mathematics 7)

Published by Heldermann Verlag, Berlin, 1985. x + 128 pp.
DM 36. ISBN 3-88538-207-5.

Combinatorial optimisation [1,2] is the branch of mathematics which tackles such problems as the travelling salesman problem, shortest path problems, matching problems and network flow problems. More specifically, if S is a non-empty finite set and f is a real-valued function on the subsets of S then combinatorial optimisation refers to the problem of maximising f on a given collection of subsets. Since S is finite the most obvious way of solving such a problem is to list all the values of f in question and to pick the largest one. This method is too naive to be of much practical use. Instead, such problems are solved by developing algorithms for finding the required solution. Combinatorial optimisation is a child of the computer age. Not only are computers used to find the solutions, but a number of the problems in the field have arisen

in research in computer design and the theory of computation. There are many "real-world" problems which can be solved by applying the techniques of combinatorial optimisation. (With regard to applications of optimisation techniques in the real world it is a salutary exercise to read the case studies in [3], e.g. "the celebrated brand X washing machine shipping catastrophe", which show how careful one must be in making decisions based on a mathematical model.)

The monograph under review discusses the following combinatorial optimisation problem: given a directed graph D with an integer "weight" on each arc, determine an acyclic subdigraph of maximum weight. An equivalent version of this *Acyclic Subdigraph Problem* (ASP) is the *Triangulation Problem*: find a simultaneous permutation of the rows and columns of a non-negative square matrix such that the sum of the entries above the diagonal of the permuted matrix is maximum. The *Triangulation Problem* has an application in economics.

Let the digraph D have n arcs. Each subset B of the arcs has a 0-1 n -dimensional incidence vector x_B associated with it. The acyclic subdigraph polytope $P_{AC}(D)$ is the convex hull of all x_B , where B runs over all acyclic arc sets in D . The ASP may then be formulated as the integer programming problem: maximise $c^t x$ subject to $x \in P_{AC}(D)$, given the non-negative vector $c \in \mathbb{Z}^n$. The ASP is an example of an NP-hard problem (see [4]) and the idea of associating a polytope with the feasible set of such a problem and then of applying linear programming techniques, has become popular in recent times. It turns out to be crucial to determine the facets ($(n-1)$ -dimensional faces) of the polytope, and the central achievement of this monograph is the determination of several classes of facets of $P_{AC}(D)$. The author expresses the confident hope that "the algorithmic exploitation of our results will in fact lead to the effective solution of large instances of real-world problems which can be formulated as an Acyclic Subdigraph Problem".

The book is well organized and well written and, as well as dealing with the ASP, it gives an excellent survey of polyhedral combinatorics, although the reader may wish to fill in the background by consulting some of the references below. The theory of the book, due to Grötschel, Jünger and Reinelt, was awarded the IBM Computer Application Prize for 1984.

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3. SALKIN, H.M.
'Integer Programming', Addison-Wesley (1975).
4. GAREY, M.R. and JOHNSON, D.S.
'Computers and Intractability: A Guide to the Theory of NP-Completeness', Freeman, San Francisco (1979).

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BOOKS RECEIVED

"MATHEMATICAL FORMULAE" (Fourth Edition)

By S. Barnett and T.M. Cronin

Published by Longman, Essex, 1986. 77 pp. Stg £3.95.
ISBN 0-582-44758-5

A reference work providing a compact collection of mathematical formulae designed specifically for engineering and science students at university or college. For this fourth edition the authors have added new sections covering such topics as z-transforms, orthogonal polynomials and Walsh functions; other additions include further properties of matrices and a useful list of symbols and notation. The tables of logarithms have been replaced by frequently used statistical tables.

"ON THE EXISTENCE OF NATURAL NON-TOPOLOGICAL, FUZZY TOPOLOGICAL SPACES"

By R. Lowen

Published by Heldermann Verlag, Berlin, 1985. xvi + 183 pp.
DM 34.00 ISBN 3-88538-211-3

This monograph presents a unified study of several important examples of natural fuzzy topological spaces; the space of probability measures on a separable metrizable topological space, the space of Radon probability measures on a linearly ordered topological space, and the hyperspace of uppersemicontinuous fuzzy sets on a uniform space.

It is shown how these spaces can be canonically equipped with non-topological fuzzy topologies, and in each case the richness of information contained in these fuzzy structures when compared to classical structures is demonstrated.