MATHEMATICAL EDUCATION

SOME THOUGHTS ON THE ROLE OF MATHEMATICS*

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1. VARIOUS ROLES OF MATHEMATICS

Mathematics appears in a variety of guises as a language, an analytical tool, a vocation.

Mathematics is the language of quantities, size, order, shape. The need to communicate in a quantified manner requires a mastery of the language, i.e. mathematics. Hence the traditional requirement for engineers and physicists to be educated mathematically and with more subjects moving towards quantification, e.g. biology, social science, psychology, economics, management science, etc. there is a growing requirement for a mathematical education for a larger section of the professional community than has been the case in the past.

Mathematics is used as an analytical tool for example by engineers to create a model of a physical system which for instance may be represented by a differential equation. The solution of the equation which yields understanding of the behaviour of the model and hence the physical system requires knowledge of certain mathematical techniques such as the Laplace Transform, Fourier Series, Bessel functions, Legendre functions, etc. As more subjects become quantified different mathematical techniques are becoming important. For example, the techniques of Operations Research are now important in quantifying management science as are the techniques of probability and combinatorics in relation to social science.

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1.1 MATHEMATICS - A VOCATION

Mathematics is a vocation when it is employed as a body of technique in solving business and industrial problems. Vocational programmes in Applied Mathematics exist at NIHE, Limerick and one in Applied Mathematical Science at NIHE, Dublin. These programmes have vocational objectives. The objective of the programme at Limerick is to produce graduates with developed analytical skills and an ability to model real industrial or business systems as situations suitable for quantitative analysis and optimization. To achieve these aims the course includes basic elements of Business Studies and Engineering Science both to facilitate effective communication with colleagues whose training is in the theory of business or the practice of engineering and to ensure that a realistic model is created of the system under investigation. Expertise in modelling is developed through modules in System Theory, Operations Research and Industrial Engineering. Analytical and computational skills are developed through modules in Mathematics, Statistics and Computer Science. A project in the final year of the course provides the opportunity to integrate the different elements in the programme in modelling and analysing a real industrial or business problem. Of the three graduating classes to date (at Limerick) totalling approximately forty students all received ready employment in a range of positions in industry and business. The type of job and type of employer ranged over management information systems at a multinational company; statistical analysis for a market survey firm; software development for a computer manufacturer; software development for a manufacturer of electrical equipment; financial services in a Semi-State company; accountancy in a chartered accounting firm; production planning in a multinational company; actuarial work with an insurance company; quality control in a manufacturing company; a variety of computer programming and software development positions in the computer industry.

This encouraging employment record is similar to the situation in the US where current demand for applied mathematics graduates is growing at such a rate that starting salaries now rival those for electrical engineers and exceed those for business graduates. The supply of graduates from those universities with established applied mathematics programmes such as Brown University, Rensselaer Polytechnic Institute and New York University does not satisfy the demand and other universities such as Clemson University have revised their mathematics curriculum towards applications of mathematics so as to prepare students for roles in business and industry.

Employers range from companies like AT & T and General Motors with large research and development sections to Standard Oil Co. of California where the Mathematics and Statistics Consulting group cut the cost of testing new locomotive oil additives from $240,000 to $3,000 by showing that one short engine test was statistically equivalent to the results of 40 longer tests previously employed. Other companies who previously did not employ mathematicians now realize according to the Recruiting Officer for the large distribution company Foremost-McKessian Inc. that mathematics is "a universal language for attacking problems". One problem this company faces is to devise the most cost-effective way to adjust distribution networks in response to an increasingly fast-changing market place.

The programme at NIHE, Dublin which has not yet graduated students has a particular emphasis on mathematical modelling. Both the programmes at Limerick and Dublin have vocational goals in contrast to those programmes which are concerned primarily with the study of mathematics.

2. CHANGING EMPHASIS IN MATHEMATICS

In many countries and examples are the UK and the US, the emphasis of the recent past on the study of mathematics as a self-contained subject is receding and the traditional connect-

ion between mathematics and its application is being restored. This movement has occurred with the advent of computers since now realistic models of problems can be formulated and analysed with the aid of a computer. Of course the move away from the study of mathematics as an autonomous discipline was also not unconnected to the declining enrollment in such courses since students found the courses unattractive and employers were uninterested in recruiting graduates from the courses. Coincident with this change in content of mathematics programmes there are appearing new technologies requiring different mathematics. For example, the mathematics which is the language of computer science consists of set theory, equivalence relations, ordering, Boolean algebra, logic networks, graph theory, combinatorics. These pure mathematics subjects of the recent past have found new applications. Interestingly much of this material is also relevant to the quantification of behavioural and social science subjects such as sociology, psychology and management science. Developments in electronics relating to digital systems require the language of discrete mathematics rather than the continuous mathematics of electrical and mechanical engineering and hence require difference rather than differential equations and Z-transforms in place of Laplace transforms.

3. SUBJECT QUANTIFICATION

There is a move towards quantification of subjects such as biology, sociology, psychology, psychiatry, management science etc. where the fundamental dynamics of the underlying systems are not yet properly understood. For example in 1986 the 4th Conference on the Mathematical Theory of the Dynamics of Biological Systems took place in the UK. The 26th British Theoretical Mechanics Colloquium at Leeds in March, 1985 heard an invited lecture by J.D. Murray (an applied mathematician) on "A New Approach to the Generation of Biological Pattern and Form". There is now a Centre for Mathematical Biology at Oxford University. J.D. Murray emphasises that for mathematics to contribute to the quantification of other subjects it is necessary for mathematicians to invest substantial time in attempting to understand the subject to which the mathematics is to be applied. Mathematicians must develop an understanding of other scientific subjects in order to interact fruitfully with scientists in quantifying subjects which were hitherto described in qualitative terms. To play a role in the move towards quantification in other subject areas, mathematicians must actively seek new areas of application for mathematics.

It is frequently found that the mathematical models which arise in the new quantification of subjects are models which have previously been studied in relation to other subjects. Also the simplest models often give most insight. For example the equation of simple harmonic motion (describing the motion of a linear spring) is

$$\frac{d^2x}{dt^2} + n^2x = 0 \qquad (1)$$

When equation (1) is altered to

$$\frac{d^2}{dt^2} + n^2\sin x = 0, \qquad (2)$$

it describes the motion of a simple pendulum (or a non-linear spring). Equation (2) can be transformed into

$$\frac{d^2x}{dt^2} + ax + bx^3 = 0, \qquad (3)$$

where a,b are constants. When two terms are added to equation (3) so that it becomes

$$\frac{d^2x}{dt^2} + k\frac{dx}{dt} + ax + bx^3 = f\cos wt, \qquad (4)$$

where k, f and w are constants, the equation is called Duffing's equation and has been widely studied in relation to the vibrations of mechanical systems. This same equation has recently been used in modelling aspects of the behaviour of the

brain (see [1]). It is, perhaps, not too surprising that Duffing's equation might model diverse phenomena since it is one of the simplest yet realistic models of movement away from an equilibrium state with d^2x/dt^2 representing acceleration, $ax + bx^3$ representing the inherent attractive force (non-linear) of the system towards equilibrium, $k\frac{dx}{dt}$ representing the resistance of the system to movement (i.e. inertia) and $f \cos wt$ representing a periodic external force attempting to create movement.

4. INTEGRATION OF MATHEMATICS WITH APPLICATIONS

In the past mathematics and its applications was integrated in a cohesive whole. In the more recent past this integration was broken with an overemphasis on the language and notational aspects of mathematics with consequent adverse effects of an educational and vocational nature. An example (in this author's view) of such an adverse educational result is the presence of set theory as a notational device in the Leaving Certificate mathematics syllabus without other material in the syllabus to which set theory can be applied. This can lead students to view mathematics only as a language divorced from applications.

In the vocational or training sense the study of mathematics involves concentration on accuracy and emphasis on the fact that there is only one correct solution to a properly posed mathematical model. This approach involves in-depth study and detailed analysis and constrasts sharply (i) with the reality of attempting to create mathematical models where sometimes there is uncertainty about the input data (for example in economic models) and (ii) with the case where a crude model is most appropriate either because an approximately correct answer (e.g. 70% - 80% correct) is sufficient or a rapid answer is required. Students therefore need to experience the necessity of producing crude simple models which yield approximately correct but rapid results. Otherwise, there is a danger that overemphasis on accuracy and 100% correct answers will lead to the production of graduates with a need to analyse every problem in depth before producing any answer.

The relative absence of a large research and development sector in the industry of this country means the absence of what is in other countries a traditional source of employment for mathematically trained graduates. However, the development of the scientific approach to management in this as well as in other countries also requires the analytical abilities which mathematically trained graduates possess. There is and will be an increasing demand from this sector for mathematicians who can model and analyse complicated management and industrial problems.

REFERENCE

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"Duffing's Equation in Brain Modelling". *Bull. IMA*, 12 (1976) 207-214.

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COMPUTERS AND THE MATHEMATICS CURRICULUM

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INTRODUCTION

Information technology will have a radical and pervasive effect on education, affecting both the aims, content and teaching and learning methods of all subjects; it will also affect the organisation of education, enabling its wider dispersal, both in terms of location and age of pupils. Teaching and learning methods, assessment and the curriculum are all bound up together, but here attention will be focused on curricular matters, specifically on the impact of computers on the mathematics curriculum, concentrating on the senior cycle of second-level education. Many of the ideas, however, will have a broader application, and will apply to mathematics education in general.

COMPUTERS AND MATHEMATICS

Firstly, a few comments on this perennial topic for debate. Although there are many connections between computers and mathematics, for example, programming may be regarded as a branch of logic (see e.g. Murphy [9]), present opinion is almost unanimous in regarding the linking of mathematics and computer studies on the curriculum as undesirable. For example: "Their view was unanimous that computer studies should not be regarded as part of mathematics but should ideally exist within a separate department." (Cockroft Report [1], par. 397.) Reasons commonly given are the need for special training in order to teach computer studies, the need to prevent such a subject becoming an elitist, and in particular a sexist one, and the fact that the linking of computer studies with mathematics inhibits its development across subject boundaries.