the inclusion of the various theoretical topics considered; without motivation, elementary graph theory tends to look like a collection of unrelated random results on graphs. My only quibble here is that Chapter 6, "the colouring of graphs", fails to make the reader fully aware of the variety of uses of graph colouring.

The book (like all graph theory texts) has a great number of definitions in its earlier pages. However, the language is very allusive and one easily absorbs this material. Why don't graph theorists agree on their basic terminology? The field cries out for some sort of rationalization. As Wilson points out on page 26, what he calls a circuit is also known in the literature as a cycle, elementary cycle, circular path and simple circuit! The most striking example of all is that the definition of a *graph* is not agreed on by everyone; some authors including Wilson permit "graphs" to have multiple edges, others don't.

Appel and Maken's computer aided proof of the four colour theorem is mentioned in a few places; obviously this edition of the book was written before serious doubts were cast on the proof, but this isn't the fault of the author. Students beware!

On page 12 two methods are described for storing graphs in computers. Perhaps the author should also mention *adjacency list representation*, which is commonly used.

At the beginning of this review, I mentioned the growing demand from computer science students for graph theory courses. Unfortunately, the present book isn't a good choice for the sort of course computer science departments usually have in mind, because it's basically theoretical. In the preface, Wilson claims his work is "suitable both for mathematicians taking courses in graph theory and also for non-specialists wishing to learn the subject as quickly as possible". The

claim is justified by this reasonably priced and very readable book.

I had found a mistake in exercise 19a on page 91 before the solutions manual arrived, but the manual to its credit had also detected the misprint. It gives answers to all the exercises, sometimes only in outline. All those I checked were correct. The preface notes that "each author wishes to make clear that any errors which occur are entirely the fault of the other"!

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"THE INS AND OUTS OF PEG SOLITAIRE" (RECREATIONS IN MATHEMATICS)

By *John D. Beasley*

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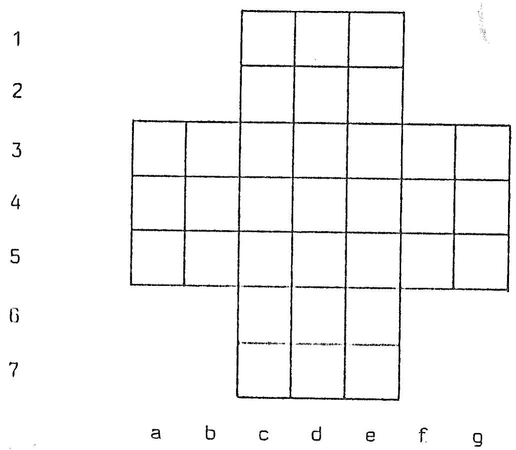
I have always carried a nagging desire to really understand the game of Solitaire, ever since a "flukey" solution on a train of "the central game" many years ago. On the standard English (or German) 33-hole board, a miraculous sequence of vertical and horizontal jumps had reduced a single vacancy at the centre to a sole survivor in that position. Many subsequent attempts to repeat the performance failed miserably. This book grants my wish completely.

Although the origins of the game are uncertain, it was known in the Western world almost three hundred years ago. The outline history of the game given by the author reveals its attraction for strategists and mathematicians alike including officers of the French Royal Artillery, Academy members, Leibniz, Crelle, Bergholt, and Conway and many others.

Crelle published a solution to "the central game" in 1852. In 1912, Bergholt published a solution consisting of 18 moves. It appeared in a weekly newspaper chiefly devoted to ladies' fashions! He later published a book on the subject called the "Complete Handbook to the Game of Solitaire on the English Board of Thirty-three holes". In 1964, Conway and others proved that Berghold's 18 move solution was the shortest possible.

If the 33-hole board is labelled according to the scheme



then allowing for duplication due to symmetry the only soluble single vacancy single survivor problems are summarised by the table overleaf. The author supplies solutions to all these problems in the number of moves listed. He invites verification of his own computer calculations that in each case the required moves are indeed minimal.

Initial Vacancy	Final Survivor	Moves Required
c1	c1, c4, c7	16
	f4	17
d1	a4	17
	d1, d4, d7	18
c2	c5	15
	c2, f5	16
d2	a5, d5	17
	d2	19
c3	c3	15
	f3	16
d3	a3, d3	16
	d6	17
d4	d1	17
	d4	18

TABLE

In general, it is not always possible to reduce any given starting position to a given target position by solitaire moves. However, it is possible to divide all solitaire positions into 16 fundamental classes with the property that if a given problem has a solution, then the starting and target positions must belong to the same class. My own favourite proof of this result is due to de Bruijn (*Journal of Recreational Mathematics* 1972) which associates an element $F_4 \times F_4$ with each solitaire position where F_4 is the field of four elements. It is included in Ian Stewart's 'Concepts of Modern Mathematics' as a nice application of abstract algebra. De Bruijn's method is not used in this book. An alternative elementary reduction is used, which is based on whether or not for a given set the number of elements on certain diagonals is "in" or "out" of phase with the parity (even or odd) of the total number in the set. On the 33-hole board, every position is in the same class as its complement. Two single-man positions are in the same class if and only if the rows and columns of the occupied holes differ by multiples of three. So if we start by vacating a single hole then we can only reach a single-man finish in the original hole or in holes at intervals of three away from it. So except for symmetry the final survivors listed in the table above are the only ones possible.

T.R. Dawson included several double-vacancy complement problems in "The Fairy Chess Review" in 1943. He stated that it was impossible "for lack of elbow room" to begin with vacancies at b4, d4 and end with exactly two survivors in those positions. Conway, Hutchings, Guy and the author developed a theory of "balance sheets" to investigate multiple vacancy complement problems. In 1961, they proved Dawson's assertion and showed that it and the similar complement problems with initial vacancies and final targets at {d2, d6}, {b4, d2} and {d1, d2} are also the only impossible ones.

A similar analysis of triple vacancy complement problems established three classes of insoluble problems:

(i) d1, d2 and any third other than a3, a5, g3 or g5,

(ii) any two middle men (b4, d2, d4, d6, f4) and any third other than an outside corner;

(iii) any three from rows 2, 4 and 6.

The theory developed is difficult to master and leaves many multiple-vacancy complement problems unresolved awaiting further research. It is expected that problems involving marked and distinguished men will receive prominence, i.e. problems similar to Bizalion's "man-on-the-watch" variation in which a nominated man remains fixed and then clears the remaining men in a final sweep to become the sole survivor. The book kindly provides solutions to all the problems set, but some of these can be difficult to follow as the notation given above is rather cumbersome to use. Take for example

the 16-move solution to the c1 complement problem given by:

e1-c1, d3-d1, f3-d3, e5-e3, d3-f3, g3-e3, b3-d3-f3, c5-e5, a5-c5, f5-d5-b5, c1-c3, a3-a5-c5, e7-e5, g5-g3-e3, d2-d5-d3-b3-b5-d5-f5-f3-d3, c7-c5-c3-e3-e1-c1.

If there is a misprint in that sequence of moves, then I have made it! I found none in the book itself.

Finally, the author considers other boards and other rules of play. The reader is introduced to fanciful symmetric boards, three-dimensional boards, hexagonal boards, even infinite boards.

All in all, this is a most enjoyable and complete account of the Solitaire Game and the book is a worthy successor to Bergholt's volume.

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