to use these information sources as well as the other career planning activities offered by the Service. Final-year students can register with the Services and be notified of vacancies - the Services are the major source of contact with employers offering graduate employment prospects at home and abroad.

Careers and Appointments Officer,
University College,
Cork.

MATHEMATICAL EDUCATION

MATRIX: COMPUTER ASSISTED MATHEMATICS TEACHING

Ted Hurley

INTRODUCTION

The main aim of this article is to describe the teaching of certain aspects of Mathematics using the MATRIX program. Before discussing this in detail, I make a few general comments on relationships between Computers and Mathematics which I hope will stimulate some discussion in this whole area.

Computer Science at most 40 years in existence, is now the best financed scientific subject, whereas Mathematics, which has been around, developing and of immense value to science, technology and society for over 3000 years is the least financed. Hardy says in *A Mathematician's Apology*, "A Mathematician, like a painter or a poet, is a maker of patterns. If his patterns are more permanent than theirs, it is because they are made with ideas." Mathematics is also the least financed of the arts. The problems of illiteracy are recognised, and rightly so, and processes have been set up to tackle the problem. Little, if anything, is being done to eliminate innumeracy, which will become even more of a problem as technology develops.

When a Mathematical idea becomes useful it appears no longer to be Mathematics. The number of "in" words associated with Computer Science continues to expand - C.I.M., C.A.M., A.I., C.A.D., C.A.L., (Exercise: State or find out the meanings!), etc. Programming is no longer simply programming but rather "software engineering". We need more "with it" words for Mathematics - how about I.E. ("Intelligence engineering") or S.E. ("Symbolic engineering)? Write your suggestions to the editor!

With the advent of the era in Computer Science of software engineering, more and more the importance of a Mathematical background, in areas such as discrete Mathematics - combinatorics and graph theory, is beginning to be recognised. Because of the intimate relationships, Computer Science has and will continue to contribute to the research, development and teaching of Mathematics. Computer Science is the first to reciprocate the help which Mathematics has given to many subjects.

U.C.G. VAX

We have on the University College Galway VAX 11/785 (U.C.G. VAX 1) two beautiful and powerful programs CAYLEY and MATRIX. These programs have been developed at Sydney over the past ten to twelve years by a group headed by John Cannon.

In a previous article [1], Pat Fitzpatrick described some of the features and uses of CAYLEY. At the moment, this program is used as a research aid but on account of the enormous number of structures available it would be our aim to develop it as a teaching aid for honours undergraduates and postgraduates in group theory, ring theory, fields, modules and possibly Lie algebras. Another area where I would envisage its use is in the teaching of group representation theory to Physicists - CAYLEY has the capacity to print out character tables.

DESCRIPTION OF MATRIX

MATRIX is suitable for many elementary and more advanced undergraduate courses. It can best be described as a *laboratory tool* for the teaching and learning of Mathematics. It is *not* a package as such (e.g. NAG, MACSYMA) where the data is simply supplied and the computer prints the answer, although it can be used in such a capacity. It has, when working as a

teaching process, essentially two components: (a) the execute part of MATRIX, and (b) the PROBLEM LIBRARY which is attached to MATRIX and from which the students derive their problems and/or instructions.

Note however that a PROBLEM in MATRIX can mean a problem in the usual sense or a teaching process or a combination of these. The PROBLEMS can perhaps be put in three categories.

p1: By a series of HINTS and DISCUSSION the student is guided through a complete explanation and possible answer.

p2: A problem, in the true sense, is produced for the student and the computer performs the computation on instruction.

p3: A problem is produced and some explanation is given if the student is stuck.

It is often possible, with the DISCUSS command, to tell the student whether or not he/she has obtained the correct answer.

The arithmetic in MATRIX is *rational*, unless otherwise required. This has obvious advantages and corresponds with the students' own arithmetic. The number 2/3 has much more meaning than 0.666667!

I list below some of the PROBLEMS I have developed with my classes this year. Other types are of course possible.

Entering matrices.

Echelon Form.

Row and column operations (used, for example, in inverting matrices or given symmetric A, finding non-singular P such that P^tAP is diagonal D with entries from {0,1,-1}).

Bases for vector spaces (for example, find a basis for the solution space of a system of equations).

Eigenvalues, eigenvectors and diagonalisation of a matrix.

Difference equations and finding large powers of a matrix.

Transition matrices.

Change of base.

Input-Output matrices.

Gram-Schmidt process, orthogonal matrices and orthogonal reduction of symmetric matrix.

Kernel and image of a linear transformation.

Systems of differential equations.

Simultaneous reduction.

Linear Programming.

It is possible to handle realistic problems using MATRIX where hand calculations are out of the question. For example, Linear Programming problems, even for a small number of variables, go quickly out of hand and teaching and examining this topic by computer seems an ideal solution. I have been able to enter a number of problems so that each student receives a different set of data for a particular type of problem. (The PROBLEM LIBRARY developed contains an infinite number of problems!)

THE STUDENTS' VIEW

No experience of programming is necessary. All the commands necessary to run MATRIX and to attach the PROBLEM LIBRARY are set up in a LOGIC COMMAND. The student simply logs in and calls up MATRIX with $MAT, or, if he/she wishes to keep a record of the session for possible printout later, with $MAT/LOG. The prompt in MATRIX is ? and so, for example, ? PROBLEM 100 prints the problem on the screen. If the student is stuck he/she may ask for HINT if this is available.

All the MATRIX commands are available on a quick reference card (which is in total one A4 size page) and this has been found to be sufficient, although a more detailed reference manual of about 50 pages comes with the program. It is not

possible to list all the commands available but they include addition, multiplication, exponentiation, transpose, dot and cross products, rowops, the standard functions (cos, log etc.), eigval, charpoly, submatrices, tableau, pivot and even HISTORY to look back over what has already been done. Procedures (subroutines) are also possible and special matrices may be called, e.g. ZERO (m,n) or HILB (n). It is possible to turn off to the student any of these commands - for example at an early stage the PIVOT command could be turned off but as the course progresses, this command would be allowed. You may perhaps allow the students to read off the eigenvalues and eigenvectors and thus proceed to more advanced problems or you may require them to work these out for themselves.

I have used the program for a third year pass course in Algebra for Science, Arts and Commerce students (about 120 students in total) and a third Engineering Mathematics Option course with about 55 students. (There were only 35 in the option until they heard about MATRIX!) It is certainly possible to use MATRIX also for first and second year pass and honours and for service courses - the classes were chosen solely because these had been assigned to me.

The MATRIX course was compulsory and each student had the terminal booked for him/her for two hours each fortnight. They were also free to work on it whenever terminals were available and many did so. Printouts of solutions (or attempts!) to particular problems had to be submitted by a particular date or the MAIL facility could be used to send their solutions to my directory.

At the end of the course, each student was asked to complete, anonymously, a questionnaire giving his or her opinion and comments on the usefulness or otherwise of MATRIX. On the question "All in all, did you think MATRIX is worthwhile?", over 70% were very much in favour (gave 9 or 10 on a scale of 0-10). It is also my view that the students learned much more

Mathematics and became more proficient on the (normal) problem sheets. There is the added experience of working on computers and many had had no such previous contact.

DIFFICULTIES

There were of course difficulties. The running and organising of such a scheme takes a tremendous amount of time and work. The main problems the students had were hardware problems, with the non-availability of terminals and printers as required. These can be overcome. The amount of time involved in problem library development should not be underestimated. Hopefully, others will become interested and it will be possible to exchange problems. This would be a simple process via the HEANET (Higher Education Authority Network), and post and transfer of files in and out of Ireland to anywhere in the world with a similar network seems to be possible. For further information on HEANET read the appropriate section of [1] and/or contact your local computer services.

The most serious Mathematical criticism is that the students' ability to calculate or manipulate expressions may be further reduced. We do have a problem with numeracy and manipulation but I argue that computer-assisted learning does not make it any worse and can on occasions be very beneficial. Many students refuse to continue when a seemingly unmanageable calculation is presented and are never able to get to the bottom of a problem. Errors often occur in arithmetic no matter what, and progress is impeded. I once felt that calculators should not be allowed at examinations.

CONCLUSION

If you would like to try out MATRIX, and I highly recommend it, you may, if you are not in Galway, do so through the HEANET. If you would like to use MATRIX in one of your courses, it would be necessary to purchase the program from Sydney by

writing to John Cannon at the University of Sydney, Sydney, New South Wales, 2006, Australia. I will be happy to correspond with anyone who is interested.

We hope very soon to implement KOENIC, a teaching program for graph theory.

REFERENCE

1. FITZPATRICK, Patrick, "CAYLEY: Group Theory by Computer", *IMS Bulletin* 16 (1986) 56-63.

Department of Mathematics,
University College,
Galway