MATHEMATICS IN FOOD ENGINEERING RESEARCH

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INTRODUCTION

Engineering has often been described as the application of mathematics to various branches of applied science. In many cases this tendency to express physical phenomena in numerical or formulae form is effected in spite of profound resistance from the mathematical principles involved. It is also true that this constant yen of engineers to evolve rules of thumb or working models of systems causes the genuine mathematician to frown. Nevertheless a good working relationship and mutual tolerance/understanding exists between these two schools of numerate thought/application. The advent and proliferation of computer systems of ever-increasing speed and memory capacity, combined with improved availability of numerical software packages, has added greatly to the common ground available to the engineer and mathematician from which they may better serve the needs of both science and industry.

In what may be called the more traditional branches of engineering - civil, mechanical, chemical, electrical and industrial for example - mathematical applications and examples are well known. It is only in the recent past, however, that the food engineer has been able to turn to mathematics and mathematicians to glean some assistance in solving the many particular problems associated with serving the food processing industry in an engineering design/research capacity. Indeed it is true to say that food engineering as a discipline is only quite recently emerging as a distinct area of professional endeavour. It owes its development to a large degree to the new tools made available to the food engineer which allow the efficient amalgamation of the skills of the many strands of the engineering sphere from which it draws its working principles.

Many of the problems encountered in the food industry have close parallels in the chemical engineering type industries. There are, however, a number of significant differences which add considerably to the complexity of the food engineer's workload. It is worth listing at this stage the main classes of problems which are the subject of both laboratory based research work and actual in-line process development. Stated briefly then, the main categories are as follows:

1. Flow problems - fluids, solids, liquid-solid solutions, and dispersions.
2. Heat transfer problems - conduction, convection and radiation, and various combinations thereof.
3. Basic process calculations - statistically based estimations of microbial lethality/quality retention.

The actual objectives in solving these various problems centre on four main 'commercial' targets:

(a) PROCESS CONTROL AND OPTIMISATION

(b) QUALITY ASSURANCE

(c) CATALOGUING THE ENGINEERING PROPERTIES OF FOODS

(d) IMPROVED DESIGN TECHNIQUES

MATHEMATICAL MODELLING

Mathematical modelling of food processes is an area which is currently of major research interest. Food process control has developed in the last decade or so to a very high level with only two major obstacles now delaying total automation:

1. Development of continuous in-line sensory devices to monitor process parameters.
2. Development of mathematical models/algorithms that relate these process measurements to finished product (quality) parameters.

Hence, provided the food engineering equipment/instrumentation companies deliver on the new microprocessor based sensory devices, it will only remain for the engineers/mathematicians to patch the resultant process data into a working model. On board a digital control system such a model can be used to either regulate a process or to view the results of proposed process changes via simulation.

In food engineering, modelling is currently employed to achieve optimum results under a number of headings:

1. Optimum microbial lethality;
2. Maximum shelf-life;
3. Maximum quality factor retention, e.g. colour, taste, vitamin content, texture;
4. Product compositional factors - moisture/fat content, density.

TYPICAL PROBLEM

A problem currently the subject of some study is that of processing particulate foods in a fluid stream. With the ever increasing market for convenience foods, solids such as meat or vegetable portions packaged in sauce or gravy mixes are becoming quite popular. To date, however, manufacturing these products is a two-stage, batch-type operation. One of the main obstacles to designing a continuous, aseptic process is the lack of a suitable mathematical model which will allow simulation of the process and hence allow prediction of lethality/quality retention for various combinations of process/product parameters.

A number of models have been proposed for the process but have included some restrictive assumptions which limit the practical application of the model, e.g. Misra *et al.* [4] and Guariguata *et al.* [2]. O'Connor *et al.* [6] have proposed a model based on finite element techniques which is further

outlined below.

The overall problem combines both fluid flow and heat transfer. To simplify the study only the 'holding tube' section of the process is considered here. This is the section of the plant directly after the heat exchanger. Entering this, it is assumed that the desired thermal processing of the fluid phase has been achieved. It is in the holding tube now that the solid particles are 'cooked'. Summarising the problems in the tube will indicate the complexity of the model. For effective simulation then, the following factors must either be known or calculated/predicted:

A. Flow profile of the fluid stream at all points in the tube.

B. Relative velocity of the solid particle(s) with respect to the fluid stream.

C. Residence time distribution of the particles in the tube.

D. Time/temperature profile within the particle and consequent microbial lethality/quality factor retention.

In the model used by the author, laminar flow is assumed in the holding tube and spherical particles are used as a representative shape for foodstuffs. Further complications may, of course, be added. Kwant *et al.* [3] developed flow profiles for fluids with temperature viscosity, for example. Obtaining a prediction of the relative velocity is extremely difficult in a laboratory situation and yet its accurate prediction is essential as it determines the surface heat transfer coefficient which controls the rate of heating of the spheres.

HEAT TRANSFER IN SPHERICAL COORDINATES

Carslaw and Jaeger [1] state the governing differential equation for heat conduction in a sphere with no heat generation as:

$$k_r \frac{\partial^2 T}{\partial r^2} + \frac{2k_r}{r}\frac{\partial T}{\partial r} + \frac{k_\psi}{r^2 \sin\psi}\frac{\partial}{\partial\psi}(\sin\psi\frac{\partial T}{\partial\phi}) + \frac{k_\phi}{r^2\sin^2\psi}\frac{\partial^2 T}{\partial\phi^2} = pc\frac{\partial T}{\partial\theta}$$

for $0 \le r \le R$; $\theta > 0$; $T = T(r,\psi,\phi,\theta)$.

When symmetry is accounted for this equation reduces further to:

$$k_r \frac{\partial^2 T}{\partial r^2} + \frac{2k_r}{r}\frac{\partial T}{\partial r} = pc\frac{\partial T}{\partial\theta}$$

where k_r = radial thermal conductivity;
T = temperature;
r = radial coordinate;
p = sphere density;
c = sphere specific heat;
θ = time.

The boundary condition for the holding tube is given as:

$$\pm\frac{\partial T}{\partial r} + h(T - T_{amb}) = 0 \text{ for } r = R,\ \theta > 0$$

where h = surface heat transfer coefficient;
T_{amb} = ambient temperature of fluid stream.

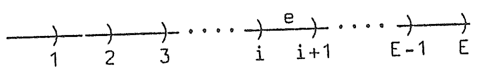
The initial condition is:

$$T = T_o \text{ for } 0 < r \le R;\quad \theta = 0.$$

FINITE ELEMENT FORMULATION

To allow for product property variation an elemental technique is used to estimate the heat transfer through the particle. This also facilitates a 'mass-average' statement of process efficiency which is much more satisfactory than the traditional single-point analysis of product treatment.

The sphere is therefore divided into a number of connective elements as shown:



The solution is obtained by finding a function T(r,0) satisfying the boundary and initial conditions stated which minimises an integral quantity called a functional. The functional is minimised at every instant of time to give an approximate temperature profile in the sphere. Misra *et al.* [4] have presented the functional as

$$I = \int_V \tfrac{1}{2}[k_r(T')^2 + 2pc\frac{\partial T}{\partial\theta}T]dV + \int_S \tfrac{1}{2}h(T - T_{amb})^2 dS$$

where $T' = \frac{\partial T}{\partial r}$ and $\int_V$ and $\int_S$ are volume and surface integrals, respectively. As the sphere is divided into E elements, this integral may be evaluated separately for each element (*c.f.* Myers [5]). Thus:

$$I = \sum_{e=1}^{E} I^{(e)}$$

where the element 'e' is chosen as the typical element. The sub-integral $I^{(e)}$ then may be calculated from the heat conduction, capacitance and convection terms as they occur in the process. Hence

$$I^{(e)} = I_k^{(e)} + I_c^{(e)} + I_h^{(e)}$$

Following the usual finite element procedure the conduction, capacitance and convection element matrices are developed from the above. These are then combined (O'Connor *et al.* [6]) to form the global matrices which form the final equation to be solved

$$\{K\}\{T\} + \{C\}\{\dot{T}\} = \{F\}$$

where {K} is made up by summing the elemental conduction and convection terms, and {C} from the capacity terms. {T} is the column matrix of nodal temperatures, $\{\dot{T}\}$ the derivative with respect to time and {F} is the force vector which accounts for heat convection at the surface. Numerical solution of this equation yields the required temperature profile.

The programme, as constructed from the above procedure, allows the user to predict time/temperature profiles for a spherical particle under varying thermal processes and including data on the product's thermal properties. The next step is to translate the temperature data into actual microbial figures, which may be done using any of a number of standard procedures (Stumbo [7]).

CONCLUSION

The above brief run through a typical process modelling is but one example of the type encountered in the food processing sector. The problems and their solutions are generally multidisciplinary in nature. In many respects there are so many variables, both in process and in raw materials, that the task seems impossible, the opportunities for error/instability quite widespread. As with many engineering problems, however, the solutions are limited to a set of finite possibilities and, in many cases, the experienced food engineer can impose constraints/guidelines as to what type of answer the model may produce.

It is to be hoped that in spite of, or maybe because of, the many complexities in biological/food systems more mathematicians may be encouraged to contribute to solving this most challenging set of problems.

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