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BULLETIN

EDITOR

Patrick Fitzpatrick

The aim of the *Bulletin* is to inform Society members about the activities of the Society and also about items of general mathematical interest. It appears three times each year: March, September and December. Deadline for copy is six weeks prior to publication date.

ASSOCIATE EDITOR

Martin Stynes

The *Bulletin* seeks articles of mathematical interest written in an expository manner. All areas of mathematics are welcome, pure and applied, old and new.

Detailed instructions relating to the preparation of manuscripts may be found on the inside back cover.

Correspondence relating to the *Bulletin* should be sent to:

Irish Mathematical Society Bulletin,
Department of Mathematics,
University College,
Cork.

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From the Editor

Please note that there will be no December issue of the Bulletin this year. The next issue is due to appear in March 1987.

As a consequence the distribution of the new membership list will be delayed until March, and we are including a mid-year update in this issue. The deadline for copy for the next issue is December 31.

Pat Fitzpatrick

LETTERS

To all readers of the Irish Mathematical Society Bulletin

Dear Colleague,

The Twenty-ninth International Mathematical Olympiad (IMO) will be held in Australia in 1988, the Bi-centenary of the European settlement of Australia. Mr Peter O'Halloran, Chairman of the 1988 IMO Executive Board belongs to the 25% of Australians of Irish extraction and is most anxious to see a team from Ireland competing in the 1988 IMO.

It was always in the back of the minds of those of us who founded the INMC that some day we would enter an Irish team in the IMO; and having received an invitation to compete in 1988, we have an opportunity to fulfil our ambition! Now seems as good a time as any to begin. Over the next two years we plan to identify mathematically gifted young people, train them and by a series of eliminating tests pick a team to represent Ireland at the Olympiad in two years time.

Because of the timing of the Olympiad - it is held in the first week of July every year - it would seem that we have to exclude from our consideration students who will be doing the Leaving Certificate, University Matriculation and Advanced Level examinations in 1988. This would seem to confine our selection of potential team-members to those who are currently in their pre-Intermediate and/or pre-Ordinary level years. We have asked teachers to identify the very good junior mathematicians in their schools and to forward their names and addresses to us. To prepare these we need to set up a network of mathematicians throughout the country who would correspond with them and hold workshops at regular intervals and generally guide them over the next two years.

We want to hear from people who would be willing to assist us in this project. We want mathematicians from RTCs, NIHEs and University Colleges to volunteer to prepare problem sets, to distribute them to students, to correspond with students and possibly to hold workshops at selected venues once a month, say. As well, we want help in raising the necessary finance to send a team to Australia.

Without your much needed support the venture may fail. Many of you have expressed concern about the adverse effects the so-called new mathematics has had on third-level students. Here is a practical way for you to become involved in correcting these deficiencies, albeit to a small gifted group.

Even if you cannot see your way to play a full part in the project, you may have ideas and suggestions that would help us see it through to a conclusion. If you have, we would love to hear from you. Please communicate your views to one of the undersigned.

Yours sincerely,

Finbarr Holland, UCC

Tom Laffey, UCD

To the Editor

Dear Sir,

Since I am interested in group theory I was very amused recently to discover that a pair of bedroom slippers I had bought bore the brand name "COSET". However there is one disturbing implication - since I now have a left coset which is not a right coset, does this imply that I am not normal?

Yours faithfully,

DESMOND MacHALE

IMS BUSINESS

IRISH MATHEMATICAL SOCIETY

Committee Meeting, 6 pm, 3rd April, 1986 at DIAS

Present: M. Newell (President, in the chair), R. Critchley, S. Dineen, B. Goldsmith, P. McGill and R. Timoney.

Apology: N. Buttimore.

1. The minutes of the previous meeting (19th December, 1985) were read and signed after one amendment.

2. No real possibilities had emerged from investigations by S. Dineen and B. Goldsmith into alternative arrangements for printing of the Bulletin.

3. The question of the European Mathematical Council's proposed database was discussed at length. The Committee felt that the project was extremely worthwhile. The sums of money involved even for funding studies preliminary to the main project were too great for the Irish Mathematical Society's meagre resources. Thus it was decided to encourage the project but to obtain the necessary funding from other sources.

   In response to a request from M.F. Atiyah for a contribution towards the cost of drawing up a detailed proposal for EEC funding, it was decided to seek support from the 3rd level Mathematics Departments in Ireland - the envisaged beneficiaries of the database's facilities. The Secretary would first obtain some documentation on the details of the scheme currently under consideration to enable local representatives to make a case to their colleges.

4. M.F. Atiyah had informed the Society, in response to a query from the Secretary, that the EMC did not have funds available for travel support for representatives attending EMC meetings. It was decided that the IMS would not be represented at the next EMC meeting in Czechoslovakia in November.

5. The Secretary presented a preliminary report on the joint meeting with the London Mathematical Society held in Dublin on March 21/22, 1986. Final financial details were not yet available.

6. The Secretary reported on correspondence with the American Mathematical Society with regard to reciprocity of membership. The AMS seemed quite willing to have such an agreement with the IMS. It was agreed that $4 would be an appropriate discount price for AMS members joining the IMS.

IRISH MATHEMATICAL SOCIETY

Ordinary Meeting, 12.15 pm, 4th April, 1986, at DIAS

The President, M. Newell, took the chair and there were 12 members present.

1. The minutes of the Ordinary meeting of 20th December, 1986, were read and signed.
2. The Secretary presented a preliminary report (appended) on the joint meeting with the London Mathematical Society held at Trinity College, Dublin, on March 21-22, 1986. Final financial details were not yet available.

   The meeting unanimously approved a motion thanking the organiser, T.T. West, for his work in making the conference such a success and thanking Trinity College for the use of their facilities.
3. It was reported that the American Mathematical Society seemed willing to have a reciprocity agreement with the Irish Mathematical Society which would result in a very substantial reduction in membership fees for IMS members joining the AMS. The Committee had decided on a discount membership rate of $4 for AMS members (not resident in Ireland) joining the IMS.
4. With regard to the proposed European Mathematical Council database, the Committee had decided to seek contributions of approximately £50 from each 3rd level institution in Ireland. This money would go towards the cost of preparing a detailed proposal for EEC funding for the project. Some of the work is being carried out at NIHE Dublin although the main centre is to be in Denmark.

IRISH MATHEMATICAL SOCIETY

Membership List Supplement 86-2

Compiled from Treasurer's Records on 10 July 1986

Amendments:

85155	Currie, Dr P.K.	Guildeland 103, 2291 VJ, Wateringen, The Netherlands.
85160	Geoghegan, Prof. R.	Department of Mathematical Sciences, State University of New York at Binghampton, N.Y. 13901, USA.

Additions:

86197	Thomas, Dr D.	University College, Swansea, SA2 8PP, Wales.
86199	Targonski, Prof. G.	University of Marburg, Lahnberge, D-3550, Marburg, Federal Republic of Germany.
86203	Cussen, J.	Patrician Academy, Mallow.
86204	Schnitzer, Prof. F.	Montan University, 8700 Leoben, Austria.
86205	Lynch, P.	Villarea Park, Glenageary, Co. Dublin.
86207	Barry, P.	R.T.C., Waterford.
86221	Shields, Prof. A.	University of Michigan, Ann Arbor, Michigan 48109-1003, USA.

R.T.C. Carlow:	86198	E. Kernan	86202	C. Clarke
U.C.D.:	86200	P. Robinson	86201	E. Cox
NIHE Limerick:	86206	T. Bradley	86220	M. Wallace

Under IMTA Reciprocity: (86208 - 86219)

Sean Ashe, Brendan McCodey, Sean Close, Frances O'Regan, John Maher, Art Anglin, Seamus McGurran, Mary Johnston, Elizabeth Oldham, Lorna Connolly, Joe Devine, Maurice O'Driscoll.

SUMMARY OF RESULTS OF THE 1986 IRISH NATIONAL MATHEMATICS CONTEST

The eighth Irish National Mathematics Contest was held on Tuesday, February 25, 1986, and attracted 1,324 entries from 75 schools. By comparison, 1,630 students from 86 schools took part last year.

As has been the case since the inception of the INMC, the paper used for the contest was supplied to us by the MAA committee on American Mathematics Competitions and was used also for the 37th Annual American High School Mathematics Examination which was taken by nearly 400,000 students throughout the world on the same day. Contestants had 90 minutes to answer 30 multiple choice type questions, which ranged from easy to extremely difficult. (Copies of the paper are available from the undersigned on request.)

A new grading scheme was in operation this year: the total mark assigned to an individual contestant was five times the number of correct answers plus two times the number of questions unanswered. The effect of this was to increase scores, while still leaving the maximum score at 150. The rationale for the change was to make the computation of scores easier, to enhance the validity of the score as a measure of mathematical ability and knowledge and, most of all, to discourage random guessing. Only intelligent guessing will be rewarded under the new scheme!

As a consequence of the new scoring formula, a lot more students scored 80 or better than in previous years. The cut-off mark for the Roll of Honour was set at 90, and 56 students registered this or higher. The top score was 105, and the names of the top ten students appear below.

This year's winner is:

Søren Pedersen,
Ashton School,
Blackrock Road,
Cork.

Team scores were considerably higher as well. O'Connell School, Dublin 1, took first place, with 293. Second place went to Blackrock College, Co. Dublin, who recorded 292 and Marist College, Athlone, took third place with 291.

The top scorers will be presented with prizes in December.

IRISH NATIONAL MATHEMATICS CONTEST 1986

Roll of Honour

	Candidate	School	Score
1	Søren Pedersen	Ashton School, Blackrock Rd, Cork	105
2	Eoin Ó Brolcháin	Blackrock College, Blackrock, Co. Dublin	103
3	Kieran Barry	Marist College, Athlone, Co. Westmeath	101
4	Michael McCauley	O'Connell School, Dublin 1	101
5	Brian Crawford	Sligo Grammar School, The Mall, Sligo	100
6	Mark Forsyth	Wilson's Hospital, Multyfarnham, Co. Westmeath	100
7	Barry Martin	C.B.C. Monkstown, Co. Dublin	100
8	William Whyte	Newpark Comprehensive School, Blackrock, Co. Dublin	100
9	Leonard Brennan	O'Connell School, Dublin 1	99
10	Eamonn Moore	Patrician College, Ballyfin	99

Finbarr Holland

SUMMARY OF RESULTS OF THE 1986 IRISH INVITATIONAL MATHEMATICS CONTEST

The Fourth Irish Invitational Mathematics Contest was held on Tuesday, March 18, 1986. Invitations to participate in this were extended to all those who had scored 85 or better in this year's INMC. Examination materials for this were also supplied by the MAA. This was an essay-type examination. Contestants had three hours in which to answer 15 questions with positive integral solutions. Partial credit was not given: a question was either marked right or wrong. Score cards were returned on behalf of 80 of the invitees.

William Whyte, who attends Newpark Comprehensive School, Blackrock, Co. Dublin, was the top scorer in this contest. William scored 8, and is to be commended for his performance.

Finbarr Holland

ERRATUM

On page 8 of Bulletin No. 16 the name of one of our new members in NIHE Dublin is misspelt: it should be Dr B. Lenoach.

PERSONAL ITEMS

Dr Frank Hodnett has been appointed Associate Professor of Applied Mathematics at NIHE, Limerick.

Dr Robert Critchley has been appointed to a Senior Lectureship in Mathematics at NIHE, Limerick

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VISCOELASTIC BOUNDARY VALUE PROBLEMS

J.M. Golden

1. INTRODUCTION

Linear viscoelastic materials are described by the hereditary constitutive relations (repeated suffix notation understood)

$$\sigma_{ij}(\underline{r},t) = 2\int_{-\infty}^{t} dt'\mu(t-t')\varepsilon_{ij}(\underline{r},t') + \int_{-\infty}^{t} dt'\lambda(t-t')\varepsilon_{kk}(\underline{r},t'), \qquad i,j = 1,2,3 \qquad (1.1)$$

in terms of the stress and strain tensors σ_{ij}, ε_{ij} at position $\underline{r} = (x,y,z) = (x_1,x_2,x_3)$ and time t, and the singular viscoelastic functions $\mu(t)$ and $\lambda(t)$, both zero for negative time in order to incorporate Causality. These are related to the relaxation functions for shear and volume deformations. In particular

$$\mu(t) = \frac{d}{dt}(G(t)H(t)) \qquad (1.2)$$

where H(t) is the Heaviside step function and G(t) is the shear relaxation function, approximated perhaps by a constant plus exponential decay terms. If one exponential is sufficient, the material is referred to as a standard linear solid. A similar relation exists between $(2\mu(t) + 3\lambda(t))/3$ and the bulk relaxation function. This latter quantity may be taken to be a constant, or alternatively, proportional to G(t), for many materials. Equation (1.1) is combined with the dynamical equations

$$\sigma_{ij,j}(\underline{r},t) + \frac{\partial^2}{\partial t^2}u_i(\underline{r},t) = 0, \qquad i = 1,2,3 \qquad (1.3)$$

where $u_i(\underline{r},t)$ are the displacements. It is sometimes possible to neglect the acceleration term in (1.3). This non-inertial approximation will be adopted henceforth. In a body occupying volume V with boundary B, the boundary conditions may for example take the form

$$u_i(\underline{r},t) = d_i(\underline{r},t), \qquad \underline{r} \in B_u(t)$$

$$\sigma_{ij}(\underline{r},t)n_j(\underline{r}) = c_i(\underline{r},t), \qquad \underline{r} \in B_\sigma(t) \qquad (1.4)$$

$$B_u(t) \cup B_\sigma(t) = B$$

where $c_i(\underline{r},t)$ and $d_i(\underline{r},t)$ are specified functions. On taking the time Fourier transform of (1.1), the hereditary integrals become products and in fact both (1.1) and (1.3) reduce to the elastic form with Lamé's constants replaced by the so-called complex moduli. If the boundary regions B_u and B_σ are constant, this observation allows one to reduce any problem to the corresponding one in Elasticity. This is the content of the Classical Correspondence Principle.

Many interesting problems are however not in this category, for example those involving loads moving over a half-space; or the Normal Contact Problem where the load is stationary but varying in magnitude; and also extending or closing crack problems. The basic complication is the following: if a displacement, for example, is known at time t on B_u, it does not follow that it is known at all previous times, so an hereditary integral over this quantity is not necessarily known. An exception to this would be if B_u is non-increasing with time, since if a point is in B_u at time t, it was in it at all previous times. Elaborations on this observation allow certain extensions of the Classical Correspondence Principle (Graham [1] and references therein).

There are many problems however where the boundary regions vary in quite a complicated manner, for example exhibiting consecutive maxima and minima. A method for tackling such problems, involving a certain decomposition of hereditary integrals, was developed a long time ago by Graham [2] and Ting [3]. The main point of the present note is to give an alternative derivation of this decomposition, recently evolved and applied by Cecil Graham and myself, and to give a simple illustration of its use. This derivation leads to a form which appears to be easier to manipulate in certain contexts.

2. DECOMPOSITION OF HEREDITARY INTEGRALS

Let the two functions $u(t)$ and $v(t)$ be related by

$$v(t) = \int_{-\infty}^{t} dt' \ell(t-t')u(t')$$
$$u(t) = \int_{-\infty}^{t} dt' k(t-t')v(t') \qquad (2.1)$$

where $k(t)$ and $\ell(t)$, both zero for negative t, are inverses of each other in the sense that

$$\int_0^t dt'\ell(t-t')k(t') = \int_0^t dt' k(t-t')\ell(t') = \delta(t) \qquad (2.2)$$

in terms of the singular delta function $\delta(t)$. Let $\theta(t)$ be the set of the present and all past times $(-\infty,t]$, which we decompose into two sets $W_u(t)$, $W_v(t)$ where $u(t')$ is given for $t' \in W_u(t)$ and $v(t')$ is given for $t' \in W_v(t)$. In certain applications $v(t')$ may not be known on $W_v(t)$ but can be usefully represented. If we could decompose $v(t)$, for example, as follows

$$v(t) = \int_{W_u(t)} dt'\Pi_u(t,t')u(t') + \int_{W_v(t)} dt'\Pi_v(t,t')v(t') \qquad (2.3)$$

then everything on the right-hand side is known, provided that the sets $W_u(t)$ and $W_v(t)$ can be specified, so that $v(t)$ is given explicitly. A decomposition of this kind can be derived in the following manner. We first define the sets $W_u(t)$ and $W_v(t)$. Let $t_1, t_2, t_3, \ldots$ be the sequence of transition times earlier than t, marking when t' changes from $W_u(t)$ to $W_v(t)$ or vice-versa. We take it that $[t_1,t] \in W_u(t)$ since otherwise $t \in W_v(t)$ and $v(t)$ is known to begin with. Let us write $v(t)$ as

$$v(t) = \int_{t_1}^{t} dt'\ell(t'-t)u(t') + \int_{-\infty}^{t_1} dt'\ell(t-t')u(t')$$
$$= \int_{t_1}^{t} dt'\ell(t-t')u(t') + \int_{-\infty}^{t_1} dt' T_1(t,t')v(t') \qquad (2.4)$$

where

$$T_1(t,t') = \int_{t'}^{t} dt'\ell(t-t'')k(t''-t') \qquad (2.5)$$

from (2.1). The procedure can be repeated to obtain finally

$$\Pi_u(t,t') = T_0(t,t')R(t';t_1,t) + T_2(t,t')R(t';t_3,t_2) + T_4(t,t')R(t';t_5,t_4) + \ldots \qquad (2.6)$$

$$\Pi_v(t,t') = T_1(t,t')R(t';t_2,t_1) + T_3(t,t')R(t';t_4,t_3) + \ldots$$

where

$$R(t;t_2,t_1) = \begin{cases} 1, & t \in [t_2,t_1] \\ 0, & t \notin [t_2,t_1] \end{cases} \qquad (2.7)$$

for all t_2, t_1 and t. Also

$$T_0(t,t') = \ell(t-t') = \int_{t'}^{t_r} dt'' T_{r-1}(t,t'')\ell(t''-t'), \quad r \text{ even}$$
$$T_r(t,t') = \int_{t'}^{t_r} dt'' T_{r-1}(t,t'')k(t''-t'), \quad r \text{ odd} \qquad (2.8)$$

The number of terms in these series depends on the number of transition times t_r. If t_n is the final transition time, then where it is the upper bound in an integral, the lower bound is $-\infty$. In a similar manner, it can be shown that $u(t)$ can be decomposed in the form

$$u(t) = \int_{W_u(t)} dt'\Gamma_u(t,t')u(t') + \int_{W_v(t)} dt'\Gamma_v(t,t')v(t') \qquad (2.9)$$

where

$$\Gamma_v(t,t') = N_0(t,t')R(t';t_1,t) + N_2(t,t')R(t';t_3,t_2) + N_4(t,t')R(t';t_5,t_4) + \ldots \qquad (2.10)$$

$$\Gamma_u(t,t') = N_1(t,t')R(t';t_2,t_1) + N_3(t,t')R(t';t_4,t_3) + \ldots$$

where the quantities $N_r(t,t')$ are given by formulae akin to (2.8) but with $\ell(t)$ and $k(t)$ interchanged.

This apparently trite formalism is actually extremely powerful in the context of non-inertial boundary value problems. The decomposition was developed as mentioned to solve the Normal Contact Problem (2.3) and applied more recently to the steady state case [4]. A form of it is the fundamental ingredient required to write down an integral equation for moving load problems (a special case of which was derived some time ago [5,6]) and perhaps more general problems also. It arises also in the case of crack problems in a manner which we shall now discuss.

3. CLOSING CRACK PROBLEM

Consider a fixed length crack lying along the x-axis occupying the region [-c,c], in an infinite viscoelastic medium. Let there be a constant pressure p(t) acting on both faces while the crack is open, where p(t) may change sign. In an elastic medium, such a sign change from positive to negative would lead to instant closure. This is not the case in a viscoelastic medium, which makes the problem non-trivial and leads to certain interesting effects. While the crack is open, this problem is one covered by the Classical Correspondence Principle and the solution may be immediately written down since the elastic solution is known [7]. In particular, we have that the gap is given by

$$g(x,t) = 2m(x)q(t) \tag{3.1}$$

where

$$m(x) = (c^2 - x^2)^{\frac{1}{2}}$$

$$q(t) = \int_{-\infty}^{t} dt'k(t-t')p(t')$$

The quantity k(t), zero for negative t, is defined by the fact that its Fourier transform $\hat{k}(\omega)$ is given by

$$\hat{k}(\omega) = (1-\hat{\nu}(\omega))/\hat{\mu}(\omega) \tag{3.3}$$

where $\hat{\nu}(\omega)$ is a generalised Poisson's ratio of the material, expressible in terms of the complex moduli $\hat{\mu}(\omega)$ and $\hat{\lambda}(\omega)$ according to the standard formula. The stress intensity factor has the form

$$K_1 = c^{\frac{1}{2}}p(t) \tag{3.4}$$

Since the crack may be open while p(t) is zero or negative, this quantity may also be zero or negative, in contrast to the elastic case. When the quantity q(t) becomes zero, the crack closes and the pressure on [-c,c] is no longer known. Thus, let p(t) be denoted by $p_o(t)$, a known quantity, when the crack is open; and by $p_c(t)$ when the crack is closed. Note however that q(t) is known when the crack is closed. It is in fact zero. But this is precisely the situation we were dealing with above, when deriving the decomposition. From (2.6) and (2.20) we can immediately write down explicit forms for $p_c(t)$ and q(t):

$$p_c(t) = \sum_{r=1,3,5..} \int_{t_{r+1}}^{t_r} dt'T_r(t,t')p_o(t')$$

$$q(t) = \sum_{r=0,2,4..} \int_{t_{r+1}}^{t_r} dt'N_r(t,t')p_o(t') \tag{3.5}$$

the second equation referring to times when the crack is open. If the crack is closed, the condition for the next time of reopening is

$$p_c(t_0) = p_o(t_0) \tag{3.6}$$

while if the crack is open at time t, the condition for the next time of next closing is

$$q(t_c) = 0 \tag{3.7}$$

These latter two equations may be used in conjunction with (3.5) to inductively determine the times of opening and clo[...] The situation simplifies if steady state conditions under a periodic load are assumed. Thus (3.5) - (3.7) constitute a complete solution to the problem, in principle, and they are a simple application of the decomposition derived above. This problem has in fact been solved for the special case of standard linear solid under a sinusoidal load in [8,9] by means of a less general machinery, and detailed results were obtained for the case of a standard linear model. It may b[e] shown, using the explicit forms for (2.6) and (2.10) given i[n] [4], that the general formulae (3.5) - (3.7) reduce to the results obtained in that paper.

Applications of the study of viscoelastic boundary valu[e] problems include the exploration of the phenomenon of hyster[-] etic friction which can be modelled by considering loads mov[ing] over viscoelastic half-spaces. This effect may be signific[ant] in many contexts, notably that of a tyre skidding on a road surface. Normal contact problems are relevant to the study of impact phenomena, while crack problems contribute insights to fracture processes in real materials.

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REALIZING RINGS AS ENDOMORPHISM RINGS - THE IMPACT OF LOGIC

B. Goldsmith

INTRODUCTION

The object of this paper is to give a brief survey of an area of Abelian group theory in which remarkable progress has been made in recent years. A second objective is to indicate how results and techniques from logic are gradually becoming important in this area of algebra; a point to which we shall return later.

Throughout the paper all *groups* shall be additively written Abelian groups and *rings* shall be unital associative rings.

A QUICK COURSE IN ABELIAN GROUP THEORY

The following concepts will be needed from Abelian group theory:

1. A group G is said to be *reduced* if G does not contain a subgroup isomorphic to the additive group of rationals, $\mathbb{Q}$, or the Prufer quasi-cyclic group $\mathbb{Z}(p^\infty)$ for any prime p [This latter group is the additively written version of the (multiplicative) group of p^nth complex roots of unity, with n running over all integers ≥ 0].

2. An element g of a group G is said to be a *torsion* element if $ng = 0$, the identity of G, for some integer $n \neq 0$. If no such integer n exists then g is a *torsion-free* element. The torsion elements form a subgroup tG of G and the quotient G/tG is a torsion-free group.

3. A group F is said to be *free* if it has the form $F = \oplus X_i$ where $X_i \simeq \mathbb{Z}$, the additive group of integers, for all $i \in I$. (These are precisely the projectives in the category of Abelian groups.)

4. The set of all endomorphisms of a group G (= homomorphisms from G to G) form a ring E(G), *the endomorphism ring of* G, if we set $(\phi_1 + \phi_2)(g) = \phi_1(g) + \phi_2(g)$ and $\phi_1\phi_2(g) = \phi_1(\phi_2(g))$ for endomorphisms ϕ_1 and ϕ_2.

5. If G is a group then the subgroups nG, where n runs through all non-zero integers, form a base of neighbourhoods of 0 for a linear topology on G. This topology is called the *$\mathbb{Z}$-adic or natural topology*. If G is torsion-free then this topology is Hausdorff precisely if G is reduced. If subgroups of the form p^nG are chosen then the resulting topology is the *p-adic topology*.

6. The completion of $\mathbb{Z}$ in its p-adic topology is the group J_p *of p-adic integers*. Elements of J_p can be regarded as formal infinite series $s_0 + s_1p + s_2p^2 + \ldots$ where $s_i \in \{0,1, \ldots, p-1\}$. We note that J_p can also carry a ring structure and that it has cardinality $2^{\aleph_0}$. (Further details may be found in, e.g. Fuchs [4]).

7. A group G is *indecomposable* if it cannot be written in the form $G = A \oplus B$ for non-zero groups A and B. Note that if $G = A \oplus B$ then E(G) has idempotents, viz. the projections.

THE REALIZATION PROBLEM.

The basic realization problem can be stated as follows:

"Given a ring A, what conditions on A will ensure that there is an Abelian group G with $E(G) \simeq A$ quâ rings."

The basic problem can be modified (and made harder!) by insisting that the resulting group G should belong to some prescribed class of groups.

The fundamental result in this area is due to A.L.S. Corner [1] in 1963. (The original paper is a beautiful example of how mathematics should be written!)

Corner's Theorem. *If A is a countable, reduced torsion-fre ring then there exists a countable, reduced torsion-free grou G with* $E(G) \simeq A$.

(We remark that properties such as reduced etc. attributed to a ring mean that the underlying group of the ring has the sai properties.)

I shall not attempt to give any proof but merely indicate tha all three conditions are necessary. Consider the following rings: (i) $\mathbb{Q} \oplus \mathbb{Q}$, (ii) $\mathbb{Z}(p) \oplus \mathbb{Z}(p)$ ($\mathbb{Z}(p)$ is the field of p elements), (iii) $J_p \oplus J_p$. Each of these rings satisfies two but not the third of the conditions on the ring A in the abov theorem. However in case (i) if $E(F) = \mathbb{Q} \oplus \mathbb{Q}$ then G would b a vector space over $\mathbb{Q}$. If dim G = n then we would need $n^2=2$ which is impossible. A similar vector space argument shows that (ii) is impossible. Finally if $E(G) = J_p \oplus J_p$ then G i naturally a p-adic module and it will have finite rank. How ever it is known that a finite rank p-adic module which is re uced (in this case $\mathbb{Q}$ must be replaced in the definition by th field of p-adic numbers) is free and this again leads after a little argument to solving $n^2 = 2$. So (iii) is also imposs- ible.

A non-algebraist might reasonably ask why the above resul is important. (I'm assuming the question comes in the conte of pure mathematics and is not related to applications!) One answer is that the result can be used to construct some amazi examples of groups. These groups show that it is practicall impossible to derive any analogue of Krull-Schmidt decompos- ition theory. We content ourselves with three examples which can be produced using Corner's result.

Example 1. *There is an indecomposable group of infinite rank.*

Take $A = \mathbb{Z}[t]$, the integral polynomials in the variable t and apply Corner's result. If $E(G) = A$ then G is indecompos- able since A has no idempotents.

Example 2. *There is a superdecomposable group of countable rank*

(i.e. a group which has no non-zero indecomposable direct summend.)

Let $\Lambda = \{\lambda_r \mid r \in \mathbb{Q},\ r \geq 0\}$ and define $\lambda_r\lambda_s = \lambda_{\max(r,s)}$. Let $A = \mathbb{Z}\Lambda$, the semigroup ring of Λ over $\mathbb{Z}$. It can be shown that the underlying group of A is freely generated by the λ_r and so A satisfies the conditions of Corner's Theorem. Hence we find a group G with $E(G) = A$. However a little calculation shows that if ε is any non-zero idempotent in A then there is a non-zero idempotent ζ such that $\zeta = \varepsilon\zeta = \zeta\varepsilon \neq \varepsilon$. But now if $G = B \oplus C$ then $B = \varepsilon(G)$ for some idempotent ε. However $D = \zeta(G)$ is then a summend of G and $\zeta(G)$ is contained in B since $\zeta(G) = \varepsilon\zeta(G)$. Thus B decomposes as $B = D \oplus E$, some E.

Example 3. *There is a countable torsion-free group A such that* $A \simeq A \oplus A \oplus A$ *but* $A \not\simeq A \oplus A$.

Take Λ to be the semigroup with 1 generated by ρ_i, σ_i (i = 0, 1,2) subject to $\rho_j\sigma_i = \delta_{ji}$. Let $R = \mathbb{Z}\Lambda$, the integral semi- group ring of Λ (identifying the 0 of the semigroup with the 0 of A). Again R is free as a group. If I denotes the principal ideal generated by $\tau = 1 - \sigma_0\rho_0 - \sigma_1\rho_1 - \sigma_2\rho_2$ then R/I is still freely generated as a group. Take $\Lambda = R/I$ and use Corner's Theorem to exhibit G with $E(G) = A$. This group G will have the desired properties. (See Fuchs [4, Vol. 2, 91.6] for more details.)

Corner extended his theorem by using topological rings and he also produced a similar type of result for p-groups in 1969. I worked on the realization problem for p-adic torsion- free modules (1974) and produced a weak realization theorem there. Apart from some modifications and slight extensions of Corner's Theorem (arising mainly from Orsatti and his school in Padova) this was the state of the Realization Problem in 1974.

ENTER SHELAH

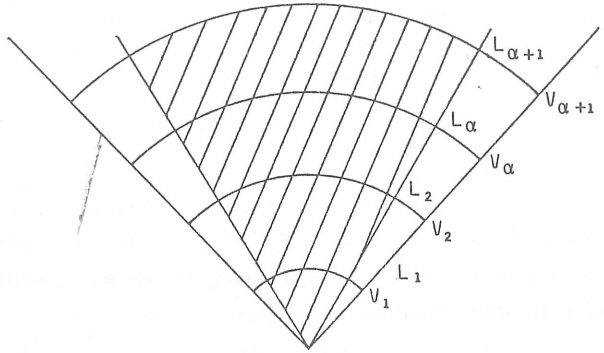
In late 1973 the situation changed dramatically. Saha Shelah applied techniques from model theory and logic to a n ber of problems in Abelian group theory [5] and produced som astonishing results. The most celebrated of these does not relate directly to the realization problem but is nonetheles important to our survey of this problem. I refer of course to his unexpected solution of the Whitehead Problem. This problem, which has its origins in topology, can be phrased a follows:

> *"If A is a torsion-free Abelian group with the property that every extension of $\mathbb{Z}$ by A splits, must A be free*

In other words if G is a group and $G/\mathbb{Z} \cong A$ implies $G \cong A \oplus \mathbb{Z}$, must A be isomorphic to $\oplus \mathbb{Z}$? Shelah's surprising answer is that the problem is undecidable in ordinary set theory!

To see what this means we must now make a small incursi into set theory. The most commonly used set theory consist of the axioms of Zermelo-Fraenkel. We do not need to consider these axioms individually; suffice it to say that they cover "naive set theory". If we include the Axiom of Choice then we have the basic everyday set theory which we denote by ZFC. To understand Shelah's answer to the Whitehead problem we need two additional axioms.

Godel's Axiom. If we let $V_0 = \emptyset$, $V_1 = \mathcal{P}(\emptyset)$, $V_2 = \mathcal{P}(V_1)$ ... and in general $V_{\alpha+1} = \mathcal{P}(V_\alpha)$ (with $V_\sigma = \bigcup_{\alpha<\sigma} V_\alpha$ for a limit ordina σ) then we have the *universe of sets* $V = \bigcup V_\alpha$ (where α runs th ough all ordinals). Alternatively if X is a set, let DefX be the family of all subsets of X of the form $\{a \in X \mid P(a)\}$ where P(a) is any property of sets expressed in the predicate calculus. Now let $L_0 = \emptyset$, $L_{\alpha+1} = \mathrm{Def}(L_\alpha)$ (with $L_\sigma = \bigcup_{\alpha<\sigma} L_\alpha$ fo a limit ordinal σ) and set $L = \bigcup L_\alpha$ (where α runs through all ordinals). L is the universe of constructible sets and in general is thought of as a "smaller" universe than V. (See Fig. 1).



$\phi = V_0 = L_0$

Godel's axiom is that $V = L$.

Martin's Axiom. This axiom arose originally in discussions of the Souslin problem. Before stating the axioms we need to recall some definitions relating to partially ordered sets.

Definitions

(i) If $(P,\leq)$ is a partially ordered set then *the elements* p,q *of* P *are compatible* if there is $r \in P$ with $p \leq r$, $q \leq r$.

(ii) A *subset of* P *is compatible* if every pair of elements is compatible.

(iii) A subset D of P is *dense in* P if for all $p \in P$, there is a $d \in D$ with $p \leq d$.

(iv) A partially ordered set $(P,\leq)$ satisfies the *countable condition* if every pairwise incompatible subset of P is countable.

We can now state Martin's axiom (MA):

Suppose (P,≤) *is a partially ordered set satisfying the coun able chain condition. If* $\{D_i\}$ ($i \in I$) *is a family of dense subsets of* P *with* $|I| < 2^{\aleph_0}$, *then there is a compatible subset* G *such that* $G \cap D_i \neq \emptyset$ *for all* $i \in I$.

An observant naive set theorist will notice that MA follows from the continuum hypothesis (CH). However it has also be shown that (ZFC + MA + negation of CH) is *consistent*. (By consistent we mean that if ZFC is free from contradictions t so also is the above.) Indeed it is also known that ZFC + (V=L) is consistent.

Shelah's answer to the Whitehead problem was this: In (ZFC + MA + negation of CH) there is a group A (of cardinali $\aleph_1$) which satisfies the conditions of Whitehead's problem bu A is not free.

The outcome is, of course, that for naive set theorists the problem is undecidable! This of course was a considerat shock to most people working in Abelian groups. (See Eklof [3] for a very readable discussion of this area.)

RECENT DEVELOPMENTS

While the Whitehead Problem is of no direct importance for the Realization Problem, the techniques developed by Shel in his 1974 paper (and subsequently extended by him) have bec ome the major tool for tackling the problem. The following results indicate some of the many recent advances made:

1. (ZFC + (V=L)). Every cotorsion-free ring is an endomor phism ring. (Dugas and Gobel, 1981).

   (A group is cotorsion-free if it is torsion-free, reduce and contains no copy of J_p, for any p.)

2. (ZFC). If A is any algebra over a complete discrete valuation ring R then there exists a R-module G having A as its "essential" endomorphism ring (Dugas, Gobel and Goldsmith, 1982).

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The state of the art for the Realization Problem (in 1984) has been very elegantly presented in a unified approach by Corner and Gobel [2]. Their results are based on a combinational technique devised by Shelah. In very recent work, Dugas and Gobel and Gobel and Goldsmith have established (in V = L) that most realizations can be obtained in classes of groups which are almost free (in the sense that all subgroups of cardinality less than the cardinal of the realizing group are free). Some of the results so obtained are undecidable in ZFC.

CONCLUDING REMARKS

One of the principal objectives in writing this paper is to convince non-logicians that set and model theory will have a role in our subjects once we deal with any uncountable structure. (Since $\mathbb{R}$ is uncountable that takes in most of us!) This impact is perhaps most apparent in Abelian group theory but the reason for this is clear - finite Abelian groups are completely classified being direct sums of cyclic groups. However other areas of algebra, topology and analysis will slowly but surely become involved also.

REFERENCES

1. CORNER, A.L.S.
   "Every Countable Reduced Torsion-Free Ring is an Endomorphism Ring", *Proc. London Math. Soc.*, (3) 13 (1963) 867-710.

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n is itself prime).

So, for example, $\mu(4) = 0$, $\mu(6) = 1$, $\mu(7) = -1$, $\mu(42) = -1$.

The function $M(n)$ is then defined as

$$M(n) = \sum_{k=1}^{n} \mu(k).$$

The following table gives the first twenty values of this function:

n	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
M(n)	1	0	-1	-2	-1	-2	-2	-2	-2	-1	-2	-2	-3	-2	-1	-1	-2	-2	-3	-3

If you were to continue the above table out into the hundreds or thousands, you would discover that the behaviour of the function $M(n)$ is quite erratic, fluctuating wildly from positive to negative. But, the value of $|M(n)|$ always appears to be less than $\sqrt{n}$, i.e.

$$|M(n)| < \sqrt{n} \qquad (1)$$

for all values of n. Since the Riemann Hypothesis follows (fairly easily) from any universally valid inequality of the form

$$|M(n)| < A\sqrt{n} \qquad (2)$$

for A a constant, Stieltjes' claim in his letter to Hermite to have proved inequality (2) for some A, would have, if true, resolved Riemann's problem at once. It was because of this claim of Stieltjes that when Hadamard wrote his now classic and greatly acclaimed paper proving the Prime Number Theorem in 1896, he apologised for publishing a proof of an already established result. (It was known that the Prime Number Theorem follows from the Riemann Hypothesis.) In fact, as we now know, Stieltjes was in all probability wrong in his claim, and his failure to ever produce a proof may indicate that he himself realised his error. But as will become clear,

it has taken the most powerful computing machinery available in 1985 to settle this matter of inequality (1) conclusively, *and even then no one has produced a specific counter example to the inequality.* Inequality (2) for $A > 1.06$ remains unsolved!

The first systematic investigation of the problem by computational means was in 1897 when F. Mertens produced (by hand calculation) a 50-page table of selected values of $\mu(n)$ and $M(n)$ for n up to 10,000. Since all his tabulated values satisfied inequality (1), he concluded that the inequality was indeed 'very probable'. Though his conclusion was wrong, it was this work which led to his name being attached to the conjecture. The *Mertens Conjecture* is the assertion that inequality (1) is valid for all values of n. (Stieltjes himself had also conjectured that the constant A in his claimed inequality (2) could be taken to be 1.)

The considerable computational evidence obtained subsequent to Mertens' work all tended to support the conjecture. In a series of papers between 1897 and 1913, R.D. von Sterneck published additional values of $M(n)$ for n up to 5×10^6, and in 1963, G. Neubauer computed all values for n less than 10^8, and selected further values for n up to 10^{10}. In 1979, M. Youinaga reached 4×10^8. All these values satisfied not only the Mertens inequality (1), but the even stronger

$$|M(n)| < 0.6\sqrt{n}. \qquad (3)$$

The first value of n for which $|M(n)| \geq 0.5\sqrt{n}$ is n=7,725,038,629, when $M(n) = 43,947$. This was obtained by Cohen and Dress in 1979, who calculated $M(n)$ for all n up to 7.8×10^9. But even they did not find any value for n for which inequality (3) is violated. Thus the numerical evidence in support of the Mertens Conjecture is quite strong - at least it seems so to the human mathematician used to dealing with much smaller numbers. (In point of fact the numerical evidence in support of the conjecture had been 'discounted' long before the final disproof was obtained: analytical evidence pointed the other way.)

The first step in attacking the Mertens Conjecture analytically involves regarding the function M as defined not just on the natural numbers but on all non-negative reals x. To do this simply let $M(x) = M([x])$, where [x] is the largest integer not greater than x (with $M(x) = 0$ if $x < 1$). Inequality (1) can now be re-written as (for all $x \geq 0$)

$$|M(x)|x^{-\frac{1}{2}} < 1 \qquad (4)$$

and inequality (2) as (for all $x \geq 0$)

$$|M(x)|x^{-\frac{1}{2}} < A \qquad (5)$$

Stieltjes claimed to have proved (5), and conjectured (4). Mertens thought that (4) was 'probable'. The present day conjecture is that, on the contrary

$$\limsup_{x\to\infty} |M(x)|x^{-\frac{1}{2}} = \infty. \qquad (6)$$

(This remains, however, an *unproved* conjecture.)

The result of te Riele and Odlyzko which disproves the Mertens Conjecture is that

$$\limsup_{x\to\infty} |M(x)|x^{-\frac{1}{2}} > 1.06 \qquad (7)$$

(No single x is produced for which $|M(x)|x^{-\frac{1}{2}} > 1.06$. The proof is indirect. The discoverers conjecture that no such x exists below at least 10^{20}.)

The connection with the Riemann Hypothesis is quite easily verified. If $\zeta(s)$ is the Riemann zeta function, then for $Re(s) > 1$ we have

$$\frac{1}{\zeta(s)} = \sum_{n=1}^{\infty} \frac{\mu(n)}{n^s},$$

and some manipulation gives (for $Re(s) > 1$):

$$\frac{1}{\zeta(s)} = s\int_1^{\infty} \frac{M(x)}{x^{s+1}}\,dx.$$

Since $M(x)$ is constant on each interval $[n,n+1)$, if inequality (4) (or (5)) held, then the integral in this last identity would define a function analytic in $Re(s) > \frac{1}{2}$, which would give an analytic continuation of $1/\zeta(s)$ to $Re(s) > \frac{1}{2}$. But then the function $\zeta(s)$ would have no zeros in $Re(s) > \frac{1}{2}$, which is the statement of the Riemann Hypothesis.

In fact the Riemann Hypothesis is probably equivalent to

$$|M(x)| = O(x^{\frac{1}{2}+\varepsilon})$$

for all $\varepsilon > 0$. (This was known to Stieltjes.)

Turning now to the specific problem of disproving the Mertens Conjecture by establishing an inequality such as (7), we begin by setting

$$x = e^y, \qquad -\infty < y < \infty.$$

Now define

$$m(y) = M(x)x^{-\frac{1}{2}} = M(e^y)e^{-y/2}.$$

The aim then is to prove that

$$\limsup_{y\to\infty} m(y) > 1.06$$

(or indeed any such inequality where the right hand side is greater than 1.)

The crux of the argument now is to define a function $h(y)$ such that

(i) for any y_0, $\limsup_{y\to\infty} m(y) \geq h(y_0)$;

(ii) it is possible to compute (in practice!) values of h;

(iii) a y_0 can be found for which $h(y_0) > 1$.

(This approach goes back to work of A.E. Ingham in 1942.)

Without going into any details, (i) and (ii) are achieved by means of the following theorem. (Just skip over this part if there are concepts unfamiliar to you.)

THEOREM Suppose that $K(y) \in C^2(-\infty,\infty)$, $K(y) \geq 0$, $K(-y) = K(y)$, $K(y) = O((1+y^2)^{-1})$ as $y \to \infty$; and suppose further that if $k(t)$ is defined by

$$k(t) = \int_{-\infty}^{\infty} K(y)e^{-ity}dy,$$

If the then $k(t) = 0$ for $|t| \geq T$ for some T, and $k(0) = 1$. zeros $\rho = \beta + i\gamma$ of the zeta function with $0 < \beta < 1$ and $|\gamma| < T$ satisfy $\beta = \frac{1}{2}$ and are simple, then for any y_0,

$$\limsup_{y\to\infty} m(y) \geq h_K(y_0),$$

where

$$h_K(y) = \sum_{\rho} k(\gamma)\frac{e^{iy\gamma}}{\rho\zeta'(\rho)}.$$

The simplest function which satisfies the conditions of the above theorem is

$$k(t) = \begin{cases} 1 - \frac{|t|}{T}, & |t| \leq T \\ 0, & |t| > T. \end{cases}$$

Using such a $k(t)$ with $T = 1{,}000$, Spira (1966) showed that

$$\limsup_{y\to\infty} m(y) \geq 0.5355.$$

The function used by te Riele and Odlyzko is

$$K(t) = g(\tfrac{t}{T}),$$

where $T = 2515.286 \ldots$ is the height of the 2,000th zero of the zeta function and

$$g(t) = \begin{cases} (1 - |t|)\cos(\pi t) + \pi^{-1}\sin(\pi|t|), & |t| \leq 1 \\ 0, & |t| \geq 1. \end{cases}$$

Finding the value of y_0 for which the corresponding function $h_K(y_0)$ is greater than 1 then involves an accurate (100 decimal digits) computation of the first 2,000 zeros of the zeta function. This was done using a Newton process and took some 40 hours of CPU time on a CDC CYBER 750 computer at SARA (the Amsterdam Computer Centre). With these values available, finding the required y_0 was achieved using a new algorithm for diophantine approximation due to Lenstra, Lenstra and Lovasz (1982) and took about 10 hours of CPU time on a CRAY-1 computer at Bell Laboratories in Murray Hill, New Jersey. (As you might imagine, the method was not a 'blind search'. Indeed the function $h_K(y)$ only 'rarely' gives a value greater than 0.5, let alone greater than 1.)

The 'magic value of y_0 that the computer found is a negative number of the order 1.4×10^{65}. For this y_0, $h_K(y_0) = 1.061$ (to three decimal places). The 'exact' values are quoted in the paper referenced below.

REFERENCE

For a much fuller account of the solution to the Mertens problem, together with an extensive bibliography on the problem, see *Disproof of the Mertens Conjecture* by A.M. Odlyzka and H.J.J. te Riele, *Journal fur die Reine und Angewandte Mathematik*, 357 (1985) pp. 138-160.

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MATHEMATICS IN FOOD ENGINEERING RESEARCH

J.F. O'Connor

INTRODUCTION

Engineering has often been described as the application of mathematics to various branches of applied science. In many cases this tendency to express physical phenomena in numerical or formulae form is effected in spite of profound resistance from the mathematical principles involved. It is also true that this constant yen of engineers to evolve rules of thumb or working models of systems causes the genuine mathematician to frown. Nevertheless a good working relationship and mutual tolerance/understanding exists between these two schools of numerate thought/application. The advent and proliferation of computer systems of ever-increasing speed and memory capacity, combined with improved availability of numerical software packages, has added greatly to the common ground available to the engineer and mathematician from which they may better serve the needs of both science and industry.

In what may be called the more traditional branches of engineering - civil, mechanical, chemical, electrical and industrial for example - mathematical applications and exampl are well known. It is only in the recent past, however, tha the food engineer has been able to turn to mathematics and mathematicians to glean some assistance in solving the many particular problems associated with serving the food processi industry in an engineering design/research capacity. Indeed it is true to say that food engineering as a discipline is only quite recently emerging as a distinct area of professio endeavour. It owes its development to a large degree to the new tools made available to the food engineer which allow the efficient amalgamation of the skills of the many strands of the engineering sphere from which it draws its working principles.

Many of the problems encountered in the food industry have close parallels in the chemical engineering type industries. There are, however, a number of significant differences which add considerably to the complexity of the food engineer's workload. It is worth listing at this stage the main classes of problems which are the subject of both laboratory based research work and actual in-line process development. Stated briefly then, the main categories are as follows:

1. Flow problems - fluids, solids, liquid-solid solutions, and dispersions.
2. Heat transfer problems - conduction, convection and radiation, and various combinations thereof.
3. Basic process calculations - statistically based estimations of microbial lethality/quality retention.

The actual objectives in solving these various problems centre on four main 'commercial' targets:

(a) PROCESS CONTROL AND OPTIMISATION

(b) QUALITY ASSURANCE

(c) CATALOGUING THE ENGINEERING PROPERTIES OF FOODS

(d) IMPROVED DESIGN TECHNIQUES

MATHEMATICAL MODELLING

Mathematical modelling of food processes is an area which is currently of major research interest. Food process control has developed in the last decade or so to a very high level with only two major obstacles now delaying total automation:

1. Development of continuous in-line sensory devices to monitor process parameters.
2. Development of mathematical models/algorithms that relate these process measurements to finished product (quality) parameters.

Hence, provided the food engineering equipment/instrumentation companies deliver on the new microprocessor based sensory devices, it will only remain for the engineers/mathematicians to patch the resultant process data into a working model. On board a digital control system such a model can be used to either regulate a process or to view the results of proposed process changes via simulation.

In food engineering, modelling is currently employed to achieve optimum results under a number of headings:

1. Optimum microbial lethality;
2. Maximum shelf-life;
3. Maximum quality factor retention, e.g. colour, taste, vitamin content, texture;
4. Product compositional factors - moisture/fat content, density.

TYPICAL PROBLEM

A problem currently the subject of some study is that of processing particulate foods in a fluid stream. With the ever increasing market for convenience foods, solids such as meat or vegetable portions packaged in sauce or gravy mixes are becoming quite popular. To date, however, manufacturing these products is a two-stage, batch-type operation. One of the main obstacles to designing a continuous, aseptic process is the lack of a suitable mathematical model which will allow simulation of the process and hence allow prediction of lethality/quality retention for various combinations of process/product parameters.

A number of models have been proposed for the process but have included some restrictive assumptions which limit the practical application of the model, e.g. Misra *et al.* [4] and Guariguata *et al.* [2]. O'Connor *et al.* [6] have proposed a model based on finite element techniques which is further

outlined below.

The overall problem combines both fluid flow and heat transfer. To simplify the study only the 'holding tube' section of the process is considered here. This is the section of the plant directly after the heat exchanger. Entering this, it is assumed that the desired thermal processing of the fluid phase has been achieved. It is in the holding tube now that the solid particles are 'cooked'. Summarising the problems in the tube will indicate the complexity of the model. For effective simulation then, the following factors must either be known or calculated/predicted:

A. Flow profile of the fluid stream at all points in the tube.

B. Relative velocity of the solid particle(s) with respect to the fluid stream.

C. Residence time distribution of the particles in the tube.

D. Time/temperature profile within the particle and consequent microbial lethality/quality factor retention.

In the model used by the author, laminar flow is assumed in the holding tube and spherical particles are used as a representative shape for foodstuffs. Further complications may, of course, be added. Kwant *et al.* [3] developed flow profiles for fluids with temperature viscosity, for example. Obtaining a prediction of the relative velocity is extremely difficult in a laboratory situation and yet its accurate prediction is essential as it determines the surface heat transfer coefficient which controls the rate of heating of the spheres.

HEAT TRANSFER IN SPHERICAL COORDINATES

Carslaw and Jaeger [1] state the governing differential equation for heat conduction in a sphere with no heat generation as:

$$k_r \frac{\partial^2 T}{\partial r^2} + \frac{2k_r}{r}\frac{\partial T}{\partial r} + \frac{k_\psi}{r^2 \sin\psi}\frac{\partial}{\partial\psi}\left(\sin\psi \frac{\partial T}{\partial\phi}\right) + \frac{k_\phi}{r^2\sin^2\psi}\frac{\partial^2 T}{\partial\phi^2} = pc\frac{\partial T}{\partial\theta}$$

for $0 \leq r \leq R$; $\theta > 0$; $T = T(r,\psi,\phi,\theta)$.

When symmetry is accounted for this equation reduces further to:

$$k_r \frac{\partial^2 T}{\partial r^2} + \frac{2k_r}{r}\frac{\partial T}{\partial r} = pc\frac{\partial T}{\partial\theta}$$

where k_r = radial thermal conductivity;
T = temperature;
r = radial coordinate;
p = sphere density;
c = sphere specific heat;
θ = time.

The boundary condition for the holding tube is given as:

$$\pm\frac{\partial T}{\partial r} + h(T - T_{amb}) = 0 \text{ for } r = R,\ \theta > 0$$

where h = surface heat transfer coefficient;
T_{amb} = ambient temperature of fluid stream.

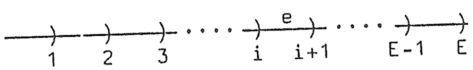
The initial condition is:

$$T = T_o \text{ for } 0 < r \leq R;\quad \theta = 0.$$

FINITE ELEMENT FORMULATION

To allow for product property variation an elemental technique is used to estimate the heat transfer through the particle. This also facilitates a 'mass-average' statement of process efficiency which is much more satisfactory than the traditional single-point analysis of product treatment.

The sphere is therefore divided into a number of connective elements as shown:



The solution is obtained by finding a function $T(r,0)$ satisfying the boundary and initial conditions stated which minimises an integral quantity called a functional. The functional is minimised at every instant of time to give an approximate temperature profile in the sphere. Misra *et al.* [4] have presented the functional as

$$I = \int_V \tfrac{1}{2}[k_r(T')^2 + 2pc\frac{\partial T}{\partial\theta}T]dV + \int_S \tfrac{1}{2}h(T - T_{amb})^2 dS$$

where $T' = \frac{\partial T}{\partial r}$ and $\int_V$ and $\int_S$ are volume and surface integrals, respectively. As the sphere is divided into E elements, this integral may be evaluated separately for each element (*c.f.* Myers [5]). Thus:

$$I = \sum_{e=1}^{E} I^{(e)}$$

where the element 'e' is chosen as the typical element. The sub-integral $I^{(e)}$ then may be calculated from the heat conduction, capacitance and convection terms as they occur in the process. Hence

$$I^{(e)} = I_k^{(e)} + I_c^{(e)} + I_h^{(e)}$$

Following the usual finite element procedure the conduction, capacitance and convection element matrices are developed from the above. These are then combined (O'Connor *et al.* [6]) to form the global matrices which form the final equation to be solved

$$\{K\}\{T\} + \{C\}\{\dot{T}\} = \{F\}$$

where $\{K\}$ is made up by summing the elemental conduction and convection terms, and $\{C\}$ from the capacity terms. $\{T\}$ is the column matrix of nodal temperatures, $\{\dot{T}\}$ the derivative with respect to time and $\{F\}$ is the force vector which accounts for heat convection at the surface. Numerical solution of this equation yields the required temperature profile.

The programme, as constructed from the above procedure, allows the user to predict time/temperature profiles for a spherical particle under varying thermal processes and including data on the product's thermal properties. The next step is to translate the temperature data into actual microbial figures, which may be done using any of a number of standard procedures (Stumbo [7]).

CONCLUSION

The above brief run through a typical process modelling is but one example of the type encountered in the food processing sector. The problems and their solutions are generally multidisciplinary in nature. In many respects there are so many variables, both in process and in raw materials, that the task seems impossible, the opportunities for error/instability quite widespread. As with many engineering problems, however, the solutions are limited to a set of finite possibilities and, in many cases, the experienced food engineer can impose constraints/guidelines as to what type of answer the model may produce.

It is to be hoped that in spite of, or maybe because of, the many complexities in biological/food systems more mathematicians may be encouraged to contribute to solving this most challenging set of problems.

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WEDDERBURN'S THEOREM REVISITED

Des MacHale

One of the most delightful results in ring theory is the following theorem of J.M. Wedderburn (1905).

THEOREM *A finite division ring is a field.*

Besides being important in algebra, Wedderburn's theorem also finds applications in geometry. For example, it provides the only known proof that in a finite geometry the configuration of Pappus is implied by that of Desargues. Herstein's TOPICS IN ALGEBRA can be recommended for its proofs and discussion of Wedderburn's theorem.

The purpose of this note is to prove the following result:

THEOREM *Let* R *be a finite ring with unity and let* T *be the set of invertible elements of* R. *If* $|T| > |R| - \sqrt{|R|}$, *then* R *is a field.*

Thus, where Wedderburn's hypothesis demands that $|T| = |R| - 1$ to force the conclusion that R is a field, we demand only that $|T| > |R| - \sqrt{|R|}$ to force the same conclusion. In a sense, our result is the best possible. Consider the ring R of residue classes mod p^2, for any prime p. This ring has exactly $\phi(p^2) = p^2 - p = |R| - \sqrt{|R|}$ invertible elements, but R is clearly not a field.

To prove our theorem we need a number of elementary lemmas. Some of these are well-known but we include proofs for convenience. We let R be a finite ring with unity $1 \neq 0$. A left zero divisor is an element $x \in R$ such that $xy = 0$ for some $y \in R$, $y \neq 0$. A right zero divisor is an element $x \in R$ such that $yx = 0$ for some $y \in R$, $y \neq 0$. In particular, 0 is a left zero divisor and a right zero divisor.

LEMMA 1 *For* $b \in R$, *if* b^n *is a left (resp. right) zero divisor for some* $n \geq 1$, *then* b *is a left (resp. right) zero divisor.*

PROOF Choose $n > 1$ minimal such that b^n is a left zero divisor and let $t \neq 0$ satisfy $b^n t = 0$. Then $b(b^{n-1}t) = 0$ implies that b is a left zero divisor, since $b^{n-1}t \neq 0$. A similar proof applies for right zero divisors.

LEMMA 2 *If* $b \in R$, *then either* b *is invertible or* b *is both a left and right zero divisor.*

PROOF Since R is finite there exist $i, j \in N$ with $b^{i+j} = b^i$. Thus $b^{i+j} - b^i = b^i(b^j - 1) = (b^j - 1)b^i = 0$.

Thus, either $b^j = 1$ or b^i is a left and right zero divisor. If $b^j = 1$ then b is invertible, and the alternative possibility implies that b is both a left and right zero divisor by lemma 1.

The key lemma is originally due to the Ganesan [1]. We prove a slightly modified version suitable for our needs.

LEMMA 3 *Suppose that* R *has* $n > 1$ *left divisors of zero. Then* $|R| \leq n^2$.

PROOF By lemma 2, R has precisely $n > 1$ right zero divisors also. For $x \in R$, let $A(x) = \{r \in R \mid xr = 0\}$. Since R has n right zero divisors and $n > 1$, there exists $x \in R$, $x \neq 0$ such that $1 < |A(x)| \leq n$. Let $y \neq 0$ be an element of $A(x)$. Then $|yR| \leq n$, since $yR \leq A(x)$ because $x(yr) = (xy)r = 0$.

Now consider the groups $\{R,+\}$ and $\{yR,+\}$. Define a function $f : R \to yR$ by $f(r) = yr$, for all $r \in R$. By the left distributive law, f is a homomorphism from R onto yR. The kernel of f is $\{r \in R \mid yr = 0\} = A(y)$, so $R/A(y)$ and yR are isomorphic as Abelian groups. In particular,

$$|R| = |A(y)||yR| \leq n.n = n^2, \text{ as claimed.}$$

We can now prove the stated theorem. Let D be the set of (left and right) zero divisors of R and suppose that $|T| > |R| - \sqrt{|R|}$. Since no zero divisor is invertible, lemma 2 implies that $|T| + |D| = |R|$. If $D = \{0\}$, then $|T| = |R| - 1$, so R is a field by Wedderburn's theorem. Otherwise, let $|D| > 1$. The conditions of lemma 3 are satisfied, so $|R| \leq |D|^2$ and $\sqrt{|R|} \leq |D|$. However, combining $|T| > |R| - \sqrt{|R|}$ and $|T| = |R| - |D|$, we get $\sqrt{|R|} > |D|$, which is a contradiction. This establishes the theorem.

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CAREERS FOR GRADUATES IN MATHEMATICS

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This article is based on a talk given earlier this year to staff of UCC's Mathematics Department. It gives an account of the careers open to graduates who have followed degree programmes in mathematical disciplines including information on the actual first destinations of Irish and UK graduates and looks at the broader career horizons open to numerate graduates with the ability to think logically and quantitatively.

1. CAREER OPTIONS - AN OVERVIEW

In recent years graduates in Mathematical disciplines have fared better than most graduates entering a difficult employment market. This is because the quality of numeracy is increasingly important in a wide range of careers and also because their specific skills in mathematics, statistics or computer science are in demand in industry and commerce - particularly in engineering, computing and finance.

This first section looks at the careers for which a Mathematics-based degree is either required or particularly appropriate.

Teaching and Lecturing

The cuts in post-primary education expenditure in early 1983 resulting in a deterioration in staff/student ratios had an immediate effect on job prospects for newly qualified teachers of all subjects. However the number of arts and science graduates entering teacher training is falling steadily so the determined graduate with the right personal qualities for teaching should not be discouraged. Some indication of the decline is given in the annual report on "first destinations of UCC's graduates" which shows that in 1985, twenty-two science graduates entered the Higher Diploma in Education, representing 13% of the graduating class. This compared with 63 (54%) ten years ago in 1976. In addition, computer studies is becoming increasingly important at second-level so that graduates who have taken some courses in computer science have an advantage. In practice few highly qualified mathematics graduates are choosing to enter teacher training, perhaps feeling over-qualified or able to find more rewarding careers elsewhere. This does not augur well for the standard of mathematics in post-primary schools.

Third-level opportunities vary depending on the type of college. Competition for university posts is extremely keen, even for the most highly qualified candidates. However the RTCs are at present experiencing a shortage of well qualified applicants for lecturing posts in mathematics and computer science. A number of these posts are being filled on a temporary basis by candidates having less than the required academic qualifications or experience. A move by undergraduate students from mathematics to engineering and computer science courses and more rewarding careers in industry and commerce is blamed for these shortages.

Scientific Research, Design and Development

Because of the nature of Irish industry, there are limited opportunities in scientific and technical R & D in Ireland, particularly for physical scientists and mathematicians, but a number go overseas. In fact in recent years there has been a rapid increase in the number of graduates including those in mathematics, going abroad to take up their first job. They work for research departments of the nuclear, aeronautical, telecommunications and computer industries and in government research organisations dealing with matters such as environmental control and defence. Recent vacancies received in the Careers Office include research on the dispersal rates of effluents and on droplet formation in chimneys with the Waren Spring Laboratory - an organisation dealing with environmental matters, which recently completed a study of smoke pollution in Dublin. GEC in the UK has 350 vacancies for maths/computer science graduates in 1986 in operational analysis, development engineering, mathematical modelling and evaluation of defence systems. Mathematicians work in process control in many types of industry, e.g. in the process control of fibre production or in the control of steel production in a rolling mill.

Much of the work in these areas involves multidisciplinary teams of graduates including engineers, physicists, mathematicians and computer scientists.

Economic and Social Research

There are limited opportunities in this country for statisticians, particularly those who have also studied economics to work on research into social, economic and medical problems. The Economic and Social Research Institute, which employs more than 30 people working directly in research offers a number of Research Assistant and postgraduate Student Fellowships each year. This type of work is developed a little further under the heading of Statistical Work below.

Management Services and Computing

Management Services personnel investigate, define and solve problems using analytical, mathematically based techniques. Operations Research, organisation and methods, work study and computer services are the main specialisations. Only large organisations employ OR specialists. The OR unit of the Department of the Public Service in Dublin does work for the whole of the Public Sector in Ireland. Entry to this work is usually through the Civil Service Administrative Officer competition, but in 1984 two graduates were recruited by direct entry. There are opportunities each Summer for mathematical science or engineering students to work in this unit. Aer Lingus advertised earlier this year for an OR specialist to work on the analysis of current airline and organisation problems as part of a team working in a wide range of management problems in every section of the airline.

Computing is the major function in most management service departments with graduates working in programming and systems analysis. The universities, financial institutions, central and local government and manufacturing industries are all major employers of computer personnel. Essentially they are using new technology to improve and speed the flow of accurate information to managers, thereby helping them to make better decisions.

Computer manufacturers offer opportunities in research and development, applications programming, software development, customer support, marketing and sales. There has been a rapid growth in manufacturers setting up software development units in Ireland - IBM in Dublin, LM Ericsson in Athlone and CPT in Cork are examples and another international computer company is about to announce a major project for Dublin. Computer consultancy - software houses, systems consultants - is another expanding and important area of employment, offering opportunities for graduates with an entrepreneurial flair, good commercial sense and possibly some work experience, to start their own business.

Computer related careers are a major growth area for graduates and this is reflected in the growth of third-level computer science courses and the number of students opting for these courses. However, many employers, particularly large ones who offer planned training to new entrants do not demand that all their graduate entrants have specialised in computer science in their degree courses. They are more interested in the potential of the graduate to be trained to do the job

they want and may use an aptitude test to measure that potential. A strong background in mathematics or statistics combined with computer studies is an ideal basis for entry to most computer related jobs. However, students are not always aware of this and are tending perhaps to opt for what they see as the more "vocational" computer courses.

Statistical Work

Most opportunities for statistical work occur in the public sector and in some commercial activities. The Central Statistics Office, employing 24 statisticians and 7 senior statisticians has a small but regular recruitment of graduates in mathematics, statistics, computer science and economics. Semi-State bodies including the ESRI, the Central Bank and the IDA are other employers and in these cases the graduate who has studied economics along with mathematics based courses is in a particularly strong position. It appears however that relatively few of the most talented school leavers seriously consider entering an arts or social science faculty to study economics and mathematics, being drawn instead to study pure science and engineering. The Central Bank offers opportunities for economists and occasional vacancies for statisticians to candidates with a good primary degree and normally a postgraduate qualification in statistics, economics or a related subject. Statistics must, however, have been a major subject in one of these degrees. Some experience of computing is a distinct advantage.

There are occasional opportunities with a small number of Market Research companies. Work here might include surveys of statistically acceptable samples of consumers, to test consumer preferences, resistance to prices, loyalty to competitors products and many other matters.

Meteorology

There is occasional recruitment to the Meteorological Office which has a staff of 45 Meteorological Officers and 10 senior Meteorological Officers. Degrees in mathematics, mathematical physics or experimental physics are the most appropriate background for this work. The most recent recruitment was in February 1985.

Actuarial Work

Actuaries apply the theory of probability and statistical methods to financial affairs, usually with life assurance and insurance companies and with actuarial consultants. While employers still employ school leavers with excellent mathematics, most now prefer to recruit graduates with good honours degrees in a mathematical discipline particularly statistics. There are more vacancies than there are interested and suitably qualified applicants and firms have had to recruit school leavers when they would have preferred to recruit graduates. Career prospects are excellent but the professional examinations are difficult and part-time study is never easy. As in accountancy, however, there is a move towards full-time initial training courses - two new postgraduate one-year courses have been introduced in the UK and some Irish life assurance companies are prepared to second trainees to these courses.

2. GRADUATE SUPPLY AND DEMAND

Because of the work of the Careers Service in the University Colleges information is now available on the type of job vacancies available to graduates and notified to the Careers Offices. Information is also available on what is happening to graduates when they complete their primary or higher degrees.

The UCC Careers Office produces a "Recruitment Activity" bulletin for internal circulation to UCC staff, giving statistical information on the vacancies received by type of employer,

type of job and type of academic discipline. The summary for the year 1985 for academic discipline is given in Table 1.

Education (H. Dip. in Ed.)	81
Engineering - Electrical	655
Engineering - Civil	14
Engineering - Mechanical	14
Dairy and Food Science	27
Meat Science	4
Food Business	9
Biological Sciences	11
Chemistry	19
Computer Science/Mathematics	203
Geology	0
Physics	26
Any Science	12
Commerce	59
Economics	2
Law	0
Psychology	5
Social Science	10
Any Social Sciences	16
Arts	5
Any Discipline	883
TOTAL	2135

TABLE 1: Vacancies notified to the UCC Careers Office 1985

N.B. Just under half of the above vacancies came from approximately 160 Irish employers. The remainder from 10 overseas employers who sent representatives to UCC to interview students and graduates.

The very big number of vacancies in the mathematics/computer science category is a good indicator of the big demand at home and abroad, dominated by opportunities in computing.

The breakdown by job type is not detailed enough for this article but the following examples give an idea of the vacancies received.

Operations Research	- Aer Lingus, Civil Service, Plessey plc.
Actuarial Work	- Stokes Kennedy Crowley, Irish Life Assurance, Shield Life, Hibernian Life, Life Association of Ireland.
Statistical Work	- ESRI, Central Bank, Civil Service, Irish Marketing Surveys.
Meteorology	- Meteorological Office.
Consultancy	- Cork Microcomputer Systems, SWS, Statistical Software Ltd, Arthur Andersen.
Software Development	- IBM, LM Ericsson, CPT, Bourns Network Division, Philips Eindhoven, Plessey plc.
Management Information Systems (MIS)	- Wang Laboratories BV, Trinity College Dublin, De Beers Industrial Diamond.
Engineering, Research, Design & Development	- Philips Eindhoven, Plessey, BBC, Schlumberger Electronics.
Lecturing	- Waterford, Dundalk and Tralee RTCs.

Graduate First Destinations

Each Spring a survey of all graduates of the previous year is carried out by the Careers Services of the third level colleges and by the NCEA. The information is published by the individual colleges and a composite report is published by the Higher Education Authority. More is known therefore about the career destinations of graduates than about any other group of young people leaving full-time education in this country. The information in the following tables relates to all mathematical disciplines combined since the various reports do not make a separate analysis of mathematics, statistics and computer science degrees.

The most recent figures available are for 1985 graduates who completed honours or higher degrees in 1985 at University College Cork and University College Dublin.

Further Study		
	Ireland	15
	Overseas	3
Teacher Training		2
Other Vocational Studies		-
Work Experience Schemes		-
Seeking Employment		2
Gained Employment		
	Ireland	37
	Overseas	22
Total		81

TABLE 2(i): 1985 "First Destinations". Maths/Stats/Comp. Primary and Higher Degrees, UCD, UCC 1985

Civil Service	-
Local Government	-
Health Boards/Hospitals	-
Education	13
Semi State Bodies	3
Industry	22
Commerce	18
Professions	3
Other Work	-
Total	59

TABLE 2(ii): "Gained Employment". Analysis by Type of Employer

The tables show a small number entering teacher training, while industry and commerce are the major employers. The great majority of those listed under "education" are working in third-level colleges - in lecturing posts in RTCs or as research assistants/demonstrators in university colleges. Of those in employment 38% are working overseas.

The UK situation is of interest because the majority of those Irish graduates who go overseas go to the UK. Table 3 gives the details.

Further Academic & Vocational Studies	521
Teacher Training	371
Seeking Employment	305
Gained Employment in UK	2231
Other Categories	704
Total	4132

TABLE 3(i): "First Destinations". Maths/Stats/Comp. Sc., Primary Degrees, UK, 1983

Management/Administration	72
Buying, Selling & Marketing	57
Operations Research	22
Computing	975
O.M. Work Study, General Management Services	133
Accountancy	308
Banking	29
Actuarial Work	180
Other Financial Work	47
Economics & Statistical Work	38
Teaching, Lecturing	49
Scientific Research, Design and Development	93
Engineering	104
Other Scientific, Technological Services	32
Others	84
Total	2231

TABLE 3(ii): "Gained Employment". Analysis by Job Type

These tables show that computing is the major area of employment but the finance category, particularly accountancy, actuarial work and banking with 564 or 25% of those in employment, has been a particularly buoyant area in recent years.

3. OTHER OPPORTUNITIES

The majority of Irish graduates tends to seek employment which is related in some way to their degree studies - to a greater extent than their counterparts in the UK where both students and employers are more flexible in their attitudes towards employment. In fact university graduates with honours degrees along with personal qualities which include good communication skills, good judgement and an enquiring mind have excellent prospects of entering and succeeding in a wide range of careers in industry, commerce and the public sector. Just a few of the careers which should be of interest to graduates in mathematics based subjects are indicated here.

Financial Work

The major accountancy firms in Ireland and overseas recruit graduates of all disciplines as trainees in auditing, taxation and management consultancy and have a particular interest in science and engineering graduates who have management and commercial potential. Non-business studies graduates can now take a one-year full-time Diploma in Accounting in NIHE Dublin which gives them major exemptions from professional examinations.

While Merchant Banks recruit a small number of graduate trainees the Associated Banks still have an upper age limit of 21 for new entrants. However within this limit there is a distinct move towards recruiting graduates and some graduates who have followed three-year degree courses will fall within the age limit. While some will follow a career in general bank management there are opportunities to specialise in economic intelligence, investment analysis and management services.

Other very occasional opportunities arise in stock broking, insurance and building societies.

The Public Sector

In both business and government many problems demand quantitative analysis. The mathematician who presents solutions convincingly and gives leadership has much to offer in the field of public administration. Reference has already been made to specialist posts in the Civil Service but the majority of honours graduates enter work concerned with general administration management and policy making. Even those who work in specialist areas like operations research often move on to other work after a time in order to broaden their career horizons and enhance their career development. The post of Tax Inspector is open to all honours graduates but those with backgrounds in mathematics and economics have very appropriate qualifications. While Civil Service recruitment has been cut back in recent years because of a recruitment embargo, an increasing number of vacancies have become available during the past year, resulting in approximately 40 appointments being made at Administrative Officer level. The major Civil Service graduate competitions are advertised each November. There are occasional opportunities for graduates to work in administration in semi-state bodies and in local government and health boards. A number of Assistant Staff Officer posts are now open to outside competition and these offer the graduate a way into local government administration. Graduates can compete for library assistant posts in the public libraries and library work or information science opportunities are also available in universities and research establishments, e.g. NBST, and occasionally in industry.

Industry

It is not possible in a few paragraphs to describe all the opportunities within "industry" because this term covers a spectrum of activities carried on in basic industries including agriculture, oil and mining, manufacturing industries including pharmaceuticals, electronics, soft drinks and clothes, and the growth area of service industries. Within these sectors there is a variety of functions all of which can offer opportunities to graduates. These include production, distribution, sales, marketing, accounting and finance, personnel and industrial relations, training, engineering, technical support and management services.

Some specialist areas such as management services have already been mentioned but graduates can find plenty of scope to use their problem solving skills in other functions. Perhaps one example will suffice.

One aspect of production is involved with the scheduling and the control of materials. Materials can account for as much as 60%-80% of manufacturing costs in some high technology industries, and large manufacturers may use up to 10,000 separate inventory items. The strategic importance of production control and materials management is therefore quite clear and sophisticated data processing techniques such as MRP (Material Requirements Planning) have been developed to manage this complex area, allowing managers to reduce expenditure on costly stocks and improve productivity. The large electronics industry in Ireland has a distinct shortage of such skilled managers.

4. FURTHER INFORMATION

The Careers Information Rooms of the University Careers Services hold a great deal of information about jobs, employers and professional training. In addition they have information on postgraduate studies for higher degrees at home and abroad and on scholarships and awards. Students should be encouraged to use these information sources as well as the other career planning activities offered by the Service. Final-year students can register with the Services and be notified of vacancies - the Services are the major source of contact with employers offering graduate employment prospects at home and abroad.

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MATHEMATICAL EDUCATION

MATRIX: COMPUTER ASSISTED MATHEMATICS TEACHING

Ted Hurley

INTRODUCTION

The main aim of this article is to describe the teaching of certain aspects of Mathematics using the MATRIX program. Before discussing this in detail, I make a few general comments on relationships between Computers and Mathematics which I hope will stimulate some discussion in this whole area.

Computer Science at most 40 years in existence, is now the best financed scientific subject, whereas Mathematics, which has been around, developing and of immense value to science, technology and society for over 3000 years is the least financed. Hardy says in *A Mathematician's Apology*, "A Mathematician, like a painter or a poet, is a maker of patterns. If his patterns are more permanent than theirs, it is because they are made with ideas." Mathematics is also the least financed of the arts. The problems of illiteracy are recognised, and rightly so, and processes have been set up to tackle the problem. Little, if anything, is being done to eliminate innumeracy, which will become even more of a problem as technology develops.

When a Mathematical idea becomes useful it appears no longer to be Mathematics. The number of "in" words associated with Computer Science continues to expand - C.I.M., C.A.M., A.I., C.A.D., C.A.L., (Exercise: State or find out the meanings!), etc. Programming is no longer simply programming but rather "software engineering". We need more "with it" words for Mathematics - how about I.E. ("Intelligence engineering") or S.E. ("Symbolic engineering)? Write your suggestions to the editor!

With the advent of the era in Computer Science of software engineering, more and more the importance of a Mathematical background, in areas such as discrete Mathematics - combinatorics and graph theory, is beginning to be recognised. Because of the intimate relationships, Computer Science has and will continue to contribute to the research, development and teaching of Mathematics. Computer Science is the first to reciprocate the help which Mathematics has given to many subjects.

U.C.G. VAX

We have on the University College Galway VAX 11/785 (U.C.G. VAX 1) two beautiful and powerful programs CAYLEY and MATRIX. These programs have been developed at Sydney over the past ten to twelve years by a group headed by John Cannon.

In a previous article [1], Pat Fitzpatrick described some of the features and uses of CAYLEY. At the moment, this program is used as a research aid but on account of the enormous number of structures available it would be our aim to develop it as a teaching aid for honours undergraduates and postgraduates in group theory, ring theory, fields, modules and possibly Lie algebras. Another area where I would envisage its use is in the teaching of group representation theory to Physicists - CAYLEY has the capacity to print out character tables.

DESCRIPTION OF MATRIX

MATRIX is suitable for many elementary and more advanced undergraduate courses. It can best be described as a *laboratory tool* for the teaching and learning of Mathematics. It is *not* a package as such (e.g. NAG, MACSYMA) where the data is simply supplied and the computer prints the answer, although it can be used in such a capacity. It has, when working as a

teaching process, essentially two components: (a) the execute part of MATRIX, and (b) the PROBLEM LIBRARY which is attached to MATRIX and from which the students derive their problems and/or instructions.

Note however that a PROBLEM in MATRIX can mean a problem in the usual sense or a teaching process or a combination of these. The PROBLEMS can perhaps be put in three categories.

p1: By a series of HINTS and DISCUSSION the student is guided through a complete explanation and possible answer.

p2: A problem, in the true sense, is produced for the student and the computer performs the computation on instruction.

p3: A problem is produced and some explanation is given if the student is stuck.

It is often possible, with the DISCUSS command, to tell the student whether or not he/she has obtained the correct answer.

The arithmetic in MATRIX is *rational*, unless otherwise required. This has obvious advantages and corresponds with the students' own arithmetic. The number 2/3 has much more meaning than 0.666667!

I list below some of the PROBLEMS I have developed with my classes this year. Other types are of course possible.

Entering matrices.

Echelon Form.

Row and column operations (used, for example, in inverting matrices or given symmetric A, finding non-singular P such that P^tAP is diagonal D with entries from {0,1,-1}).

Bases for vector spaces (for example, find a basis for the solution space of a system of equations).

Eigenvalues, eigenvectors and diagonalisation of a matrix.

Difference equations and finding large powers of a matrix.

Transition matrices.

Change of base.

Input-Output matrices.

Gram-Schmidt process, orthogonal matrices and orthogonal reduction of symmetric matrix.

Kernel and image of a linear transformation.

Systems of differential equations.

Simultaneous reduction.

Linear Programming.

It is possible to handle realistic problems using MATRIX where hand calculations are out of the question. For example, Linear Programming problems, even for a small number of variables, go quickly out of hand and teaching and examining this topic by computer seems an ideal solution. I have been able to enter a number of problems so that each student receives a different set of data for a particular type of problem. (The PROBLEM LIBRARY developed contains an infinite number of problems!)

THE STUDENTS' VIEW

No experience of programming is necessary. All the commands necessary to run MATRIX and to attach the PROBLEM LIBRARY are set up in a LOGIC COMMAND. The student simply logs in and calls up MATRIX with $MAT, or, if he/she wishes to keep a record of the session for possible printout later, with $MAT/LOG. The prompt in MATRIX is ? and so, for example, ? PROBLEM 100 prints the problem on the screen. If the student is stuck he/she may ask for HINT if this is available.

All the MATRIX commands are available on a quick reference card (which is in total one A4 size page) and this has been found to be sufficient, although a more detailed reference manual of about 50 pages comes with the program. It is not

possible to list all the commands available but they include addition, multiplication, exponentiation, transpose, dot and cross products, rowops, the standard functions (cos, log etc.), eigval, charpoly, submatrices, tableau, pivot and even HISTORY to look back over what has already been done. Procedures (subroutines) are also possible and special matrices may be called, e.g. ZERO (m,n) or HILB (n). It is possible to turn off to the student any of these commands - for example at an early stage the PIVOT command could be turned off but as the course progresses, this command would be allowed. You may perhaps allow the students to read off the eigenvalues and eigenvectors and thus proceed to more advanced problems or you may require them to work these out for themselves.

I have used the program for a third year pass course in Algebra for Science, Arts and Commerce students (about 120 students in total) and a third Engineering Mathematics Option course with about 55 students. (There were only 35 in the option until they heard about MATRIX!) It is certainly possible to use MATRIX also for first and second year pass and honours and for service courses - the classes were chosen solely because these had been assigned to me.

The MATRIX course was compulsory and each student had the terminal booked for him/her for two hours each fortnight. They were also free to work on it whenever terminals were available and many did so. Printouts of solutions (or attempts!) to particular problems had to be submitted by a particular date or the MAIL facility could be used to send their solutions to my directory.

At the end of the course, each student was asked to complete, anonymously, a questionnaire giving his or her opinion and comments on the usefulness or otherwise of MATRIX. On the question "All in all, did you think MATRIX is worthwhile?", over 70% were very much in favour (gave 9 or 10 on a scale of 0-10). It is also my view that the students learned much more

Mathematics and became more proficient on the (normal) problem sheets. There is the added experience of working on computers and many had had no such previous contact.

DIFFICULTIES

There were of course difficulties. The running and organising of such a scheme takes a tremendous amount of time and work. The main problems the students had were hardware problems, with the non-availability of terminals and printers as required. These can be overcome. The amount of time involved in problem library development should not be underestimated. Hopefully, others will become interested and it will be possible to exchange problems. This would be a simple process via the HEANET (Higher Education Authority Network), and post and transfer of files in and out of Ireland to anywhere in the world with a similar network seems to be possible. For further information on HEANET read the appropriate section of [1] and/or contact your local computer services.

The most serious Mathematical criticism is that the students' ability to calculate or manipulate expressions may be further reduced. We do have a problem with numeracy and manipulation but I argue that computer-assisted learning does not make it any worse and can on occasions be very beneficial. Many students refuse to continue when a seemingly unmanageable calculation is presented and are never able to get to the bottom of a problem. Errors often occur in arithmetic no matter what, and progress is impeded. I once felt that calculators should not be allowed at examinations.

CONCLUSION

If you would like to try out MATRIX, and I highly recommend it, you may, if you are not in Galway, do so through the HEANET. If you would like to use MATRIX in one of your courses, it would be necessary to purchase the program from Sydney by

writing to John Cannon at the University of Sydney, Sydney, New South Wales, 2006, Australia. I will be happy to correspond with anyone who is interested.

We hope very soon to implement KOENIC, a teaching program for graph theory.

REFERENCE

1. FITZPATRICK, Patrick,
"CAYLEY: Group Theory by Computer", *IMS Bulletin* 16 (1986) 56-63.

Department of Mathematics,
University College,
Galway

BASIC MATHEMATICAL SKILLS OF U.C.C. STUDENTS

(A report on a test administered in October 1985)

Donal Hurley and Martin Stynes

1. INTRODUCTION AND SUMMARY

A recent article [1] in this journal gave evidence of serious deficiencies in the mathematical skills of many students entering Cork R.T.C. in Autumn 1984. We wish to report here similar evidence showing that many students entering Science (and a small number entering Arts) in U.C.C. in Autumn 1984 displayed the same disturbing inability to cope with short basic mathematical problems. We discuss the implications of these results.

2. THE STUDENTS; THE TEST

First year Science students in U.C.C. must take one of the subjects Mathematics (M) or Mathematical Methods (MM). The choice is based on the student's planned career path in later years at U.C.C. On the evidence of Leaving Certificate grades, M students are on average mathematically superior to MM students.

It was decided to administer a test to 1st Science students in Autumn 1985 to determine which mathematical deficiencies (if any) were widespread among them. Such deficiencies would then be tackled by, for example, assigning specific students to small group tutorials. As MM already contained a remedial mathematics component, the test was only given to M students. This group also included some 1st Arts students who were taking computer studies and/or honours mathematics.

The test was given at the end of the second week of classes. It consisted of thirty multiple choice questions, with four possible answers per question, to be completed in one hour.

The students were warned in advance of the test and were provided with copies of a similar test given to the previous year's 1st Science students. The questions were based on material common to both the Higher and Ordinary Level Leaving Certificate course in Mathematics, with the exception of Q.29 which is exclusively Higher Level. Use of calculators or of mathematical tables was not allowed.

We used the following marking scheme. Each correct answer received 1 mark. Each incorrect answer received $-\frac{1}{3}$ mark. No marks were added or subtracted for questions not attempted. Students were advised not to guess answers.

While no pass mark for the test was set in advance, it was felt that, considering the basic nature of the material, a score of at least 20 marks should be expected from any competent student. In the event, 219 students (165 Science, 54 Arts) sat the test and the average score was 16.7 marks. Appendix A lists the test questions and Appendix B the students' success rate for each question (the number of students answering correctly being expressed as a percentage of the number who attempted the question). These results show that logarithms (Q.21, 27) were not understood by the majority of the students (a familiar complaint of third-level lecturers). On the more elementary technique of conversion of units (Q.26) only a dismal 31% of the students answered correctly. The success rates for Q.1, 10, 12, 22 and 23 are also very poor given the basic level of these questions.

3. IMPLICATIONS OF THE RESULTS

The results of this test are similar to those of tests administered to First Science students for the past four years. In discussing the situation with colleagues teaching mathematics at other third-level institutions in Ireland, it is clear that the same weaknesses are prevalent among students nationally. We believe that if similar tests were given to students in other colleges, the results would be the same. Some colleagues might wish to try.

This is disturbing at two levels. First there is the lack of numerate skills which are required by all adults to cope with everyday living, not to mention studying science at College. Because of the wider use of calculators, microcomputers and computers, the trend is to rely more and more on quantitative presentation of information. The evidence suggests that we can't be confident that all First Science students at U.C.C., let alone the population at large, have mastered the requisite numerate skills.

The second level of concern about the results of these tests is related to studies being undertaken by First Science students at U.C.C. In their other courses (Physics, Chemistry and Biology), it is assumed that they have a basic understanding of topics such as trigonometry, logarithms and exponentials. Furthermore, analysis of laboratory results and the representation of data by graphical methods are also causing difficulties.

Why do students not have the desired basic skills and what can be done about it? While we decry the state of affairs, we must try to determine the causes and rectify them. Recently the Irish Mathematical Society and the Irish Mathematics Teachers' Association agreed to formal links. One area which could be investigated by both immediately, in their publications, is this problem. It is also time for those of us teaching mathematics at third-level institutions to take an interest in and play an active role in designing the mathematics curriculum at first and second levels.

It is not enough to complain about the poor performance of our first year students; we have a responsibility to discuss the issues with mathematics teachers and learn of their difficulties and efforts to cope with a syllabus that many feel is in need of reform.

APPENDIX A

No. 1 $\sqrt{x^2 + 25}$ is equal to

A) $5x$ B) $x + 5$ C) $\sqrt{(x-5(x+5)}$ D) none of these

No. 2 $\sqrt{x} - x\sqrt{x}$ is equal to

A) $\sqrt{x}(1 - x)$ B) $-x$ C) $\frac{x^{\frac{1}{2}}}{x^{\frac{3}{2}}}$ D) $\frac{1}{2}(x - x^2)$

No. 3 $\cos^2\sqrt{1+x}$ is equal to

A) $\sqrt{1+x}\cos\sqrt{1+x}$ B) $(\cos\sqrt{1+x})(\cos\sqrt{1+x})$ C) $\cos(1+x)$ D) $2\cos\sqrt{1+x}$

No. 4 $(\sqrt{x})^3$ is equal to

A) $x^{\frac{1}{3}}$ B) $x^{\frac{7}{2}}$ C) $(x^3)^{\frac{1}{2}}$ D) $x^{\frac{1}{8}}$

No. 5 $\log x + \log x^2$ is equal to

A) $2\log^2 x$ B) $\log 2x^2$ C) $3\log x$ D) $\log 3x$

No. 6 Which of the following is a solution to the equation $x^2 - 0.04 = 0$?

A) .02 B) .002 C) -.2 D) none of these

No. 7 If $x = 10^k$ and $y = 10^{-m}$, then $\log_{10}xy$ is

A) mk B) $\frac{k}{m}$ C) 10^{k-m} D) $k-m$

No. 8 $\frac{.0032 \times 5.71}{4 \times .04}$ is equal to

A) .1042 B) 1.142 C) .1142 D) none of these

No. 9 Which of the following is a solution of the equation $x^3 - (x-2)(3-x) - 8 = 0$?

A) 2 B) 0 C) -2 D) none of these

No. 10 The solution of $\frac{3x^2}{x^2 - 4} = 0$ is

A) $x = \pm\sqrt{3}$ B) $x = 0$ C) $x = \pm 2$ D) $x = \pm 2$ or 0

No. 11 The area of an equilateral triangle of side 3 cm is

A) $\sqrt{3}$ cm^2 B) 4.5 cm^2 C) $9\sqrt{3}$ cm^2 D) $\frac{9\sqrt{3}}{4}$ cm^2

No. 12 Only one of the following is true. Which is it?

A) $\pi = 3.14$ B) $\frac{9}{12} < \frac{4}{5} < \frac{5}{6}$ C) $\tan 45° = \frac{\sqrt{3}}{2}$ D) $\frac{7}{12} < \frac{8}{15} < \frac{2}{5}$

No. 13 $\frac{0.00125 \times 10^{-5}}{10^{-8}}$ is equal to

A) 125 B) 12.5 C) 1.25 D) 0.125

No. 14 If $v = u + at$, express a in terms of u, v and t.

A) $\frac{v - t}{u}$ B) $\frac{v}{u} - t$ C) $\frac{uv}{t}$ D) $\frac{v - u}{t}$

No. 15 Given that $s = \frac{p^3 - q^3}{r^2}$ find s when $p = -1$, $q = 3$ and $r = 8$

A) $-\frac{7}{16}$ B) $-\frac{7}{64}$ C) $-\frac{13}{32}$ D) $-\frac{1}{8}$

No. 16 Solve the following equation to find the value of x:

$$\frac{x-1}{x-3} + 3 = 5.$$ The solution is

A) 5 B) -1 C) $\frac{17}{4}$ D) none of these

No. 17 $2^n + 2^n$ is equal to

A) 2^{n+1} B) 2^{2n} C) 4^n D) none of these

No. 18 $\sqrt{0.00016}$ is equal to

A) 0.0126 approx. B) 0.04 C) 0.00126 approx. D) 0.004

No. 19 Simplify $\left[\frac{(t^{-3})(t^{-2})}{t}\right]^2$

A) t^{18} B) t^{-2} C) t^{-4} D) t^6

No. 20 The area of a circle of diameter d is

A) πd^2 B) $2\pi d$ C) $\pi d^2/2$ D) $\pi d^2/4$

No. 21 $\frac{\log 20}{\log 5}$ equals

A) log 4 B) log 15 C) 4 D) none of these

No. 22 2π radians = 360°, so 60° is closest to

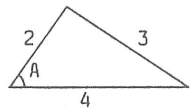
A) 0.5 radians B) 1 radian C) 1.5 radians D) 2 radians

No. 23 The value of 4×10^3 divided by 8×10^7 is

A) 5×10^{-5} B) 5×10^{-3} C) 2×10^4 D) 2×10^{-4}

No. 24 $\frac{\sin \frac{\pi}{2}}{2}$ is equal to

A) $\sin \frac{\pi}{4}$ B) $\sin \pi$ C) 0 D) $\frac{1}{2}$

No. 25 In the triangle 

, cos A is equal to

A) 3/4 B) 2/4 C) 2/3 D) none of these

No. 26 Express 0.01 m^3 in cm^3

A) 1 cm^3 B) 10^3 cm^3 C) 10^4 cm^3 D) none of these

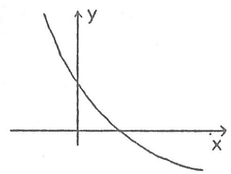
No. 27 Given that $\log_{10}5 = 0.699$ (approx.), $\log_{10}0.005$ is approximately

A) -3.699 B) -3.301 C) -2.301 D) 0.00699

No. 28 $\sin^2 x - \cos^2 x$ is equal to

A) -1 B) $2\sin^2 x - 1$ C) $\sin x - \cos x$ D) $1 + 2\cos^2 x$

No. 29 For the graph



A) $\frac{dy}{dx} > 0$ and $\frac{d^2y}{dx^2} > 0$ B) $\frac{dy}{dx} > 0$ and $\frac{d^2y}{dx^2} < 0$

C) $\frac{dy}{dx} < 0$ and $\frac{d^2y}{dx^2} > 0$ D) $\frac{dy}{dx} < 0$ and $\frac{d^2y}{dx^2} < 0$

No. 30 If $x^2 < 4$, then

A) $x < \pm 2$ B) $x < 2$ C) $x > 0$ D) none of these

APPENDIX B

	Number Choosing Each Answer A	B	C	D	Number of Correct Answers	Total Number of Students Attempting Question	% Success Rate (See Text)
QUESTION 1	0	26	39	144	144	209	69
QUESTION 2	189	9	1	3	189	202	94
QUESTION 3	2	159	6	13	159	180	88
QUESTION 4	10	16	163	6	163	195	84
QUESTION 5	18	9	128	4	128	159	81
QUESTION 6	10	0	153	47	153	210	73
QUESTION 7	1	12	26	129	129	168	77
QUESTION 8	5	17	145	38	145	205	71
QUESTION 9	185	1	0	22	185	208	89
QUESTION 10	5	136	25	30	136	196	69
QUESTION 11	0	45	17	93	93	155	60
QUESTION 12	47	149	7	5	149	208	72
QUESTION 13	1	6	194	6	194	207	94
QUESTION 14	0	0	1	207	207	208	100
QUESTION 15	202	2	5	1	202	210	96
QUESTION 16	196	1	3	10	196	210	93
QUESTION 17	101	46	14	46	101	207	49
QUESTION 18	136	7	13	40	136	196	69
QUESTION 19	168	10	8	2	168	188	89
QUESTION 20	7	1	38	159	159	205	78
QUESTION 21	19	62	10	75	75	166	45
QUESTION 22	59	111	11	5	111	186	60
QUESTION 23	133	19	8	41	133	201	66
QUESTION 24	22	11	19	141	141	193	73
QUESTION 25	8	64	4	116	116	192	60
QUESTION 26	83	21	57	25	57	186	31
QUESTION 27	62	5	62	25	62	154	40
QUESTION 28	16	123	6	14	123	159	77
QUESTION 29	17	19	39	30	39	105	37
QUESTION 30	86	58	0	60	58	204	28

REFERENCE

1. Report on the Basic Mathematical Skills Test of First Year Students in Cork R.T.C. in 1984. *I.M.S. Newsletter* 14 (1985) 33-43.

Mathematics Department,
University College,
Cork.

HISTORY OF MATHEMATICS

On the following pages we include brief biographies of three of the most famous Irish mathematicians. They are reprinted by kind permission of the Royal Irish Academy from the joint RIA/NBST publication "Some People and Places in Irish Science and Technology", edited by Charles Mollan, William Davis and Brendan Finucane.

We are interested in establishing a collection of such articles devoted entirely to Irish mathematicians (in their various guises). Readers are invited to contribute biographies to this collection, which should conform as closely as possible to the format illustrated. Also, we would be pleased to hear from anyone who can provide us with brief outline information on prominent Irish mathematicians.

Some suggestions have already been received for the first half of the 19th century: James Thomson (1786-1849), Matthew O'Brien (1814-1855), Robert Murphy (1806-1843), John Thomas Graves (1806-1870), James McCullagh (1809-1847).

We will be happy to provide editorial advice and assistance in this project if necessary.

Pat Fitzpatrick
Martin Stynes

Sir William Rowan Hamilton with one of his sons (circa 1845)
Courtesy Trinity College Dublin

Born: Dublin, Midnight 3-4 August 1805.

Died: Dunsink Observatory, 2 September 1865.

Family:
Married: Helen Bayly, 1833 (she died in 1869).

Children: Two sons and one daughter.

Distinctions:
Andrews Professor of Astronomy, Trinity College, Dublin, and Royal Astronomer of Ireland, Dunsink Observatory (these appointments were made while Hamilton was still an undergraduate at Trinity College) 1827;
Member of the Royal Irish Academy 1832;
Cunningham Medal of the Royal Irish Academy 1834, 1848;
Knighted 1835;
Royal Medal of the Royal Society for his work in optics 1836;
President of the Royal Irish Academy 1837-1846;
First Foreign Associate of the American National Academy of Sciences 1863.

Addresses:
1805-1808 Dominick Street, Dublin;
1808-1823 Trim, Co. Meath;
1823-1827 South Cumberland Street, Dublin;
1827-1865 Dunsink Observatory.

A line with a direction in space is called a *vector*. A *scalar*, on the other hand, is a magnitude without direction. It was William Rowan Hamilton who introduced these terms. He is best known for his method of *quaternions,* which was a solution to the problem of multiplying vectors in three dimensional space. His method, which employs the imaginary number $\sqrt{-1}$, involves three unit imaginaries i, j, k (with $i^2 = j^2 = k^2 = -1$) such that each of them is perpendicular to the others. The product of any two of them corresponds to a 90° rotation of one about the other in a direction depending on the order in which they are multiplied. Thus:

$$ij = k \quad jk = i \quad ki = j$$

$$\text{but} \quad ji = -k \quad kj = -i \quad ik = -j$$

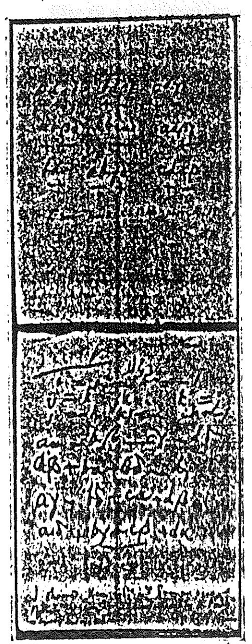
The product of two vectors consists therefore of a real scalar part plus three terms in i, j, and k. Thus it has four terms in all, hence the name *quaternion* (from the Latin *quaternio*, a set of four).

The revolutionary aspect of Hamilton's discovery, which deeply influenced later developments in algebra, is that it does not correspond with traditional multiplication. If we multiply a real number a by another real number b, the product is ab. If we multiply b by a, the answer is the same; that is ab = ba. However, in the case of Hamilton's imaginaries i, j, k, we find that ij = -ji, jk = -kj, and ki = -ik (see above).

Dunsink Observatory, Hamilton's home from 1827-1865 Courtesy P.A. Wayman

Quaternions played a seminal role in the invention of vector analysis, and have found applications in physics. Hamilton's major treatises on the subject are *Lectures on Quaternions* (Dublin, 1853) and *Elements of Quaternions* (London, 1866).

Among Hamilton's other important works are his early optical researches, in which he sought to make optics a mathematical science based on general principles. This work enabled him to predict (1832) that under certain conditions a light ray undergoes an unusual kind of refraction, called 'conical refraction', on passing through biaxial crystals (see entry no. 12). Hamilton's search for general principles extended to dynamics. His reformulation of the equations of motion of Joseph-Louis Lagrange (1736-1813) became – and remains – a powerful tool in classical mechanics and in modern wave mechanics.



The pocket book in which Hamilton first entered the fundamental formulas of quaternions Courtesy Trinity College Dublin

Hamilton had quite extraordinary linguistic gifts. Under his uncle's special tuition at Trim, he could read Greek, Latin, and Hebrew at the age of 3 or 4, and it seems he had acquaintance with some 15 languages by the time he was 10. When at Trinity College he was unbeaten in every examination in both Classics and Science in which he entered, achieving the highest grade in every case. He could also perform prodigious feats of mental calculation.

He was a habitual scribbler: he would scribble his ideas on literally anything – scraps of paper (he always carried a notebook with him), his fingernails, even the shell of his morning egg! His most famous 'scribble' came on 16 October 1843, while walking to the Royal Irish Academy with his wife along the Royal Canal. As he was passing Brougham Bridge, the idea of quaternions suddenly came to him. He stopped, took out his penknife, and on the stone parapet of the bridge scratched the fundamental formulae of his quaternion algebra.

His personal life was burdened with anguish: the real, unattained love of Hamilton's life was Catherine Disney, whom he first met on 17 August 1824, but who married a Rev. William Barlow in May 1825.

Further reading:

Thomas L. Hankins: *Sir William Rowan Hamilton*, Johns Hopkins University Press, Baltimore and London, 1980.

Alan Gabbey,
Department of History and Philosophy of Science,
The Queen's University,
Belfast.

Born: Lincoln, 2 November 1815.
Died: Cork, 8 December 1864.

Family:
Married: Mary Everest (1832-1916). Her uncle George was the surveyor after whom Mount Everest is called.

Children: Five daughters including

Alicia (1860-1940) a notable mathematician;
Lucy (1862-1905) the first woman professor of chemistry in England;
Ethel (1864-1960) a novelist whose book *The Gadfly* has sold more copies than any other book written by an Irish-born author.

Boole's grandson, Geoffrey Taylor F.R.S., was a noted applied mathematician who worked on the development of the atomic bomb. His great-grandson, Howard Hinton F.R.S., was one of the world's foremost entomologists.

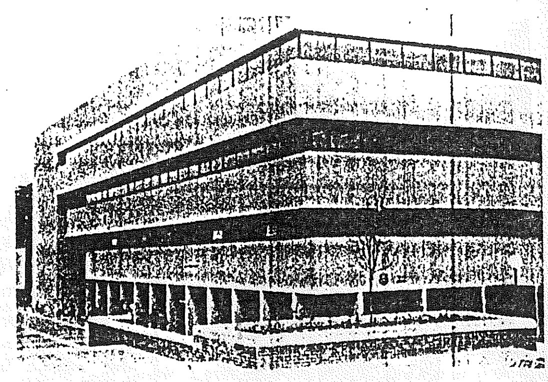
Addresses:
1849-1855 5 Grenville Place, Cork;
1855-1857 Sunday's Well, Cork;
1857-1862 Castle Road, Blackrock, Cork;
1862-1864 Lichfield Cottage, Ballintemple, Cork.

Distinctions:
Awarded the first ever gold medal for mathematics by the Royal Society 1844;
Awarded honorary LL.D. by Trinity College, Dublin 1851;
Elected president of the Cuvierian Society 1855;
Elected Fellow of the Royal Society 1857;
Awarded the Keith prize by the Royal Society of Edinburgh 1857;
Elected member of the Cambridge Philosophical Society 1858;
Honorary degree of D.C.L. from Oxford University 1859.

Boole was a pioneer in mathematics whom Bertrand Russell described as the 'founder of pure mathematics'. He invented a new branch of mathematics – invariant theory – and made important contributions to operator theory, differential equations and probability. However, his most significant discoveries were in mathematical logic. Boole was a deeply religious man, a Unitarian, whose ambition was to understand the workings of the human mind and to express the 'laws of thought' in mathematical form. He invented a new type of algebra, called boolean algebra, which today's engineers and scientists have found to be ideal for the design and operation of electronic computers. Perhaps in some uncanny way Boole foresaw that the human brain behaves like a very complicated computer. Boolean algebra is also essential for the design and operation of the electronic hardware responsible for today's technology. Much of the 'new mathematics' now taught in schools can be traced back to Boole's work – for example, set theory, binary numbers and probability.

George Boole's family – Mary Boole (seated), their five daughters (standing) and some grandchildren

Boole was the eldest son of a struggling Lincoln shoemaker who was more interested in building microscopes than mending shoes. When his father's business failed, George left school at fourteen and became a junior teacher to support his family. Later he opened a school in Lincoln, helped by his sister and brothers. In his spare time he taught himself Latin, Greek, French, Italian and German. Later he studied optics and astronomy and finally he turned to mathematics. In addition, he read and wrote poetry and supported movements for adult education and social reform.


The Boole Library, University College, Cork

In 1849 Boole was appointed first professor of mathematics at Queen's College (now University College) Cork, despite being almost entirely self-taught and having neither secondary schooling nor a university degree. While in Cork he produced his greatest work *The Laws of Thought* which earned him the title 'Father of Symbolic Logic'. This book contains the mathematics of today's computer technology. Boole was an excellent and devoted teacher and met his death after walking in the pouring rain to give a lecture. He is buried beside St Michael's Church of Ireland in Blackrock near Cork City.

Further reading:
George Boole: *The Laws of Thought,* Dover, 1958 (Reprint of 1854 publication by Walton and Maberley, London).
Desmond MacHale: Boolean Algebra, in *The Handbook of Applicable Mathematics,* Ledermann Wiley (ed.), Wiley, 1980.
Desmond MacHale: *George Boole — his Life and Work,* Boole Press, Dublin, 1985. Illustrations are taken, with kind permission, from this publication.

Desmond MacHale,
Department of Mathematics,
University College,
Cork.

Born: Dublin, 1 July 1840.

Died: Cambridge, 25 November 1913.

Family:
Married: Francis Elizabeth Steele, 1868.

Children: Four sons and two daughters, including his biographer W. Valentine Ball.

Nephew of Mary Ball — see entry no. 18.

Addresses:
1840-1854 3 Granby Row, Rutland Square, Dublin;
1865-1867 Birr Castle, Co. Offaly;
1874-1892 Dunsink Observatory.

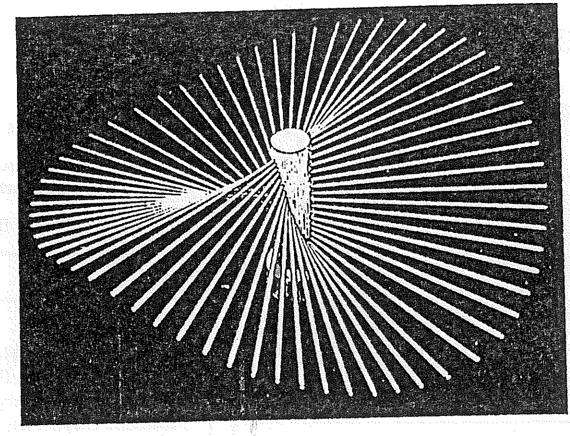
Distinctions:
Member Council Royal Irish Academy 1870 (Secretary 1877-1880; Vice-President 1885-1892);
Fellow Royal Society 1873 (Council Member 1897-1898);
President Royal Astronomical Society 1897-1899;
President Mathematical Association 1899-1900;
President Royal Zoological Society, Ireland 1890-1892;
Cunningham Medal of Royal Irish Academy (for mathematical research) 1879;
Knighted 1886.

Son of a distinguished Cork naturalist, Robert Stawell Ball received his early education at a school kept by Dr J. Lardner Burke in North Great George's Street, Dublin, and at Abbotsgrange, Tarvin, near Chester. In 1857 he entered Trinity College, Dublin where, in a distinguished career, he gained a scholarship, the Lloyd Exhibition, a University Studentship, two gold medals, and prizes in three successive fellowship examinations, 1863-1865.

In the latter year, he accepted a post as tutor to the younger sons of the Earl of Rosse, on the understanding that he would be allowed to observe with the great six-foot reflector at Birr Castle, then the largest telescope in the world. Between 1866 and 1867 he made observations with this instrument of the positions of many faint nebulae, correcting his measurements for instrumental errors to an accuracy not previously achieved by other users of this telescope. In 1867, on the recommendation of the Earl of Rosse, he was appointed Professor of Applied Mathematics at the then new Royal College of Science in Dublin. A compendium of his excellent lectures to College students on Experimental Mechanics was published in 1871.

In 1874 he was appointed Royal Astronomer of Ireland and Andrews Professor of Astronomy in the University of Dublin at Dunsink Observatory. In this dual capacity, he sought to develop the existing Dunsink tradition of measuring stellar distances, using a large sample of stars rather than specially selected objects. Although the method he adopted was later found to be inappropriate for the task, his findings served to identify special problems in making extensive sky surveys and anticipated the later development of more accurate investigative methods. In 1892 he was appointed Lowndean Professor of Astronomy and Geometry at Cambridge but the sad circumstance of the deterioration of his eyesight from 1883, culminating in the loss of his right eye in 1897, gradually brought a halt in this period to his activity as a visual observer.

The work for which he is chiefly remembered, his classic researches on screw motions, was developed over more than thirty years in a series of important communications, contributed in great part to the Royal Irish Academy from 1871. He developed a powerful geometrical method to treat the problem of small movements in rigid dynamics, investigating in particular the behaviour of rigid bodies having different degrees of freedom. In the case where there are two degrees of freedom, he demonstrated that the cylindroid shown in the figure represents the cubic surface locus of the screw axis for all possible twists. Thereafter, he took a special interest in exploring the detailed properties of this kind of surface. In the course of his investigations he made independent discovery of certain theorems concerned with the theory of linear complexes in line geometry, a topic which, in his day, was only in its infancy, and he is now ranked among the leaders of nineteenth century mathematics for his contributions to the geometry of motion and force.



Ball's cylindroid: a model for screw motions

Of genial temperament, he was an outstanding public lecturer and his popular works on astronomy (thirteen volumes published between 1877 and 1908), including *The Story of the Heavens* and a university textbook *A Treatise on Spherical Astronomy*, enjoyed a considerable vogue.

Further reading:
W. Valentine Ball (ed.): *Reminiscences and Letters of Sir Robert Ball*, Cassell and Co. Ltd, London, 1915.
Sir Robert S. Ball: *A Treatise on the Theory of Screws*, Cambridge University Press, 1900.
O. Henrici: *The Theory of Screws*, Nature No. 1075, 42, 127-132, 1890. (Review of 'Theoretische Mechanik Starrer System' by H. Gravelius, published Berlin, Reimer, 1889, an important German treatise based mainly on Ball's work.)

Susan McKenna-Lawlor,
Experimental Physics Department,
Maynooth College,
Co. Kildare.

BOOK REVIEWS

"MEASURE AND INTEGRATION FOR USE"

By *H.R. Pitt, FRS*

Published by *Clarendon Press*, Oxford, 1985. Stg £9.95.
ISBN 0-19-853608-9

Integration is an essential tool for doing mathematics, yet its teaching leaves much to be desired. We are told that the Lebesgue integral is 'too heard' and any analyst knows that the Riemann theory has poor manipulative properties. One solution is that of Henstock*; but prejudice against such simplicity seems too strong. The only other solution is to teach Lebesgue's integral.

This book goes some way towards convincing me that this is possible and desirable. Indeed the main difficulty is that the treatment stays too close to the Riemann integral, presumably so that the reader feels secure. The author sets out to show that the Lebesgue integral is not so difficult to learn, and is of such importance in applications that it simply cannot be ignored. He concentrates on the applications of the theory; notably to harmonic analysis and to probability theory where its ubiquity is most impressive. The point is that one uses the Lebesgue integral for its economy, for its structural integrity and because it unifies.

The book is divided into two parts. The first concerns the theory of integration. Formulated initially in an abstract setting, it focuses mainly on reaching the main results needed to integrate in Euclidean space. There is little digression on measure theoretic pathology. An irritating feature is the constant comparison with the Riemann integral, but this is mostly excusable on cultural grounds. In fact he takes care to answer many of the questions I remember worrying about

* R. Henstock: *Theory of Integration*, Butterworths, 1963

when I first studied the Lebesgue theory; such as integrability of continuous functions, that it generalises the Riemann integral, the appropriate fundamental theorem of calculus etc. The second part introduces the applications and attempts to convince the reader that the Lebesgue integral is the natural tool for tackling certain basic problems.

I have several reservations. One is that the book is basically concerned with measures on R^k, which leads to a fundamental loss of clarity in the section on probability theory since path space has infinite dimension. Another difficulty is that there are no exercises. This is a major drawback in a book of this type. I also noted several throwaway remarks where whole subject areas are surveyed in a few lines. For example there are some comments on filtering theory whose value to the beginner is doubtful. Against this I would commend the inclusion of proofs of Heisenberg's inequality, the Riemann-Lebesgue lemma, the individual ergodic theorem, and Liapounoff's theorem.

Do we need another book on integration? The answer is surely no, but books like this are transitional and are to be welcomed. The difficulty lies with a prejudice which retains the Riemann integral as a first year undergraduate essential. In my opinion this can no longer be justified.

Paul McGill,
Department of Mathematics,
Maynooth College,
Co. Kildare,
Ireland

"WORKED EXAMPLES IN MATHEMATICS FOR SCIENTISTS AND ENGINEERS"

By *G. Stephenson*

Published by *Longman*, 1985, Stg £4.95. ISBN 0-582-44684-8

I must confess immediately that I find it difficult to warm to any book aimed at students - whatever its stated objective - which contains no exercises. Students need to be encouraged to be active, not passive, and a collection of worked examples, however well - chosen or elegantly solved (as they generally are in this text), is unlikely to do this. The author sees a need for a book such as this because "lecture courses usually tend to concentrate ... on the theory rather than examples". This may be so, but a quick perusal of standard and popular mathematics textbooks would tend to suggest that the necessity of providing an adequate number of worked examples is well recognised by most lecturers. What we do not always provide, however, are examples relevant to the interests of our listeners, and the book under review is certainly open to criticism in this regard: despite being aimed at scientists and engineers I failed to find even a token reference to an electrical circuit or to Boyle's Law.

Turning to the content of the book, the chapter headings run from "Functions", "Inequalities", "Limits", through "Partial differentiation", "Matrix algebra", "Ordinary differential equations" to "Contour integration", "Fourier transforms" and "Calculus of variations", and the author's purpose is to cover "most of the topics met in ancillary mathematics courses". As well as the worked examples, the book also contains occasional and brief (very brief) synopses of basic results. Most of the examples in the book are a little harder than the general run of examples encountered in standard texts and a number of them have been taken from examination papers set for courses at Imperial College (London University). The text and solutions are always concise, sometimes indeed too concise:

"But the exponential number e is defined by

$$\lim_{k\to\infty} (1 + 1/k)^k$$

Hence $(1 + 1/k)^k < e$." (p. 9)

"A set of functions $f_1(x)$, $f_2(x)$, ..., $f_n(x)$ is linearly independent if $c_1f_1 + c_2f_2 + .. + c_nf_n = 0$ for all x and for values of the constants c_i which are not all zero. Hence the functions are linearly dependent if their Wronskian determinant vanishes." (p. 73)

Two other points caught my eye: the suggestion that the basic result $\lim_{x\to 0} \sin x/x = 1$ be "proved" by l'Hopital's rule (p. 11) and that the result

$$\frac{d}{dx}\int_a^x f(u)du = f(x)$$

be "derived" from a more general formula for $\frac{d}{dx}\int_a^b f(x,u)du$ where a and b are functions of x (p. 27).

The alert and intelligent student will enjoy the book (a good read for a good student?), but the less able student, if he were to follow the suggestion of the author and use this book as a "means of revision for examinations", could well find the experience a little alarming.

J.B. Twomey,
Mathematics Department,
University College,
Cork

"INTRODUCTION TO GRAPH THEORY" (3RD EDITION)

By *Robin J. Wilson*

Published by *Longman*, Harlow, Essex, 1985, viii + 168 pp.
Stg £5.95. ISBN 0-582-44685-6

"HINTS AND SOLUTIONS MANUAL FOR INTRODUCTION TO GRAPH THEORY"

By *Robin J. Wilson and W.J.G. Wingate*

Published by *Longman*, Harlow, Essex, 1985, 62 pp. ISBN 0-582-44703-8

Make no mistake - graph theory is coming! Computer science departments are realizing that the traditional calculus sequence is largely irrelevant to their needs. They are beginning to demand its replacement by various topics from discrete mathematics. Among these, graph theory comes high on the list. Given the large numbers of students which computing now attracts, mathematics departments can expect a lot of pressure to teach graphs (the non-calculus type). Since Euler first begat graph theory in Konigsberg in 1736, it has been a relatively minor branch of mathematics (the first textbook didn't appear until 1936). Today, 250 years on, it's finding its feet.

Wilson approaches his subject from a theoretical rather than an applied viewpoint. Proofs are generally given, except for some deeper results where a reference is supplied instead. The arguments are usually clear, two exceptions being those of Corollary 13D and Theorem 13G on pages 67 and 68 respectively, where some elaboration is needed. The style is pleasant and holds the reader's interest.

Most of the nine chapters contain a short section on applications. These sections go a long way towards justifying the inclusion of the various theoretical topics considered; without motivation, elementary graph theory tends to look like a collection of unrelated random results on graphs. My only quibble here is that Chapter 6, "the colouring of graphs", fails to make the reader fully aware of the variety of uses of graph colouring.

The book (like all graph theory texts) has a great number of definitions in its earlier pages. However, the language is very allusive and one easily absorbs this material. Why don't graph theorists agree on their basic terminology? The field cries out for some sort of rationalization. As Wilson points out on page 26, what he calls a circuit is also known in the literature as a cycle, elementary cycle, circular path and simple circuit! The most striking example of all is that the definition of a *graph* is not agreed on by everyone; some authors including Wilson permit "graphs" to have multiple edges, others don't.

Appel and Maken's computer aided proof of the four colour theorem is mentioned in a few places; obviously this edition of the book was written before serious doubts were cast on the proof, but this isn't the fault of the author. Students beware!

On page 12 two methods are described for storing graphs in computers. Perhaps the author should also mention *adjacency list representation*, which is commonly used.

At the beginning of this review, I mentioned the growing demand from computer science students for graph theory courses. Unfortunately, the present book isn't a good choice for the sort of course computer science departments usually have in mind, because it's basically theoretical. In the preface, Wilson claims his work is "suitable both for mathematicians taking courses in graph theory and also for non-specialists wishing to learn the subject as quickly as possible". The

claim is justified by this reasonably priced and very readable book.

I had found a mistake in exercise 19a on page 91 before the solutions manual arrived, but the manual to its credit had also detected the misprint. It gives answers to all the exercises, sometimes only in outline. All those I checked were correct. The preface notes that "each author wishes to make clear that any errors which occur are entirely the fault of the other"!

Martin Stynes,
University College,
Cork

"THE INS AND OUTS OF PEG SOLITAIRE" (RECREATIONS IN MATHEMATICS)

By *John D. Beasley*

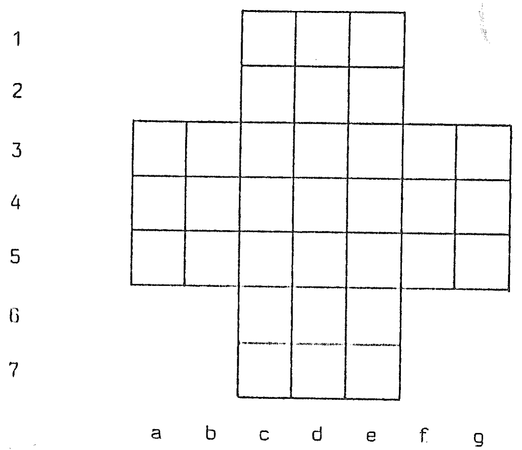
Published by *Oxford University Press*, Oxford, 1985, Stg £12.50.
ISBN 0-19-853203-2

I have always carried a nagging desire to really understand the game of Solitaire, ever since a "flukey" solution on a train of "the central game" many years ago. On the standard English (or German) 33-hole board, a miraculous sequence of vertical and horizontal jumps had reduced a single vacancy at the centre to a sole survivor in that position. Many subsequent attempts to repeat the performance failed miserably. This book grants my wish completely.

Although the origins of the game are uncertain, it was known in the Western world almost three hundred years ago. The outline history of the game given by the author reveals its attraction for strategists and mathematicians alike including officers of the French Royal Artillery, Academy members, Leibniz, Crelle, Bergholt, and Conway and many others.

Crelle published a solution to "the central game" in 1852. In 1912, Bergholt published a solution consisting of 18 moves. It appeared in a weekly newspaper chiefly devoted to ladies' fashions! He later published a book on the subject called the "Complete Handbook to the Game of Solitaire on the English Board of Thirty-three holes". In 1964, Conway and others proved that Berghold's 18 move solution was the shortest possible.

If the 33-hole board is labelled according to the scheme



then allowing for duplication due to symmetry the only soluble single vacancy single survivor problems are summarised by the table overleaf. The author supplies solutions to all these problems in the number of moves listed. He invites verification of his own computer calculations that in each case the required moves are indeed minimal.

Initial Vacancy	Final Survivor	Moves Required
c1	c1, c4, c7	16
	f4	17
d1	a4	17
	d1, d4, d7	18
c2	c5	15
	c2, f5	16
d2	a5, d5	17
	d2	19
c3	c3	15
	f3	16
d3	a3, d3	16
	d6	17
d4	d1	17
	d4	18

TABLE

In general, it is not always possible to reduce any given starting position to a given target position by solitaire moves. However, it is possible to divide all solitaire positions into 16 fundamental classes with the property that if a given problem has a solution, then the starting and target positions must belong to the same class. My own favourite proof of this result is due to de Bruijn (*Journal of Recreational Mathematics* 1972) which associates an element $F_4 \times F_4$ with each solitaire position where F_4 is the field of four elements. It is included in Ian Stewart's 'Concepts of Modern Mathematics' as a nice application of abstract algebra. De Bruijn's method is not used in this book. An alternative elementary reduction is used, which is based on whether or not for a given set the number of elements on certain diagonals is "in" or "out" of phase with the parity (even or odd) of the total number in the set. On the 33-hole board, every position is in the same class as its complement. Two single-man positions are in the same class if and only if the rows and columns of the occupied holes differ by multiples of three. So if we start by vacating a single hole then we can only reach a single-man finish in the original hole or in holes at intervals of three away from it. So except for symmetry the final survivors listed in the table above are the only ones possible.

T.R. Dawson included several double-vacancy complement problems in "The Fairy Chess Review" in 1943. He stated that it was impossible "for lack of elbow room" to begin with vacancies at b4, d4 and end with exactly two survivors in those positions. Conway, Hutchings, Guy and the author developed a theory of "balance sheets" to investigate multiple vacancy complement problems. In 1961, they proved Dawson's assertion and showed that it and the similar complement problems with initial vacancies and final targets at {d2, d6}, {b4, d2} and {d1, d2} are also the only impossible ones.

A similar analysis of triple vacancy complement problems established three classes of insoluble problems:

(i) d1, d2 and any third other than a3, a5, g3 or g5,

(ii) any two middle men (b4, d2, d4, d6, f4) and any third other than an outside corner;

(iii) any three from rows 2, 4 and 6.

The theory developed is difficult to master and leaves many multiple-vacancy complement problems unresolved awaiting further research. It is expected that problems involving marked and distinguished men will receive prominence, i.e. problems similar to Bizalion's "man-on-the-watch" variation in which a nominated man remains fixed and then clears the remaining men in a final sweep to become the sole survivor. The book kindly provides solutions to all the problems set, but some of these can be difficult to follow as the notation given above is rather cumbersome to use. Take for example

the 16-move solution to the c1 complement problem given by:

e1-c1, d3-d1, f3-d3, e5-e3, d3-f3, g3-e3, b3-d3-f3, c5-e5, a5-c5, f5-d5-b5, c1-c3, a3-a5-c5, e7-e5, g5-g3-e3, d2-d5-d3-b3-b5-d5-f5-f3-d3, c7-c5-c3-e3-e1-c1.

If there is a misprint in that sequence of moves, then I have made it! I found none in the book itself.

Finally, the author considers other boards and other rules of play. The reader is introduced to fanciful symmetric boards, three-dimensional boards, hexagonal boards, even infinite boards.

All in all, this is a most enjoyable and complete account of the Solitaire Game and the book is a worthy successor to Bergholt's volume.

Martin Newell,
University College,
Galway

PROBLEM PAGE

Recent issues of the London Mathematical Society Newsletter have carried items by David Singmaster on the following pattern, which was first contemplated, apparently, by Kepler.

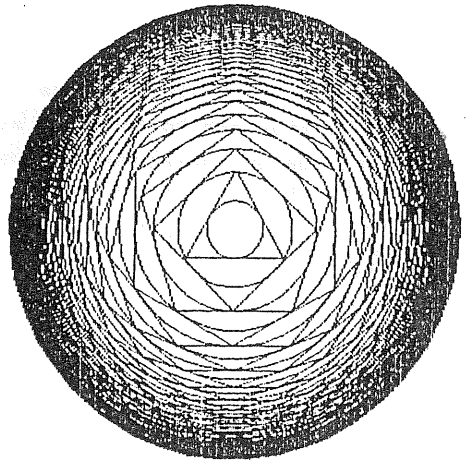


FIGURE 1

It is not hard to convince yourself that the radii of the expanding circles do *not* tend to infinity, but the exact value of the limiting radius seems to be unknown. If the inner radius is 1 then the limiting radius is approximately 8.7 and this number has been calculated to 55 decimal places by Herman P. Robinson in *Popular Computing* (Oct. 1980).

The first problem this time is related, but much easier.

1. If, at each stage, we *double* the number of sides of the escribed polygons, then the limiting radius can be found explicitly. What is it?

I heard the next problem from John Reade at Manchester. He came across it while setting questions on curve-sketching.

2. If $f(x) = p(x)e^x$, where p is a quadratic with integer coefficients, is it possible for f, f' and f'' to have rational zeros?

The final problem must be an old chestnut, but it has a very pretty answer.

3. Which integers can be expressed as the sum of two or more consecutive positive integers?

Now for the solutions to two earlier problems.

1. A rectangle R is partitioned into a finite number of rectangles $R_1, R_2, \ldots, R_n$, each of which has the property that at least one side is of integer length. Prove that R has the same property.

According to Bob Vaughan at Imperial College, this problem originated in France. It was mentioned by J-M. Deshouiller at the conference in honour of Professor K. Roth at Imperial College in 1985, and has spread far and wide since then.

It has a remarkable solution based on the following fact: if:

$$R = \{(x,y) : a \le x \le b,\ c \le y \le d\},$$

then

$$\iint_R e^{2\pi i(x+y)}\,dxdy = 0,$$

if and only if at least one of the numbers $b-a$ and $d-c$ is an integer. This is true because

$$\iint_R e^{2\pi i(x+y)}\,dxdy = \left[\int_a^b e^{2\pi ix}dx\right]\left[\int_c^d e^{2\pi iy}dy\right]$$

$$= \left[\frac{e^{2\pi ib} - e^{2\pi ia}}{2\pi i}\right]\left[\frac{e^{2\pi id} - e^{2\pi ic}}{2\pi i}\right].$$

To solve the rectangle problem, we simply write

$$\iint_R e^{2\pi i(x+y)}dxdy = \sum_{k=1}^{n} \iint_{R_k} e^{2\pi i(x+y)}dxdy = 0,$$

since each of $R_1, R_2, \ldots, R_n$ has at least one side of integer length. Hence R has the same property.

Notice that the method generalises to higher dimensional boxes.

2. A rod moves so that its end points lie on a convex curve Γ_1 in $\mathbb{R}^2$ and a point P, which divides the rod into lengths a and b, then describes a closed curve Γ_2. Prove that the area lying between Γ_1 and Γ_2 is πab.

This strange fact is known as Holditch's Theorem. It can be proved using Green's formula. First parameterize as follows.

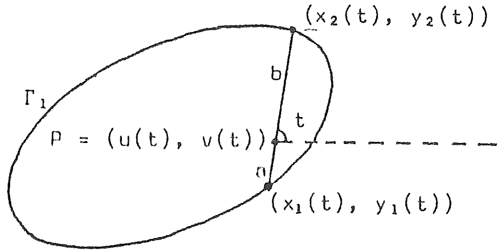


FIGURE 2

If we assume, as we may, that a+b = 1 then $x_2 - x_1 = \cos t$, $y_2 - y_1 = \sin t$, and

$$u = bx_1 + ax_2, \qquad v = by_1 + ay_2.$$

The areas A_1, A_2 lying inside Γ_1, Γ_2 can then be obtained from Green's formula as follows:

$$A_1 = \tfrac{1}{2}\int_0^{2\pi}(x_1\dot{y}_1 - y_1\dot{x}_1)dt = \tfrac{1}{2}\int_0^{2\pi}(x_2\dot{y}_2 - y_2\dot{x}_2)dt;$$

$$A_2 = \tfrac{1}{2}\int_0^{2\pi}(u\dot{v} - v\dot{u})dt$$

$$= \tfrac{1}{2}\int_0^{2\pi}[(bx_1 + ax_2)(b\dot{y}_1 + a\dot{y}_2) - (by_1 + ay_2)(b\dot{x}_1 + a\dot{x}_2]dt$$

$$= (a^2 + b^2)A_1 + \tfrac{1}{2}ab\int_0^{2\pi}(x_1\dot{y}_2 + x_2\dot{y}_1 - y_1\dot{x}_2 - y_2\dot{x}_1)dt$$

$$= A - \tfrac{1}{2}ab\int_0^{2\pi}[(x_2 - x_1)(\dot{y}_2 - \dot{y}_1) - (y_2 - y_1)(\dot{x}_2 - \dot{x}_1)]dt$$

$$= A_1 - \tfrac{1}{2}ab\int_0^{2\pi}(\cos^2 t + \sin^2 t)dt$$

$$= A_1 - \pi ab.$$

Phil Rippon,
Open University,
Milton Keynes

CONFERENCE REPORTS

GROUPS IN GALWAY 1986

There were 23 participants at this year's Group Theory Conference, sponsored by the Royal Irish Academy, the Irish Mathematical Society, and University College, Galway, which was held on Friday and Saturday 9th and 10th May 1986, at University College, Galway.

David Hughes (UCG) opened the Conference with his talk on "Modules and Root Systems". He described Gabriel's characterisation of hereditary algebras having a finite number of indecomposable modules, and the use of almost split sequences to categorise these modules.

Radoslav Dimitric (Kevin St), in his lecture entitled "Abelian Groups and Axiomatic Set Theory", explained Shelah's result that if Martin's Axiom holds and the Continuum Hypothesis fails, then there exist non-free abelian groups satisfying Whitehead's Condition, but that if the Axiom of Constructibility holds, then every Whitehead group is free.

Joseph Rotman (Illinois/London) spoke on "Finite Projective Planes, Graphs, and Symplectic Groups". He gave reasons for his conjecture that the collineation group of a projective plane of even order n is an intersection of conjugates of the symplectic group of degree n^3 over the field with 2 elements, and showed how this might lead to a proof that there is no projective plane of order 10.

Russell Higgs' (UCD) title was "Degrees of Projective Representations". He proved that if a finite group has all its projective representations of degree a power of a fixed prime, then it is soluble, and he also mentioned other related results.

David Redmond (Maynooth) discussed the use of "Groups in Chemistry". He considered the decomposition of the action of the symmetry group of a molecule on homogeneous polynomials, and the notation and methods developed by chemists.

Colin Campbell (St Andrews) delivered the last lecture of the Conference on "Presentations for Certain Perfect Groups". He gave uniform presentations with 2 generators and 3 relations for all projective special linear groups of degree 2 over fields of odd prime order, and also for certain other groups (for instance when the field has order 8, 16, 25, 27, 49, ...).

In addition, Martin Edjvet (Brunel, London) spoke on "Group Presentations and Simplicial Complexes", and Michael Barry (Carysfort) gave a talk on "Order Conjugacy in Groups of Lie Type". Ted Hurley made the group theoretical computer package CAYLEY available to those attending the Conference.

We would like to thank the lecturers, the sponsors, and the participants for their continued support.

Rex Dark

PROTEXT III CONFERENCE

The Third International Conference on Text Processing Systems
22-24 October 1986, and the related
PROTEXT III SHORT COURSE
Computer-Aided Text Processing - An Introduction, 20-21 October 1986
Both events will be held in the
University Industry Centre, University College, Belfield, Dublin
and are co-sponsored by the Institute for Numerical Computation and Analysis
Dublin

For further information, contact: PROTEXT III, Conference Management Services, P.O. Box 5, 51 Sandycove Road, Dun Laoghaire, Co. Dublin, Ireland. Tel. 808025.