CAL laboratory. The original course has been reprogrammed and is now available in colour to students on DEC PRO-350 microcomputers or DEC VT 220 terminals linked to a Microvax 11. The increased computing power has reduced the time taken by students and the greater availability has meant that large classes can now be accommodated. Anyone interested in seeing (or purchasing!) the course should contact the authors.

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UNDERGRADUATE PROJECTS IN GROUP THEORY: AUTOMORPHISM GROUPS

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In my earlier note [3], I described a project undertaken by a 3rd year student involving commutativity ratios. The basic tools needed were an intuitive idea of a presentation of a group, and some modular arithmetic. That project was the equivalent of a half paper in the final exams. The following year the system was modified and projects were enlarged so as to be equivalent to a full paper in the final exams. Here I will describe briefly a project involving calculation of automorphism groups from a presentation. The groups studied were the dihedral groups, which have a fairly easy presentation readily available:

$$D_n = \langle x,y : x^n = y^2 = (xy)^2 = e\rangle \text{ for } n \geq 3.$$

The idea of the project was as follows:

If $\alpha: D_n \to D_n$ is an automorphism then

$$\alpha(x) = x^i y^j$$
$$\alpha(y) = x^k y^\ell$$

for some $0 \leq i,j \leq n-1$ and $0 \leq j,\ell \leq 1$ since any element of D_n has a representation in the form $x^a y^b$, with $0 \leq a < n$, $0 \leq b < 2$. If one wants to build automorphisms, therefore, one may attempt to do so by picking "suitable" i,j,k,ℓ. Of course saying that α is a homomorphism and specifying $\alpha(x)$ and $\alpha(y)$ will say where each $x^a y^b$ is to go provided that the relations are compatible with the choice of i,j,k,ℓ. By this we mean that

$$x^n = e \Rightarrow (\alpha(x))^n = e \text{ that is } (x^i y^j)^n = e$$

$$y^2 = e \Rightarrow (\alpha(y))^2 = e \text{ that is } (x^k y^\ell)^2 = e$$

and the last relation implies $(x^i y^j x^k y^\ell)^2 = e$.

As one knows that

$$yx = x^{-1}y$$

(coming simply from $(xy)^2 = e$), one can simplify these expressions to get relations between i, j, k and ℓ.

Thus just using modular arithmetic, one can identify which i,j,k,ℓ correspond to an endomorphism, α, defined on generators as above. (The basic detailed abstract theory behind this depends on von Dyck's theorem, see Johnson [1] and was summarised by the student in her dissertation.) Now one has merely to observe that an endomorphism α: $D_n \to D_n$ is an automorphism if and only if Ker α is trivial to have a method of extracting which (i,j,k,ℓ) correspond to automorphisms.

The problems in the modular arithmetic led the student to reduce the problem to the closely related one of calculating $Aut(C_n)$ for any n. As the aim was to give a presentation of $Aut(D_n)$, it was clearly insufficient to note merely that $Aut(C_n) \cong U_n$ the Abelian group of units in the ring $\mathbb{Z}/n\mathbb{Z}$, one needed a presentation of U_n. A search through many group theory books produced no easily readable discussion of this, however from various sources, the student pieced together a reasonable account. In her write-up of this, she included tables illustrating some of the principal differences between the various cases: for example n a power of 2, n a product of odd prime powers, etc. These tables listed explicit generators for $Aut(C_n)$ and presentations for all n up to 32.

The student then returned to studying $Aut(D_n)$. With some help on using *Mathematical Reviews*, the student had found a reference to a paper by G.A. Miller [2] dealing precisely with automorphisms of D_n. The solution of the problem for $Aut(C_n)$ cleared the way for finding a presentation for $Aut(D_n)$. That done, she found which of the automorphisms were inner. At this point a discrepancy was noticeable between her calculations and Miller's description. She found that D_3, D_4 and D_6 are the only dihedral groups, which are their own groups of automorphisms. Miller only mentions D_3 and D_4 in this context. Again she found that there are only three dihedral groups (D_4, D_5 and D_6) with outer automorphism group isomorphic to C_2; Miller seems only to mention D_4 and a metacyclic group of order 20. (For non-group theorists $Out(G) = Aut(G)/Inn(G)$ is the outer automorphism group of G.)

She continued with the study, giving explicit generators, and presentations for $Out(D_n)$ for all n, again tabulating the results for $n \le 32$.

At various points in the project, it became useful to explore semi-direct products, parts of the theory of free groups (e.g. Johnson [1], Ch. 1), inner and outer automorphisms of groups in general, holomorphs, and some not so elementary modular arithmetic. Some of this material is usually considered too abstract by group theorists to be introduced in undergraduate courses, but here one was faced with a need for certain terminology, notation and ideas to simplify the description of $Aut(D_n)$. Perhaps this material is only viewed as being too abstract because it is often presented without it being necessary for the development or simplification of a solution to a problem. When one considered that it is this sort of situation that leads to new ideas and new concepts in mathematics, it is worth wondering if a small change in emphasis might not allow students some insight into the reason for the "menu" rather than being shown only the "finished meal" in mathematics courses.

Perhaps I should mention a slight disadvantage about projects of the form I have described in these two notes. After a student has been exposed to this sort of mathematics, where concepts are seldom studied, or introduced, unless necessary for further development, synthesis, simplification etc., it can happen that the usual style of lecture course seems to them hopelessly unmotivated, irrelevant and needlessly abstract. Even though one might like all pure mathematics courses to be presented in a better way, realistically one has to be cynical and warn a student, who knows how to do mathematics, but not necessarily how to remember unmotivated chunks of theory, that not all lecture courses in group theory are presented in this way. (I admit to exaggerating here to make a point. I should also mention that group theory is probably not the worst offender in this way.)

Finally I would mention that another student, this year, is attempting a similar analysis of automorphism groups of dicyclic groups. Also a glance through some of the older (pre 1930) group theory books provides some idea of the wealth of material in this general area which may be useful when planning out projects in group theory. (I suspect the same is true for other areas as well but my personal experience of projects has been more or less solely in this area.)

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BOOKS RECEIVED

"INTRODUCTION TO DIFFERENTIAL GAMES AND CONTROL THEORY"

By *V.N. Lagunov*

Published by *Heldermann Verlag*, Berlin, 1985, vii + 285 pp.
DM 88. ISBN 3-88538-401-9

The main aim of the present book is to give a game-theoretic introduction to zero-sum two-person differential games. It is elementary and concise, not demanding from the reader any preliminary game-theoretic preparation and not requiring mathematical knowledge exceeding the modern technical-college course of higher mathematics.

To make it easier for the beginner to understand such a complex mathematical subject as a differential game the material is initially divided into two parallel streams: the elements of the general theory of games and the elements of the mathematical theory of optimal control. In the subsequent treatment both streams merge into a single channel: differential games.

"SECOND-ORDER SYSTEMS OF PARTIAL DIFFERENTIAL EQUATIONS IN THE PLANE"

By *L.K. Hua, W. Liu And C.-Q Wu*

Published by *Pitman Publishing*, London, 1985, 291 pp.
Stg £16.50. ISBN 0-273-08645-6

This research note presents new results in the theory of pairs of second-order partial differential equations in the plane, with applications. Second-order systems of PDEs are reduced to their canonical form, from which the systems can be easily classified as elliptic, hyperbolic, parabolic