LETTER TO THE EDITOR

Dear Editor,

My book "The Joy of Understanding and Solving problems: A Guide to School Mathematics" was recently reviewed in the IMS Bulletin. Unfortunately the reviewer, Dr Brennan, did not get round to giving details of either the contents or of special features of the book, such as, for instance, its inclusion of historical remarks. These details are usually expected in a review. I am writing to very briefly remedy this lacuna.

I should make it clear that the joy of understanding stated in the title of my book will not come from simply reading a sentence or two of my book but involves a process of work and concentration. Once it happens it is a transformative experience.

Let me illustrate by a few examples what I have done in the book. There are altogether 15 chapters that cover what can be called core mathematics. They include numbers, equations, geometry, coordinate geometry, permutation/combinations, complex numbers, exponentials and logs, trigonometry, calculus, group theory, probability and statistics. There are additional topics such as quaternions, boolean algebra, diophantine equations and so on. Each topic is introduced by some motivational remarks, and there are short biographies of great contributors.

A broad and very brief picture of what is done at the level of concepts in four sections: numbers, equations, geometry and calculus is as follows: Mathematics is first presented as the science of significant forms and then the operations with numbers which underpin the subject are explained in geometrical terms. This converts, for example, the algebraic results such as $(a+b)^2 = a^2 + b^2 + 2ab$ into a geometrical property of a square of side $(a+b)$. Understanding concepts is crucial for making progress in mathematics. Thus we explain the basis for the rule $(-)(-) = +$ and carefully present fractions is parts of a whole i.e as the ratio of positive integers, while rational numbers have an additional rule which determines the sign of the ratio of signed integers. If this is not clear we have the paradox that $\frac{+1}{-1}$ has to be greater than one as $-$ is less than $+1$.

Moving to geometry we point out that from the parallel line axiom a measurable prediction, namely, that the three angles of a triangle add to 180° follows. This is a stunning result which can be appreciated only when explained using words. William Rowan Hamilton, when young, wrote a lovely discussion between Papus and Euclid in which the method for choosing axioms was discussed. Such discussions give students a sense of appreciation of the human element of the subject responsible for creating the structures of mathematics introduces in school. The formula that relates the sum of the three angles of a great circle triangle on a sphere of given radius to its area is mentioned in the book to show students that other geometries with elegant properties are possible. Understanding of concepts comes from highlighting hidden structures. They need words.

Turning to calculus we stress that calculus introduces a new operation into mathematics: the operation of taking a "limit" This new operation acts on functions just as addition subtraction etc are operations that act on numbers. The example of the way an infinite series can represent the number 2 is used to gently make the notion of this "limit" operation clear. New ideas need words to be properly understood.

There are more many examples of this nature in the book, and many more results to excite interest such as, for instance a way to find transcendental numbers, how to use group theory to get Kepler's law and how trigonometry on its way to measure areas using angles and ends up with the beautiful idea of a periodic function.

I believe this style of presentation is not present in current Irish school text books. I also believe if a generation, used to exploring and finding things out for themselves from the internet, have their imagination restricted by a narrow view regarding what is mathematics a great disservice will be done to the young.

My book covers material required for school leavers in India and I am using it in my enthusiastic interaction with trainee teachers and students when I am in India.

Siddhartha Sen
CRANN, Trinity College Dublin