# BOOK REVIEW

"GEORGE BOOLE.   HIS LIFE AND WORK" (Profiles of genius, No.2)

By *Desmond MacHale*

*Boole Press*, 1985.    xv + 304 pp.    £25.00

I    The Boole Press achieves a notable publication in this second book in their biography series.    The name of the house itself reflects the esteem in which George Boole (1815-1864) is held in Ireland; and this book meets the expectations that the reader might hold, for Dr MacHale has produced a fine and rounded portrait of one of the great thinkers of the 19th century.    Boole is a remarkable example of a man who mastered his disadvantaged origins to take the founder chair in mathematics at Queen's College Cork and, in his research, to lay down a main line of study in logic and also systematise and extend a knowledge of differential equations.    MacHale's book is a major achievement, unlikely to be eclipsed as a general biography, and deserves to be a best-seller.

In addition to a general description of the works, MacHale provides much information, often little-known, on Boole's life and career: birth in Lincoln to a poor but intelligent cobbler (Ch. 1); proprietor of his own schools in the area from his 20th to his 35th years (Ch. 2); and then extensive accounts, on which hitherto virtually nothing was known, on the period at Cork (mainly Chs 5,6,8,11,14).    He also pays due attention to Boole's attempts to write poetry (Ch. 12), and the extremely important religious components of his thought (Ch. 14).    A variety of portraits and photographs adorn the text, which ends with a full bibliography of Boole's writings (a summary description of the *Nachlass* in the archives of the Royal Society would have been welcome[1]), a selection of related writings on Boole (supplemented in the bibliography attached to this review), and an excellent index.

In the final chapter of the book, which might have been better called an appendix, MacHale elaborates upon the family tree of p. 4 to describe Boole's family.    They comprise his wife Mary Everest Boole (1832-1916), a notable figure in (Boolean) educational psychology in her own right; her five daughters, who include a mathematician, a chemist, and a novelist; and various descendants, who count among their number the mathematician C.H. Hinton[2], his grandson the entomologist H.E. Hinton and niece the mathematician Joan Hinton, and the applied mathematician Sir G.I. Taylor.

II    On the prehistory of Boole's achievements, MacHale is less strong, making Boole appear more isolated than was the case and even attributing to him some achievements of others.    The principal main source of the mathematical traditions to which he contributed is the work of Lagrange, who tried to algebrise mathematics, and in the context of the calculus initiated the study of differential operators and functional equations. MacHale notes this tradition but mishandles it by attributing the operator form (d/dt)x to Leibniz (p. 45): in fact, with Leibniz it is d which is the operator, working on x to produce dx, and 'dx/dt' means there 'dx ÷ dt'.

Lagrange's approach was developed in the early years of the 19th century by several minor but interesting French figures (Arbogast, the Français brothers, Brisson, Servois), but died out there in the 1820s when the methods of Cauchy took over.    However, they became a major attraction in England, especially the differential operators, when the general situation in mathematics improved after the reforms of the 1810s and 1820s occurred at Cambridge University (Becher 1980).    In addition, logic was also emphasised there, giving it a prominence new to English thought[3].

III Boole's researches belong to both these areas.   In his
spare time from school-teaching he worked especially on diff-
erential operators, systematising the theory in his great paper
of 1844 'On a general method in analysis' (pp. 61-66).
Here he stressed the laws of commutativity and distributivity ,
and also the index law

$$\pi^m \pi^n (\underline{u}) \quad = \quad \pi^{m+n}(\underline{u}) \tag{1}$$

where $\pi$ was some differential (or difference) operator.   Then,
a few years later, inspired by a disagreement on logic between
Sir William Hamilton and Augustus de Morgan[5], he outlined his
programme for 'a mathematical analysis of logic' (pp. 68-72),
giving again the laws of commutativity and distributivity but
replacing (1) by the index law

$$x^2 = x, \text{ or } x(1-x) = 0. \tag{2}$$

I am very surprised that MacHale endorses Russell's quite
mistaken view that Boole's Boolean algebra made him the 'dis-
coverer of pure mathematics'[6]; for it is clear even from the
titles of his books that Boole saw his work as mathematics
*applied* to logic, and especially to mental processes.   Thus
*The laws of thought* (1854) means exactly what it says: a math-
ematical psychology[7].   The processes described in (2) and
their consequences could apply to any area of thought, includ-
ing mathematics itself: MacHale rather misses the importance
for Boole of the example of singular solutions to differential
equations (pp. 220-222), since they (as a 1) have the dual
properties of solving the equations (like an x) but lying out-
side the general solution (à *la* 1-x).

Indeed, mathematics and logic themselves seem to be com-
plementary parts of a 'universal calculus of symbols', to use
the happy phrase of Laita (1977).   On its basis I presented
in (1982, 37) the following theme for Boole's ideas, and one
which moreover has a Boolean structure, as self-reference would
demand (see Fig. 1).   The view that the mind has the power
to pass from the particular to the general, marked by the cen-
tral arrow in the diagram, is crucial to his theory: it seems

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to have informed the rather rigid style of his school-teaching
(pp. 41-43), was stressed explicitly in a lecture of 1851
(quoted on p. 99) and may have motivated his practice of work-
ing at night in the dark (p. 166, repeated on p. 228), freeing
the mind from the geometrical and visual and allowing symbols
free rein.

FIGURE 1

Representation of Boole's System,
together with the corresponding Boolean structure
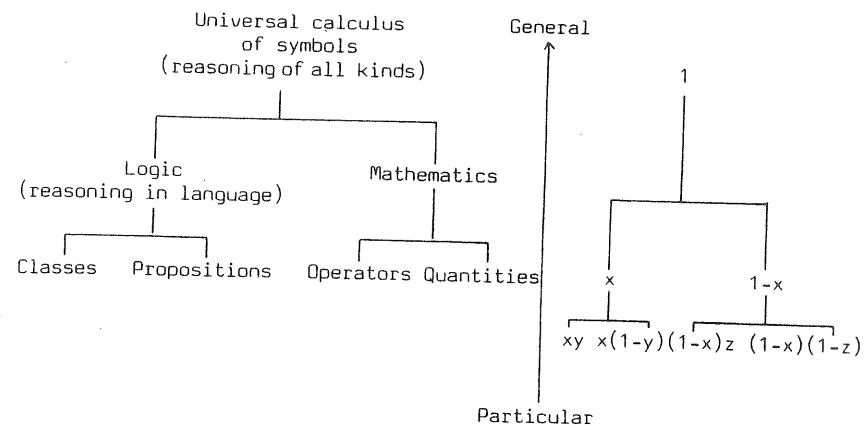


In addition, Boole's logic carried with it a strong rel-
igious connotation, in that the universe 1 was reflected in
the ecumenical views of the time, especially of F.D. Maurice,
Boole's hero in his later years (p. 206), whose portrait was
laid before him as he died (pp. 240-241).   These sides of
Boole's idea rapidly died with him, despite their advocacy by
his widow (or perhaps, considering the eccentricity of her
style, because of it).

During his last years Boole concentrated once again on
mathematics, producing important text books on differential
and on difference equations, in which both differential oper-

ators and functional equations were prominent; in addition, he wrote papers in these areas, and also in probability, where he continued a concern launched in the later chapters of *The laws of thought*. MacHale devotes his Ch. 15 to these areas, perhaps a little lightly; for example, the significance of his contributions to probability is not easy to assess. He notes Mrs Boole's involvement in the textbooks (pp. 219-220), but his later general judgement that 'she had very little knowledge of mathematics and little more than a superficial understanding of her husband's work on logic' (p. 258) seems grossly unfair: Laita (1980) argues persuasively for the *general* correctness of her testimony. In many ways we are indebted to her for the prosecution of his later studies, as well as for the line of genius which she and her husband bequeathed to the world.


## BIBLIOGRAPHY

In addition to listing the items cited above, this bibliography includes secondary literature of note on Boole which is not included in MacHale's 'additional references' on pp. 296-297. All such pieces are marked by an asterisk. There are, in addition, accounts of varying degrees of detail and penetration in other articles and in histories of mathematics and logic.

ALBURY, W.R.
'Introduction', in his (trans.), *Etienne de Condillac. La Logique. Logic*, New York (Abaris) (1980) 7-34, 350-354.

AUROUX, S.
"Condillac ou la Vertu des Signes", in his (ed.) *Condillac. La Logique de Calculs*, Lille (Presses Universitaires (1981) i-xxx.

BECHER, H.
"William Whewell and Cambridge Mathematics", *Hist. Stud. Phys. Sci.*, 11 (1980) 3-48.

*CORCORAN, J. and WOOD, S.
"Boole's Criteria for Validity and Invalidity", *Notre Dame J.*

*Formal Logic*, 21 (1980) 609-638.

GRATTAN-GUINNESS, I.
'Dear Russell - Dear Jourdain ...*, London, Duckworth (1977).

GRATTAN-GUINNESS, I.
"Psychology in the Foundations of Mathematics: the Cases of Boole, Cantor and Brouwer", *Hist. Phil. Logic*, 3 (1982) 33-53; also in *'Psicoanalisi e Storia delle Scienze'*, Florence: Olschki (1983) 93-121.

*HAILPERIN, T.
*'Boole's Logic and Probability'*, North Holland, Amsterdam (1976).

*HAILPERIN, T.
"Boole's Abandoned Propositional Logic", *Hist. Phil. Logic*, 5 (1984) 39-48.

*HESSE, M.
"Boole's Philosophy of Logic", *Ann. Sci.*, 8 (1952) 61-81.

HINTON, C.H.
*'The Fourth Dimension'*, London (Sonnenschein) and New York (Lane) (1904).

*LAITA, L.
"The Influence of Boole's Search for a Universal Method on his Logic", *Ann. Sci.*, 34 (1977) 163-176.

*LAITA, L.
"Influences on Boole's Logic: the Controversy Between William Hamilton and Augustus de Morgan", *Ann. Sci.*, 36 (1979) 45-65.

*LAITA, L.
"Boolean Algebra and its Extra-Logical Sources: the Testimony of Mary Everest Boole", *Hist. Phil. Logic*, 1 (1980) 37-60.

MERRILL, D.D.
"De Morgan [*sic*], Peirce and the Logic of Relations", *Trans. C.S. Peirce Soc.*, 14 (1978) 247-284.

MUSGRAVE, A.
"George Boole and Psychologism", *Scientia*, 107 (1972) 593-608.

PYCIOR, H.M.
"The Three Stages of Augustus De Morgan's Algebraic Work",

*Isis*, 74 (1983) 211-226.

RICHARDS, Joan L.
"The Art and Science of British Algebra ...", *Hist. Math.*, 7 (1980) 343-365.

*RICHARDS, John,
"Boole and Mill: Differing Perspectives on Logical Psychologism", *Hist. Phil. Logic*, 1 (1980) 19-36.

SERVOIS, F.J.
"Essai sur un Nouveau Mode d'Exposition de Principes du Calcul Differentiel", *Ann. Math. Pures. Appl.*, 5 (1814) 93-140.

*SMITH, G.
'Boole's Annotations on "The Mathematical Analysis of Logic"', *Hist. Phil. Logic*, 4 (1983) 27-38.

*VAN EVRA, J.W.
"A Reassessment of George Boole's Theory of Logic", *Notre Dame J. Formal Logic*, 18 (1977) 363-377.

VAN EVRA, J.W.
"Richard Whately and the Rise of Modern Logic", *Hist. Phil. Logic*, 5 (1984) 1-18.

## FOOTNOTES

1.   Hesse (1952), not in the bibliography, describes some materials in the *Nachlass*; see also footnote 6.   G.C. Smith is working on the correspondence between Boole and W. Thomson (later Lord Kelvin), held at Kelvin's end in the Cambridge and Glasgow University Libraries.

2.   MacHale discusses on pp. 259-260 aspects of C.H. Hinton's book *The fourth Dimension* (Hinton 1904).   However, Hinton's geometrical treatment of syllogistic reasoning - in which he drew on sister-in-law Alicia Stott Boole's ideas (Hinton, p. 90) - presents a symmetry which surely should not obtain (p. 102, where the box AEO should be dropped from the scheme).

3.   See Van Evra (1984).   In a curious detail of non-transmission of thought, the French 'logique' of Lagrange's time did not come over.   A form of semiotics (to us) in the hands of men such as Condorcet and Condillac, it linked specifically to algebra (Albury 1980, Auroux, 1981), and to other things, such as the education of the deaf.   However, the general concern with signs is evident in England especially with Babbage, and to some extent in Boole.

4.   These words were introduced, in connection with functional equations, by Servois (1814), one of several French figures whose work refutes MacHale's claim that 'Boole was the first person to define clearly the concept of an operator' (p. 65). Further, I know of no information to back his claim on p. 218 that 'Boole's premature death alone prevented him from being enrolled by the French Academy of Sciences'.   Finally, it is disheartening to see the mathematician V.A. Lebesgue called 'Lebesque' on pp. 46-48, 54, and even indexed on p. 301 as 'Lebesque, Henri', which misidentifies the mathematician as well as mis-spells the name.

5.   On this influence, see Laita (1979).   MacHale shows himself on p. 285, n.6 not to be abreast of the current interest in de Morgan: see especially Joan Richards (1980) on the lead up to de Morgan's position on algebra, Pycior (1983) on its development, and Merrill (1978) on his contributions to logic.

6.   See pp. 130, 217; and also p. 224.   In an improbable move, Mrs Boole hoped around 1905 that Russell might edit Boole's manuscripts for publication; he declined, and recommended Couturat, but nothing was done (see my (1977), 137).

7.   On Boole as a 'psychologist logician' as opposed to Mill as a 'logical psychologist', see John Richards (1980).   The apologies for Boole's psychologism given in Musgrave (1972) are based on misunderstandings.

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