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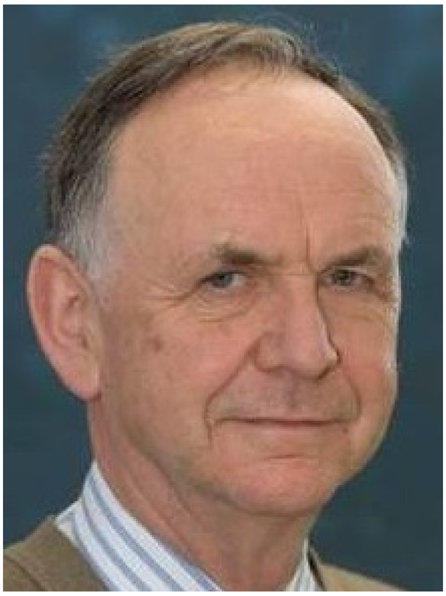
Patrick Denis Barry: 1934–2021

TOM CARROLL, FINBARR HOLLAND, DONAL HURLEY, TONY O'FARRELL, PHIL RIPPON



P.D. Barry

Preamble *by* Tom Carroll

Paddy Barry held the Chair in Mathematics at UCC from 1964 until his retirement in 1999. As a student at UCC in the early 1980s, I was fortunate to have many excellent lecturers. First Year Honours mathematics was taught in 1980–81 by Tony Seda who taught us analysis, Paddy Barry who taught us abstract algebra and number systems, and Finbarr Holland who taught us matrices and linear algebra. I should say that 'us' here includes Stephen Buckley and Pat McCarthy, both now at Maynooth, and Jerry Murphy now at DCU. In our third year, Paddy taught a course on differential equations. Paddy's lectures were meticulously prepared and each covered a lot of ground. Some academics take the course they've been teaching for many years and in time turn their lecture notes into a book; in Paddy's case it felt like the book was already written and, indeed, the notes on the board came with chapter headings!

When I had the good fortune to return to Cork in 1990 as a member of staff, Paddy was no longer my lecturer but my Head of Department. Departmental meetings were held in his office in Aras na Laoi. Everyone got to have his or her say and Paddy listened patiently, even to the new lecturer who had very little of value to add. Paddy set the tone for the department and led by example. Only gradually, as I got to know

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other lecturers across the university, did I realise how fortunate and even exceptional it was to work in such a supportive and collegial department.

Paddy passed away peacefully on 2^{nd} January 2021. He was a true gentleman. This tribute, written by his colleagues, covers both his life and his mathematics - I have simply compiled and edited the result. *Ar dheis Dé go raibh a anam dílis*.

Paddy's life and times *by* Finbarr Holland

In 1964, when he was aged thirty, Patrick Barry (Paddy to everybody who knew him) was appointed Professor of Mathematics at UCC in succession to Paddy Kennedy who had moved to take up the Chair of Mathematics at the recently established University of York.

As it happened, 1964 coincided with the centennial of the death of George Boole, Cork's first Professor of Mathematics, and on 25^{th} May that year, UCC hosted a meeting of members of the Royal Irish Academy (RIA) in his honour, at which J. L. Synge gave the commemorative lecture. Later on, Cork University Press issued *George Boole: A Miscellany*, a booklet of essays edited by Paddy which *inter alia* contains Synge's lecture. In it, also, Paddy draws a comparison between what the department was like in Boole's day and his own.

Three years earlier Paddy had returned home to Cork from Stanford University, where he had spent a year as an Instructor of Mathematics, to begin his teaching career at an institution that had changed very little since his undergraduate days. At the time, only a very few members of the teaching staff had individual offices, and secretarial support was limited to say the least, though some departments had the luxury of sharing secretarial support and – legend has it – the same typewriter which had to be carried across campus from office to office! In my final degree year, along with about ten other students, I took a course on differential equations from him. The following year he taught Lebesgue Integration à la Burkill's Cambridge Tract to a small number of postgraduates, including myself. As well, he prepared me for the 1962 Travelling Studentship Examination, teaching me the theory of Entire Functions on a one-to-one basis in the College Rest!

Paddy took up his professorship at a time when he had the assistance of only one full-time staff member, namely the late Siobhán Vernon, and his teaching and examining load would have been heavy. Between them, Siobhán and himself would have delivered lectures at various levels to students in the Faculties of Arts, Commerce, Engineering, Medicine and Science. But following the launching of Sputnik by the Soviets, times were changing: the student population began to swell with students taking STEM subjects, leading especially to an increase in numbers studying Electrical Engineering, Chemistry and Mathematical Science.

UCC in the 1960s was a vibrant and carefree place for those privileged enough to be there. A more liberal attitude emerged with staff and students mingling more freely outside the classroom especially after meetings of Clubs and Societies. The wearing of gowns by students was no longer a requirement for attendance at class, roll calls were taken only sporadically, and student members of religious orders were no longer easily identifiable by their clothes. Scholarship became a much respected attribute to possess: poets, dramatists and musicians, whose works were widely acclaimed, emerged from the ranks of the staff; more students began to pursue postgraduate studies and, between 1960 and 1970, about 30 UCC students secured NUI Travelling Studentships in a variety of subjects, eight in Mathematical Science. Students also excelled on the sporting field: for instance, the UCC Hurling Club contested the Cork County Final about five times in that period, winning it in 1962 and 1970.

The pace of change was accelerated in 1967 with the appointment of President Donal McCarthy (1967–78), who set about modernising the College. Under McCarthy's reforming zeal, students flocked to UCC from the Munster region, new degree programmes were introduced, young scholars with newly minted PhDs from abroad were appointed to teach and examine them, library holdings were expanded, subscriptions were taken out for research journals, and new buildings were erected to accommodate an increase in student and staff numbers and library stock.

Paddy was central to McCarthy's ambitious plans. Known for his acuity and probity, following a sabbatical at Imperial College, London, his peers elected him to UCC's Governing Body, and he was made its first Vice-president (1974–76). As such, with his analytical skills, eye for detail and fairness, he developed objective criteria that led the way in streamlining the appointments system, served on numerous interview boards and helped to acquire degree-awarding status for Mary Immaculate College, Limerick, which trained primary school teachers.

His tenure as Head of Department (1964–1999) was one of harmony and collegiality. Being even-tempered, thoughtful and non-confrontational by nature, he always managed to coax consensual decisions at meetings he chaired. Having to cater for a broad range of student ability, interest and class size, and deliver a large amount of service teaching, he appointed staff with a diverse range of specialisms, and gave each the freedom to develop his or her own courses and research. At the beginning of each academic year, teaching duties were equitably assigned subject to timetable constraints and individual preferences, to the mutual satisfaction of all concerned. Towards the end of his role as HoD, he introduced an innovative part-time two-year postgraduate degree course in Mathematical Education for secondary-school teachers of Mathematics. This was offered on two occasions in a ten year period, and was availed of by a total of about forty teachers from the Munster area; It raised these teachers' profiles, earned them an extra salary increment and was hugely beneficial to their students.

Paddy had a life-long passion for classical geometry and loved to teach it. In 2001, as Professor Emeritus, he published *Geometry with Trigonometry* [1], a rigorous account of Euclid's geometry suitable for teachers of school mathematics and based on Birkhoff's approach. His treatment is the foundation of the geometry section of the current Leaving Certificate mathematics syllabus. A second edition of the book appeared in 2015 [2]; cf. Anca Mustaţă's thought-provoking review [30]. In his declining years, he wrote up extensive notes under the heading 'Some Generalization in Geometry', which form the basis of his third book *Advanced Plane Geometry*, published in 2019 by Logic Press [3]. This last is accessible to anyone who has mastered [2] and is one of the few texts published in Ireland since Mac Niocaill's [29] that is accessible to teachers, and guides them skilfully beyond the basics.

Patrick Denis Barry was born in Ballynacargy, Co. Westmeath, on 20$^{\text{th}}$ October 1934. His father, also called Patrick Denis, was a Garda sergeant; his mother a National School teacher. He was the fifth in a family of three boys and four girls. When Paddy was two the family moved to Co. Cork, first to the village of Glenville before taking up permanent residence in Mallow in 1945, a town then linked by road and rail to the main Irish cities. There he received his secondary education at the Patrician Academy, sitting the Leaving Certificate examination in 1952, his performance winning him a university scholarship from Cork County Council. In the same year, he achieved first place in the UCC Entrance Scholarship examination, and second place in the examination for the Irish Civil Service.

During his schooldays, Paddy played cricket, soccer, tennis and badminton, the latter a sport at which he was particularly skilful and which he continued to play late in life. Incidentally, by playing such sports at a time when the GAA operated its infamous ban on 'foreign' games, he showed early signs of having an independent mindset, and a steely determination to follow his own inclination, something that was characteristic of him.

Having won two scholarships, he became the first pupil from his school to go to university when, in October 1952, he enrolled in UCC to study Mathematical Science, commuting to the College by train. He graduated in 1955 with a First Class Honours BSc, majoring in Mathematics and Mathematical Physics. That same year he represented Ireland at badminton at Under–21 level.

He continued to study in UCC until 1957 when he obtained his master's degree and a Travelling Studentship from the National University of Ireland.

But having already accepted the position of Research Assistant to Walter Hayman FRS at Imperial College, London, and being scrupulous, he declined this award, which passed to Diarmuid Ó Mathúna, a contemporary of his at UCC, who used it to obtain his PhD at MIT. Hayman, whose first PhD student had been Paddy Kennedy, now directed Paddy's doctoral studies at I.C. He earned a Diploma from I.C., and a PhD from the University of London, for a thesis entitled *On the minimum modulus of integral functions of small order*, which was an outgrowth of his first research paper 'The minimum modulus of certain integral functions', published in the Journal of the LMS in 1958 [22]. Indeed, in his autobiography *My Life and Functions* [26, Chapter 5], Hayman writes that Paddy Barry was 'the only student I ever had who came to me with a PhD problem already prepared. It was on the minimum modulus of small integral and subharmonic functions [20] a subject on which Barry became the world expert.'

On receipt of his doctorate he spent a year at Stanford University as a Mathematics Instructor before returning to his *alma mater* in 1961 where he was appointed first a lecturer in the Mathematics Department and, in 1964, Professor and Head of Department, positions he occupied until his mandatory retirement in 1999, when he became Professor Emeritus of his subject.

Shortly after returning to Cork, Paddy met and married Frances King, a vivacious young woman from Belfast whose sister's boyfriend had a post in UCC's English Department, and was Paddy's flat-mate! Fran became a secondary teacher of Mathematics, and later acquired a PhD in group theory for a thesis written under the guidance of Des MacHale, to become one of the few Irish secondary teachers with a doctorate.

Paddy loved cooking and liked to show off his culinary skills at dinner parties hosted by Fran and himself, producing a variety of exotic dishes, made – according to his children – with mathematical precision! His speciality was a delicious cinnamon-infused apple pie.

Paddy died in a Dublin nursing home, from Covid-19, Fran having pre-deceased him by fifteen years. They are survived by their children: Conor, a film maker in Dublin; Una, who practises general medicine in Calgary, Canada; and Brian, a surgeon in Cork.

Paddy's early research on entire (integral) functions
by Finbarr Holland

Over a span of about 40 years, beginning in 1957, when he was still a postgraduate student in UCC, Paddy wrote 11 research papers on growth problems associated with either slowly growing entire or subharmonic functions. In this section we review some of his main results that have as common theme the growth of the ratio of the minimum and maximum modulus of an entire function of small order. The maximum modulus on the circle of radius r of an entire function f is defined by $M(r, f) = \max\{|f(z)| : |z| = r\}$ while the minimum modulus is defined, as function of r, by $m(r, f) = \min\{|f(z)| : |z| =$

$r\}$. The (upper) order $\rho(f)$ of an entire function f is defined by

$$\rho(f) = \limsup_{r\to\infty} \frac{\log\log M(r,f)}{\log r}.$$

It's familiar that the function $r \to \log M(e^r, f)$ belongs to the class Ψ of continuous non-decreasing functions that are convex on $(-\infty,\infty)$; members of Ψ feature in the hypotheses of many of the theorems enunciated in Paddy's papers. The main object of interest in Paddy's work on entire functions is the relationship between $m(r,f)$ and $M(r,f)$. Subject to various restrictions on the size of f, he gives corresponding estimates for the distribution function of this ratio with respect to logarithmic measure μ defined on Lebesgue measurable sets of $(0,\infty)$ by $\mu(E) = \int_E \frac{1}{t}\,dt$. The key idea he needs to obtain such estimates is an extension of the elementary pigeonhole principle according to which, if A is the average of a finite number of positive numbers and $0 < \lambda < 1$, then the proportion of numbers greater than or equal to λA doesn't exceed $1/\lambda$. Paddy achieves his objectives by first defining a majorant ϕ of $\log\big(M(r,f)/m(r,f)\big)$, and identifying the integral $\int_{[0,r]} \phi\,d\mu$ with the image of the counting function of the zeros of f, viz., $n(t) = \#\{z : f(z) = 0,\ |z| \le t,\}$, under a certain linear operator. Paddy carries this strategy through to a successful conclusion for entire functions of genus 0 that are not polynomials.

His work is of a very general nature, applying not merely to a single function, but to members of a class of functions satisfying similar conditions at infinity. It clearly demonstrates his appetite and aptitude for meticulous attention to detail, which was his forte. He loved to examine the subject of his interest in minute detail at every step of his analysis. He sets out to obtain best-possible results at every opportunity and, by employing intricate reasoning and skilful manipulations, obtains sharp estimates and exact constants wherever possible. He produces explicit examples to show that the results he obtains, subject to the underlying assumptions, are best possible. In his long paper [20], for instance, he manages to eschew rough estimates – big Oh hardly sees the light of day – which is surprising in a paper on classical analysis. His work has an air of finality about it; his was the last word on the subject he analysed, it would appear. But he did leave something unfinished: this was a conjecture that remained open for about 20 years, which I'll come to below. A summary of his main results is presented in [21]; the detail is given in [20].

To give some idea of his achievements, suppose that f is an entire function of genus zero, that $f(0) = 1$, and that $\{z_n\}$ are its zeros arranged in order of increasing moduli so that $\sum_{n=1}^{\infty} \frac{1}{|z_n|} < \infty$. Then, by Hadamard's factorization theorem,

$$f(z) = \prod_{n=1}^{\infty} \Big(1 - \frac{z}{z_n}\Big).$$

If n is the associated counting function then, by Jensen's formula,

$$\int_{[0,r]} n\,d\mu = \frac{1}{2\pi}\int_0^{2\pi} \log|f(re^{i\theta})|\,d\theta \le \log M(r), \quad 0 < r < \infty.$$

Paddy supplements this with the following attractive identity (cf. [21, Lemma 2]) which is fundamental to his purpose. He chooses a convenient majorant $\phi(r)$ for $\log\big(M(r,f)/m(r,f)\big)$, viz.

$$\phi(r) = \log\Big(\prod_{n=1}^{\infty} \frac{|1 + r/r_n|}{|1 - r/r_n|}\Big), \quad r_n = |z_n|.$$

The fundamental identity that he obtains can be stated as a linear integral equation connecting n and ϕ. It involves the following nonnegative kernel function

$$K(s,t) = \log \frac{1+\kappa(t/s)}{1-\kappa(t/s)}, \quad 0 < s,\, t < \infty,$$

where, for $u > 0$, $\kappa(u) = \min(u, 1/u)$. This generates a linear operator – which we'll call the Barry transform – under which the image G of a function g is given by $G(s) = \int_0^\infty K(s,t)g(t)\,d\mu(t)$.

Lemma 0.1. *If* $0 < r < \infty$, *then*

$$\int_{[0,r]} \phi\, d\mu = \int_{[0,\infty)} K(r,\cdot)\, n\, d\mu.$$

Using delicate estimations and intricate reasoning, this result is employed to obtain an array of interesting theorems that are shown to be sharp in some respects. The following results may give a flavour of what interested him and what he achieved.

Theorem 0.2. [21, Theorem 5] *Suppose that* f *is an entire function such that*

$$\limsup_{r\to\infty} \frac{\log M(r,f)}{\log^2 r} = \sigma < \infty. \tag{1}$$

If $0 < \delta < 1$ *and* $\epsilon > 0$ *then, for* $r \geq r_0(\epsilon)$,

$$\mu\Big([0,r] \cap \Big\{t : \log \frac{M(t,f)}{m(t,f)} \geq \delta^{-1}2\pi^2(\sigma+\epsilon)\Big\}\Big) \leq \delta \log r.$$

We sketch the proof. First of all, if r is sufficiently large,

$$n(r) \leq \frac{\log M(r^2,f)}{\log r} \leq \frac{(\sigma+\epsilon)(\log(r^2))^2}{\log r} = 4(\sigma+\epsilon)\log r.$$

Next, since the Barry transform of $\log t$ is equal to $(\pi^2/2)\log s$, an application of the lemma shows that

$$\int_0^r \phi(t)\, d\mu(t) < 2\pi^2(\sigma+\epsilon)\log r, \quad r \geq r_0(\epsilon),$$

whence the result follows readily.

(Defining the upper logarithmic density of a Lebesgue measurable set $E \subset (0,\infty)$ as

$$\text{upper log-dens}(E) = \limsup_{r\to\infty} \frac{\mu\big((0,r]\cap E\big)}{\log r},$$

the conclusion of the theorem tells us that the upper logarithmic density of the set $\{t : \log \frac{M(t,f)}{m(t,f)} \geq \delta^{-1}2\pi^2(\sigma+\epsilon)\}$ doesn't exceed δ. In fact, as a reading of his work shows, the conclusions of Paddy's theorems are generally expressed in terms of logarithmic densities of one kind or another.)

This result applies, in particular, to the functions considered by Paddy in his first paper [22].

Paddy also proved in [20] the following result for entire functions satisfying (1): if $\log M(r,f) = \mathrm{o}(\log^2 r)$ as $r \to \infty$, and $\epsilon > 0$, then

$$\lim_{r\to\infty} \frac{\mu\Big((0,r] \cap \big\{r : m(r,f) > (1-\epsilon)M(r,f)\big\}\Big)}{\log r} = 1.$$

He also sought the best possible lower bound for

$$\limsup_{r\to\infty} \frac{m(r,f)}{M(r,f)}$$

for the class of entire functions satisfying the hypothesis (1). He proved in [20] that it is not less than any of the numbers

$$e^{-\pi^2\sigma}, \quad \prod_{k=1}^{\infty} \tanh^2\left(\frac{2k-1}{8\sigma}\right), \quad \frac{e^{1/4\sigma}-3}{e^{1/4\sigma}+1},$$

and conjectured that it was in fact equal to

$$C = \prod_{k=1}^{\infty} \tanh^2\left(\frac{2k-1}{8\sigma}\right).$$

In support of this he showed that

$$\limsup_{r\to\infty} \frac{m(r, f_0)}{M(r, f_0)} \le C,$$

where

$$f_0(z) = \prod_{k=1}^{\infty} \left(1 - ze^{-k/2\sigma}\right),$$

the latter confirming that the constant $e^{-\pi^2\sigma}$ is sharp for large σ. This conjecture lasted for the best part of twenty years. It was settled in the affirmative in 1979 by A.A. Gol′dberg [25], and independently by P.C. Fenton [24] two years later.

Paddy later revisited this topic in a long paper [13] published in the Proceedings of the Royal Irish Academy. There he replaces the condition (1) by the stronger condition

$$\limsup_{r\to\infty} \frac{\log M(r, f)}{\psi(r)} = \sigma < \infty, \tag{2}$$

where $\psi(r) = o(\log^2 r)$ as $r \to \infty$, and obtains a variety of results on the size of the set of r where $m(r, f)/M(r, f)$ is close to 1.

The papers *On a theorem of Besicovitch* and *On a theorem of Kjellberg* *by* Phil Rippon

Several of Paddy's papers are widely cited and continue to be highly influential. For example, the paper [19] includes an extremely strong $\cos\pi\rho$-type theorem. The origins of such types of result lie in the classical work of Wiman and of Valiron, in particular their result that if the order ρ of an entire function is less than 1, then

$$\limsup_{r\to\infty} \frac{\log m(r, f)}{\log M(r, f)} \ge \cos\pi\rho.$$

This statement shows, for example, that for any function of order $\rho < 1/2$, there is an unbounded sequence of values of r for which the minimum modulus $m(r, f)$ is greater than $M(r, f)$ raised to a fixed *positive* power, and in particular that $m(r, f)$ must be unbounded for such functions.

Paddy's remarkable work in [19] *On a theorem of Besicovitch* strengthens the above result to show that if $0 \le \rho < 1$ and $\rho < \alpha < 1$, then the inequality

$$\frac{\log m(r, f)}{\log M(r, f)} \ge \cos\pi\alpha$$

holds for all values of r in a set E that has lower logarithmic density at least $1 - \rho/\alpha$. Besicovitch had earlier proved a weaker result of this type with E a set of upper linear density at least $1 - \rho/\alpha$. Though just ten pages long, Paddy's paper [19] is highly technical, highly ingenious, and as ever beautifully explained. His result immediately implies the above theorem of Wiman and Valiron, and it shows moreover that for functions of order $\rho < 1/2$, there is a number $\sigma \ge 2$ (for example $\sigma = 1/(1/2 - \rho)$)

such that $m(r, f)$ is greater than $M(r, f)$ raised to a fixed positive power for at least one value of r in each interval of the form $[R, R^\sigma]$ for R sufficiently large.

The latter result has made Paddy's $\cos \pi\rho$-type theorem a favourite tool in complex dynamics for studying a conjecture of Noel Baker that an entire function f of order less than 1/2 cannot have unbounded components of its Fatou set, the open set where the iterates of f form a normal family; see [27] for a survey on the history of Baker's conjecture and [31] for more recent developments.

Baker's conjecture is still open but there are many partial results, frequently building on the fact that whenever f has order less than 1/2 and $M(r, f)$ behaves in a fairly regular manner, then Paddy's theorem tells us that any curve which stretches sufficiently far radially must have an image that also stretches about the same amount radially, in a certain precise sense, and this repeated radial stretching under iteration of f is incompatible with the curve lying in a component of the Fatou set.

I recall his modest surprise, sometime around 2013 on a visit to UCC as external examiner, when Paddy learnt that his paper [19] was often cited by complex dynamics authors. Paddy then took advice on how to look up citations and was excited to find the large number on MathSciNet (currently 50 citations for his paper [19]) and then a day later the larger number on Google Scholar (currently 132 citations)!

The paper [18] *On a theorem of Kjellberg* is a partner to [19] and is also still widely cited, though less so in complex dynamics. Here Paddy combined his techniques from [19] with earlier techniques of Kjellberg to prove a result in which the hypothesis that the order of f is less than 1 is replaced by the weaker hypothesis that the lower order of f is less than 1, and in the conclusion lower logarithmic density is replaced by the weaker conclusion of upper logarithmic density. Once again the proofs are ingenious and elegantly expressed, with meticulous credit given to other authors.

Paddy's work on classification of functions *by* Donal Hurley

Paddy's work on differential equations [10–12], joint with D.H., was motivated by a wish to classify functions that arise in analysis and enable properties to be developed. He considered functions satisfying homogeneous linear differential equations of operator format. The operator format equations were based on two operators introduced by George Boole, and are defined as follows; for any function $w(z)$

$$\pi w(z) = zw'(z) \quad \text{and} \quad \rho w(z) = zw(z).$$

Operator format differential equations are then formed by equating to zero finite sums of the form

$$\sum_l \sum_m a_{l+1,m+1} \pi^l \rho^m w(z)$$

where $a_{l+1,m+1}$ are independent of z. Assuming that the solutions are of the form

$$w(z) = \sum_{n=-\infty}^{\infty} k_n \left(\frac{z}{\lambda}\right)^n$$

where λ is a constant, on substituting this expression for $w(z)$ into the operator format differential equation, one arrives at a homogeneous linear difference equation for the coefficients $\{k_n\}$.

A basic differential classification of functions, generated by the coefficients $\{k_n\}$, is determined by the order of the differential equation, the order of the difference equation for the $\{k_n\}$, and the number of non-zero coefficients k_n in the difference equation. Many familiar functions were encountered in his work. However, computations rapidly become very complicated which necessitated using computer software packages.

The publication in Proceedings of the AMS [12] is a result obtained as a byproduct of this work on classification of functions.

Paddy Barry and Tony O'Farrell with a copy of *Advanced Plane Geometry* (first published in 2019)

Paddy Barry and School Geometry *by* Tony O'Farrell

MacDonald [28] quotes Plato, who said of Education: *If it ever leaves its proper path and can be restored to it again, to this end everyone should always labour throughout his life with all his powers.* No-one ever took up this challenge with such determination and energy as Paddy Barry.

There have been six main revisions of the Irish school geometry syllabus in the past sixty years. Syllabus I was in force 1934-1968, and the revisions came into junior-level exam papers as follows: II:1969, III:1976, IV:1990, V:1995, VI:2003, VII:2015 (affecting first-year students three years earlier in each case). All this change took place during Paddy's active career.

Possessed of deep learning, and a strong sense of duty, he took seriously the responsibility of university mathematicians to monitor and assist with developments in the schools' programme. He was particularly concerned about changes to the geometry syllabus that took place in the nineteen-sixties. These changes were seriously misguided. The whole sorry story is almost unbelievable, and is documented in Susan MacDonald's PhD thesis [28]. Paddy was tireless and relentless over a long period in his efforts to correct the problem. Of his writings about school geometry, the most significant is his book *Geometry with Trigonometry* [1, 2]. This text was eventually adopted by the National Committee for Curriculum and Assessment (NCCA) as the bedrock underlying the geometry programme in the Irish secondary Mathematics syllabus. It is a fully rigorous text on Euclidean geometry, goes substantially beyond the schools' programme, and is suitable for study by university undergraduates and practising teachers.

Paddy's contributions to school geometry went on throughout his career. The fact that Leaving Certificate (LC) mathematics could substitute for the Matriculation Examination gave the universities some leverage over the Department of Education examiners – viz., the chief inspector until the creation of the State Examination Commission. The NUI Senate could, in principle, refuse to accept a pass in the leaving certificate examination as satisfying the matriculation requirement in Mathematics. As a result the draft leaving certificate papers were sent in advance to NUI (statutory) professors for comment, and Paddy always paid attention.

The view in the Department was that there was no reason to consult university people about the junior cycle programme. This reflected a failure to understand that catastrophe could result from tinkering with the logical structure underpinning the geometry programme. As a result, university people had no power in relation to syllabus revision, and enthusiastic engagement with new ideas combined with inadequate understanding on the part of those who did have the power landed us in trouble.

Paddy used every opportunity available to him to sound the alarm in advance, and to press for corrective action. He was not inclined to work with a megaphone. He tried to bring pressure through Royal Irish Academy committees, the Irish Mathematics Teachers' Association (IMTA), and the NUI Senate, and tried to advise the syllabus committees and inspectors. His efforts were blocked for a long time.

Paddy worked in parallel to provide the ingredients for a return to a sound programme, and to educate anyone who would listen, by means of his geometry book [1, 2] and his own course materials intended for classroom use.

The incoherent hybrid Syllabus II did terrible damage to geometrical teaching and study, and we are still some way from recovering. It continued to taint all succeeding versions, until Syllabus VI, an outcome of the NCCA'a Maths Project. In this last Paddy's case was finally accepted, and the geometry programme is again coherent, based on the foundation [2], the Level 2 account [32], and the syllabus [23]. This acceptance was assisted by the fact that some people educated or infuenced by Paddy were serving on relevant committees. However the geometry content at higher-level remains impoverished, compared with the best international standards, there is a persistent issue with textbooks, and it remains to be seen how long it may take us to get back to a stable position.

The adoption of Paddy's *Geometry with Trigonometry* as the Level 1 account underpinning school geometry led to the sale of all copies remaining in print, a second printing, and publication of a new second edition by a subsidiary of Elsevier.

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Tom Carroll is Senior Lecturer in Mathematics at UCC and current President of the Irish Mathematical Society.

Finbarr Holland is Emeritus Professor of Mathematics at UCC.

Donal Hurley is Emeritus Senior Lecturer in Mathematics at UCC.

Tony O'Farrell is Emeritus Professor of Mathematics at Maynooth University and Editor of this Bulletin.

Phil Rippon is Professor of Pure Mathematics at The Open University. He held a Lectureship in Mathematics at UCC for a period in the early 1980s.

(Carroll, Holland, Hurley) School of Mathematical Sciences, University College Cork.

(O' Farrell) Department of Mathematics, Maynooth University, W23 HW31.

(Rippon) School of Mathematics and Statistics, The Open University, Milton Keynes, UK.

E-mail address, T. Carroll: t.carroll@ucc.ie
E-mail address, F. Holland: f.holland@uw.ie
E-mail address, D. Hurley: d.hurley@ucc.ie
E-mail address, A. O'Farrell: anthony.ofarrell@mu.ie
E-mail address, P. Rippon: phil.rippon@open.ac.uk