Book Review

p-adic Numbers. An Introduction

Universitext Series

Fernando Q. Gouvêa

Springer-Verlag 1993, 284 pp.

ISBN 3-540-56844-1

Price DM 58.00.

Reviewed by Brendan Goldsmith

The role of p-adic numbers and p-adic analysis has become central in many areas of mathematics in the century or so since their introduction by Hensel. Despite this, they are still not part of the 'toolkit' of many mathematicians and are often not talked about (or perhaps just given as a somewhat bizarre example) at undergraduate level. One of the objectives of this book is to make the subject more accessible to an undergraduate audience, 'taking its readers for a short promenade along the p-adic path'. In this it succeeds admirably. I have used parts of the book as background material at a beginning graduate seminar and found that students had little difficulty with it.

There are several standard approaches to p-adic theory with the most popular being either via valuation theory or via absolute values (the two are, of course, intimately related). Here the author has chosen the latter and proceeds to completions and Hensel's Lemma. (Incidentally, there is a nice application of this lemma to the determination of p-adic roots of unity). The initial 'algebraic' approach finishes with some interesting aspects of 'local-global' arguments and applications of the Hasse-Minkowski Theorem, although, not surprisingly, no proof of the theorem is given.

After the introductory appetizer, we are treated to some elementary p-adic analysis focusing on sequences and series. There

is no attempt to develop a p-adic theory of integration but suitable references are given for such a development. Having laid this analytical ground work, the book then switches to p-adic vector spaces and field extensions, finishing off with a nice discussion of normed vector spaces over complete valued fields and showing that the algebraic closure, $\overline{Q}_p$, of the field of p-adic numbers is not complete with respect to the (extended) p-adic absolute value. This leads, rather naturally to a discussion of the completion C_p.

A delightful feature of the book is the large number (329, to be exact!) of exercises which form an integral part of the text. Equally useful is the section 'Hints and Comments on the Problems' where hints of varying degrees of detail are given! The book concludes with a brief discussion on the literature, some comments on software for doing p-adic calculations and a sensible bibliography.

I found the book a pleasure to read and while one might occasionally wish to delve deeper into some topics, one must accept that the author's 'aim is sightseeing, rather than a scientific expedition'. However, this must not be interpreted incorrectly; this is a serious book looking at important mathematics and definitely worthy of a place in the prestigious Springer Universitext series.

Brendan Goldsmith,
Dublin Institute of Technology,
Fitzwilliam House,
30 Upper Pembroke Street,
Dublin 2.